Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/42-1.2.2.5-P-x-a+b-x^2+c-x^4-^p

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September 28, 2022 Compiled on September 28, 2022 at 12:57am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [111]. This is test number [42].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (111)	0.00 (0)
Mathematica	100.00 (111)	0.00 (0)
Maple	100.00 (111)	0.00 (0)
Mupad	95.50 (106)	4.50 (5)
Giac	95.50 (106)	4.50 (5)
Fricas	81.98 (91)	18.02 (20)
Maxima	74.77 (83)	25.23 (28)
Sympy	40.54 (45)	59.46 (66)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

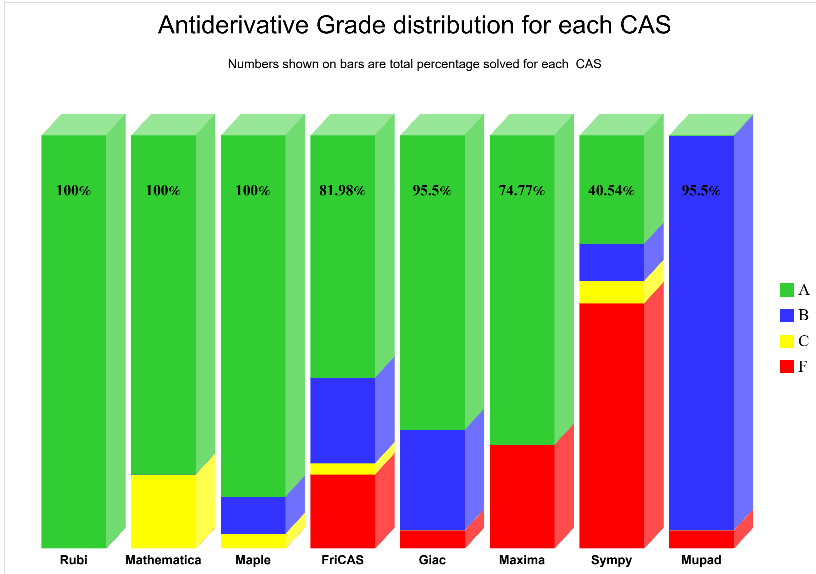
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

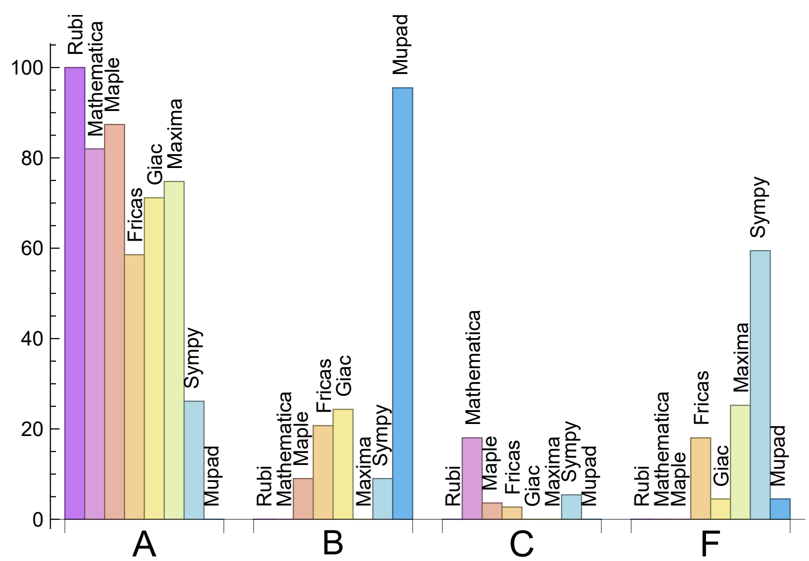
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Maple	87.39	9.01	3.60	0.00
Mathematica	81.98	0.00	18.02	0.00
Maxima	74.77	0.00	0.00	25.23
Giac	71.17	24.32	0.00	4.50
Fricas	58.56	20.72	2.70	18.02
Sympy	26.13	9.01	5.41	59.46
Mupad	N/A	95.50	0.00	4.50

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	20	0.00 %	100.00 %	0.00 %
Giac	5	80.00 %	0.00 %	20.00 %
Maxima	28	100.00 %	0.00 %	0.00 %
Sympy	66	12.12 %	87.88 %	0.00 %
Mupad	5	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

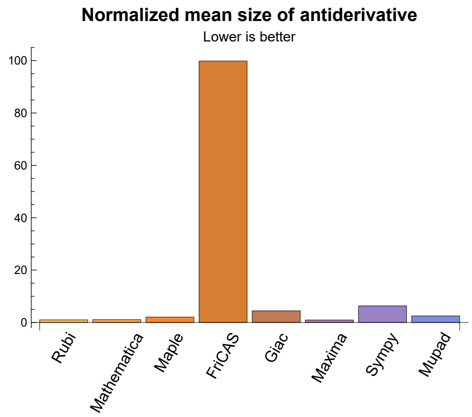
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

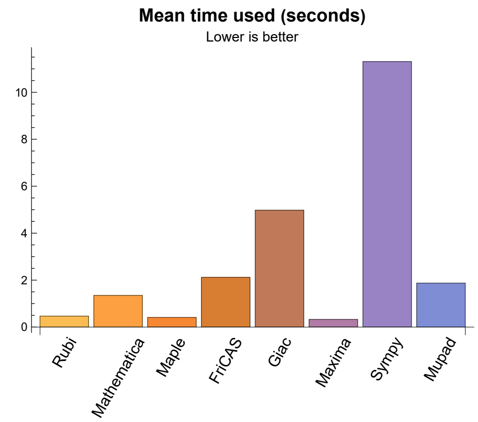
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.47	217.75	1.00	140.00	1.00
Mathematica	1.35	260.32	1.08	146.00	1.02
Maple	0.41	356.73	2.02	154.00	1.07
Maxima	0.33	103.67	0.91	92.00	0.91
Fricas	2.12	20483.04	99.82	138.00	1.16
Sympy	11.31	620.93	6.32	122.00	1.19
Giac	4.98	2170.14	4.44	126.00	1.00
Mupad	1.87	700.78	2.47	132.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
100
80
60
40
20
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

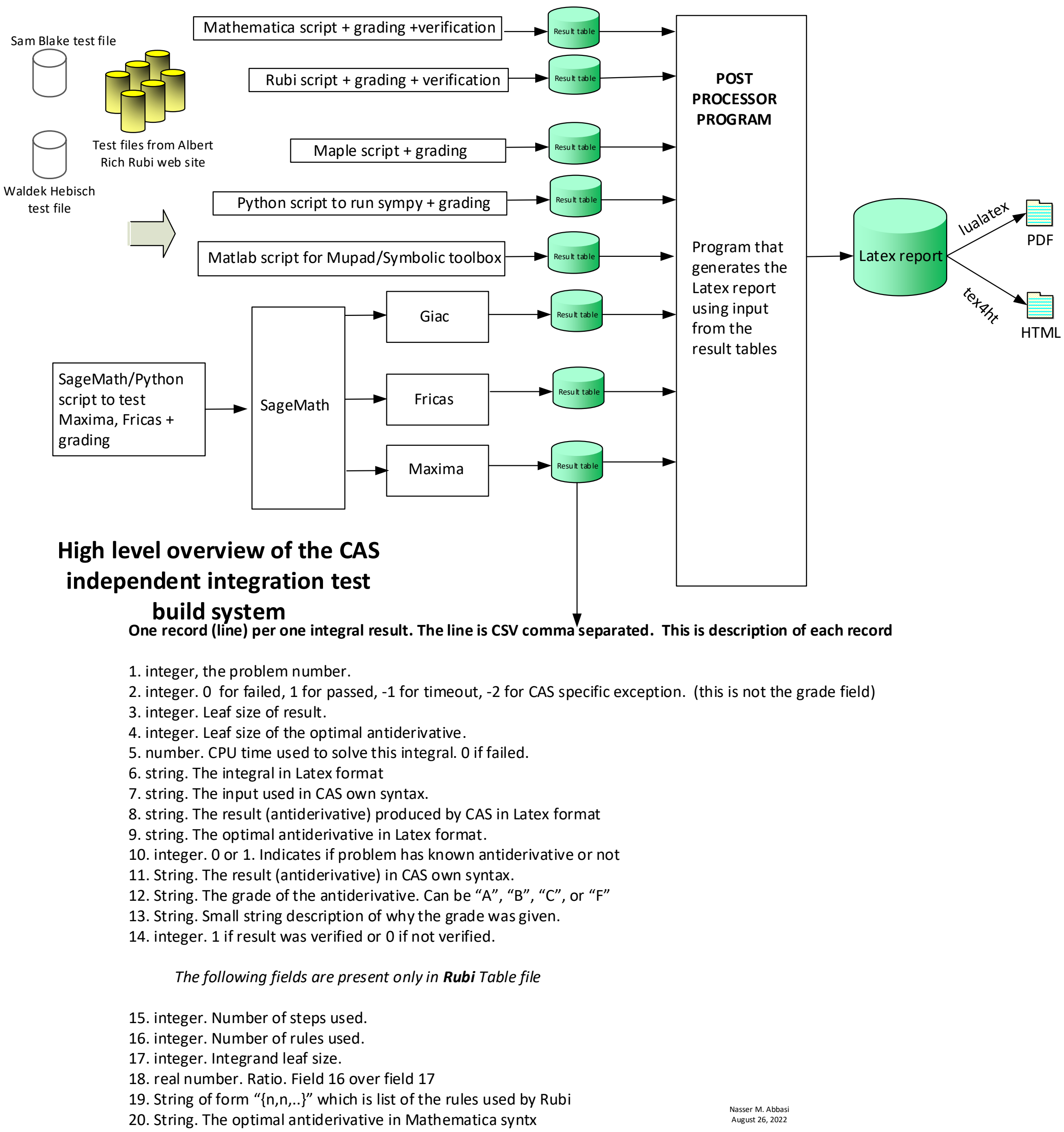
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111 }

B grade: { }

C grade: { 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 47, 48, 49, 50, 51, 103, 104, 105, 106, 107 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105 }

B grade: { 38, 52, 53, 54, 60, 66, 103, 104, 106, 107 }

C grade: { 108, 109, 110, 111 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { 20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 103, 104, 105, 106, 107 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 31, 32, 33, 34, 35, 47, 48, 49, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 103, 104, 105, 106, 108, 109, 110, 111 }

B grade: { 26, 27, 28, 29, 30, 42, 43, 44, 45, 46, 50, 51, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 107 }

C grade: { 20, 21, 64 }

F grade: { 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 65, 66 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 85, 91, 97 }

B grade: { 10, 11, 26, 42, 80, 81, 82, 86, 92, 98 }

C grade: { 15, 16, 31, 32, 47, 48 }

F grade: { 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 83, 84, 87, 88, 89, 90, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 60, 61, 62, 63, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102 }

B grade: { 20, 21, 22, 23, 24, 25, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 108, 109, 110, 111 }

C grade: { }

F grade: { 103, 104, 105, 106, 107 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 109, 110, 111 }

C grade: { }

F grade: { 103, 104, 105, 106, 107 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	41	43	40	46	43	40
N.S.	1	1.00	1.00	0.82	0.86	0.80	0.92	0.86	0.80
time (sec)	N/A	0.028	0.002	0.011	0.281	0.790	0.008	4.117	0.027

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	60	57	65	64	59
N.S.	1	1.00	1.00	0.84	0.87	0.83	0.94	0.93	0.86
time (sec)	N/A	0.032	0.014	0.041	0.279	0.394	0.008	3.973	0.033

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	75	77	74	83	85	78
N.S.	1	1.00	1.00	0.85	0.88	0.84	0.94	0.97	0.89
time (sec)	N/A	0.047	0.013	0.083	0.269	0.381	0.009	5.212	0.662

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	92	89	102	106	95
N.S.	1	1.00	1.00	0.86	0.88	0.85	0.97	1.01	0.90
time (sec)	N/A	0.069	0.023	0.129	0.279	0.399	0.010	4.713	0.659

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	105	106	104	121	124	112
N.S.	1	1.00	1.00	0.86	0.87	0.85	0.99	1.02	0.92
time (sec)	N/A	0.077	0.027	1.032	0.286	0.395	0.011	3.557	0.058

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	95	102	94	116	106	94
N.S.	1	1.00	0.87	0.85	0.91	0.84	1.04	0.95	0.84
time (sec)	N/A	0.090	0.032	0.064	0.279	0.399	0.015	3.682	0.056

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	146	138	165	157	138
N.S.	1	1.00	1.00	0.90	0.95	0.90	1.07	1.02	0.90
time (sec)	N/A	0.091	0.031	0.088	0.277	0.354	0.018	4.236	0.697

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	183	188	182	209	208	182
N.S.	1	1.00	1.00	0.93	0.96	0.93	1.07	1.06	0.93
time (sec)	N/A	0.117	0.037	0.086	0.284	0.382	0.019	3.824	0.716

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	234	219	224	218	258	259	220
N.S.	1	1.00	1.00	0.94	0.96	0.93	1.10	1.11	0.94
time (sec)	N/A	0.163	0.055	0.137	0.273	0.388	0.023	4.481	0.114

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	50	47	43	515	51	51
N.S.	1	1.00	1.11	1.11	1.04	0.96	11.44	1.13	1.13
time (sec)	N/A	0.023	0.012	0.024	0.283	0.387	1.805	4.850	0.709

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	58	62	55	51	2195	59	63
N.S.	1	1.00	1.14	1.22	1.08	1.00	43.04	1.16	1.24
time (sec)	N/A	0.038	0.016	0.028	0.267	0.415	97.911	3.274	0.715

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	68	74	65	61	0	69	75
N.S.	1	1.00	1.19	1.30	1.14	1.07	0.00	1.21	1.32
time (sec)	N/A	0.051	0.022	0.033	0.290	0.567	0.000	4.218	0.742

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	81	89	76	72	0	80	90
N.S.	1	1.00	1.27	1.39	1.19	1.12	0.00	1.25	1.41
time (sec)	N/A	0.104	0.027	0.041	0.277	1.295	0.000	4.721	0.815

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	98	107	85	88	0	89	108
N.S.	1	1.00	1.29	1.41	1.12	1.16	0.00	1.17	1.42
time (sec)	N/A	0.122	0.041	0.046	0.276	6.516	0.000	4.496	1.190

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	98	68	67	65	923	67	118
N.S.	1	1.00	1.07	0.74	0.73	0.71	10.03	0.73	1.28
time (sec)	N/A	0.050	0.109	0.057	0.486	0.415	1.815	3.363	0.242

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	121	82	77	75	3589	77	159
N.S.	1	1.00	1.16	0.79	0.74	0.72	34.51	0.74	1.53
time (sec)	N/A	0.061	0.087	0.095	0.490	0.420	72.275	5.515	0.951

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	150	90	85	83	0	85	199
N.S.	1	1.00	1.18	0.71	0.67	0.65	0.00	0.67	1.57
time (sec)	N/A	0.070	0.292	0.115	0.508	0.552	0.000	5.778	1.127

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	165	99	94	92	0	94	1209
N.S.	1	1.00	1.21	0.73	0.69	0.68	0.00	0.69	8.89
time (sec)	N/A	0.102	0.377	0.177	0.502	1.272	0.000	4.732	6.108

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	187	117	103	106	0	103	1509
N.S.	1	1.00	1.24	0.77	0.68	0.70	0.00	0.68	9.99
time (sec)	N/A	0.123	0.364	0.212	0.492	4.468	0.000	4.754	7.805

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	194	200	0	398481	0	1437	1308
N.S.	1	1.00	1.03	1.06	0.00	2108.37	0.00	7.60	6.92
time (sec)	N/A	0.149	0.168	0.044	0.000	3.342	0.000	5.382	1.316

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	240	0	723401	0	1714	2500
N.S.	1	1.00	1.11	1.14	0.00	3428.44	0.00	8.12	11.85
time (sec)	N/A	0.185	0.141	0.052	0.000	20.399	0.000	4.705	2.139

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	280	285	0	0	0	3274	2500
N.S.	1	1.00	1.14	1.16	0.00	0.00	0.00	13.36	10.20
time (sec)	N/A	0.122	0.185	0.046	0.000	0.000	0.000	6.753	2.539

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	383	353	0	0	0	5203	2500
N.S.	1	1.00	1.32	1.22	0.00	0.00	0.00	17.94	8.62
time (sec)	N/A	0.543	0.315	0.062	0.000	0.000	0.000	5.074	1.749

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	441	408	0	0	0	6100	2500
N.S.	1	1.00	1.37	1.27	0.00	0.00	0.00	19.00	7.79
time (sec)	N/A	0.391	0.410	0.066	0.000	0.000	0.000	7.707	2.030

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	816	829	0	0	0	11833	2500
N.S.	1	1.00	1.50	1.52	0.00	0.00	0.00	21.71	4.59
time (sec)	N/A	2.993	0.825	0.621	0.000	0.000	0.000	7.860	4.306

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	90	106	89	169	604	93	84
N.S.	1	1.00	0.96	1.13	0.95	1.80	6.43	0.99	0.89
time (sec)	N/A	0.038	0.038	0.036	0.274	0.409	2.157	4.495	0.088

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	112	130	112	217	0	115	107
N.S.	1	1.00	0.97	1.13	0.97	1.89	0.00	1.00	0.93
time (sec)	N/A	0.093	0.054	0.041	0.275	0.519	0.000	5.729	0.104

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	134	154	133	262	0	136	128
N.S.	1	1.00	0.97	1.12	0.96	1.90	0.00	0.99	0.93
time (sec)	N/A	0.101	0.035	0.046	0.300	0.968	0.000	3.635	0.136

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	159	178	151	304	0	158	146
N.S.	1	1.00	1.06	1.19	1.01	2.03	0.00	1.05	0.97
time (sec)	N/A	0.146	0.048	0.049	0.287	1.884	0.000	5.239	0.870

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	185	202	157	346	0	168	164
N.S.	1	1.00	1.14	1.25	0.97	2.14	0.00	1.04	1.01
time (sec)	N/A	0.152	0.058	0.059	0.297	7.429	0.000	4.131	0.583

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	146	124	100	154	952	100	149
N.S.	1	1.00	1.04	0.89	0.71	1.10	6.80	0.71	1.06
time (sec)	N/A	0.065	0.308	0.074	0.505	0.442	2.211	3.200	0.252

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	186	154	124	212	4106	128	201
N.S.	1	1.00	1.13	0.93	0.75	1.28	24.88	0.78	1.22
time (sec)	N/A	0.091	0.251	0.109	0.506	0.448	89.502	2.655	0.315

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	200	172	136	239	0	142	237
N.S.	1	1.00	1.12	0.96	0.76	1.34	0.00	0.79	1.32
time (sec)	N/A	0.099	0.241	0.128	0.525	0.568	0.000	2.722	1.154

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	234	196	144	255	0	155	1547
N.S.	1	1.00	1.25	1.05	0.77	1.36	0.00	0.83	8.27
time (sec)	N/A	0.117	0.356	0.179	0.502	1.640	0.000	5.439	5.349

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	243	214	148	279	0	163	1894
N.S.	1	1.00	1.25	1.10	0.76	1.44	0.00	0.84	9.76
time (sec)	N/A	0.139	0.336	0.214	0.506	5.465	0.000	4.622	8.177

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	341	489	0	0	0	3434	2382
N.S.	1	1.00	1.03	1.48	0.00	0.00	0.00	10.41	7.22
time (sec)	N/A	0.525	0.495	0.136	0.000	0.000	0.000	7.234	1.504

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	398	579	0	0	0	5164	2500
N.S.	1	1.00	1.08	1.57	0.00	0.00	0.00	14.03	6.79
time (sec)	N/A	0.587	0.740	0.152	0.000	0.000	0.000	6.866	1.709

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	421	714	0	0	0	5580	2500
N.S.	1	1.00	1.09	1.85	0.00	0.00	0.00	14.46	6.48
time (sec)	N/A	0.320	0.798	0.163	0.000	0.000	0.000	7.616	1.771

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	489	665	0	0	0	7501	2500
N.S.	1	1.00	1.11	1.51	0.00	0.00	0.00	17.09	5.69
time (sec)	N/A	1.313	1.160	0.088	0.000	0.000	0.000	6.511	2.306

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	524	724	0	0	0	19735	2500
N.S.	1	1.00	1.12	1.55	0.00	0.00	0.00	42.17	5.34
time (sec)	N/A	0.730	1.268	0.092	0.000	0.000	0.000	7.140	3.116

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	770	935	1317	0	0	0	20187	2500
N.S.	1	1.00	1.21	1.71	0.00	0.00	0.00	26.22	3.25
time (sec)	N/A	5.650	3.492	1.325	0.000	0.000	0.000	8.852	13.909

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	128	162	129	307	668	123	118
N.S.	1	1.00	0.90	1.13	0.90	2.15	4.67	0.86	0.83
time (sec)	N/A	0.055	0.069	0.047	0.275	0.605	2.213	3.801	0.092

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	161	198	163	389	0	157	151
N.S.	1	1.00	0.92	1.13	0.93	2.22	0.00	0.90	0.86
time (sec)	N/A	0.147	0.084	0.056	0.274	0.586	0.000	5.782	0.113

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	193	234	196	470	0	190	182
N.S.	1	1.00	0.95	1.15	0.96	2.30	0.00	0.93	0.89
time (sec)	N/A	0.156	0.057	0.053	0.279	0.848	0.000	5.227	0.847

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	231	270	222	544	0	224	209
N.S.	1	1.00	1.03	1.21	0.99	2.43	0.00	1.00	0.93
time (sec)	N/A	0.203	0.078	0.058	0.280	1.659	0.000	4.233	0.248

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	261	306	230	616	0	244	233
N.S.	1	1.00	1.09	1.28	0.96	2.58	0.00	1.02	0.97
time (sec)	N/A	0.215	0.083	0.066	0.285	6.680	0.000	6.042	0.616

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	186	158	143	278	1103	131	185
N.S.	1	1.00	1.01	0.85	0.77	1.50	5.96	0.71	1.00
time (sec)	N/A	0.083	0.475	0.085	0.519	0.437	2.160	5.782	0.260

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	235	202	179	384	4496	171	249
N.S.	1	1.00	1.05	0.91	0.80	1.72	20.16	0.77	1.12
time (sec)	N/A	0.149	0.349	0.120	0.501	0.500	142.245	4.011	1.008

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	259	232	200	435	0	198	295
N.S.	1	1.00	1.07	0.95	0.82	1.79	0.00	0.81	1.21
time (sec)	N/A	0.152	0.359	0.147	0.494	0.609	0.000	3.621	1.170

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	303	262	217	485	0	228	1611
N.S.	1	1.00	1.15	1.00	0.83	1.84	0.00	0.87	6.13
time (sec)	N/A	0.181	0.515	0.188	0.498	1.361	0.000	3.688	5.453

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	325	292	225	521	0	246	1963
N.S.	1	1.00	1.21	1.09	0.84	1.94	0.00	0.91	7.30
time (sec)	N/A	0.197	0.512	0.228	0.502	5.429	0.000	2.962	8.217

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	488	889	0	0	0	3399	2500
N.S.	1	1.00	1.03	1.88	0.00	0.00	0.00	7.17	5.27
time (sec)	N/A	1.471	1.186	0.294	0.000	0.000	0.000	8.692	2.344

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	625	1311	0	0	0	5292	2500
N.S.	1	1.00	1.01	2.11	0.00	0.00	0.00	8.52	4.03
time (sec)	N/A	3.263	2.184	0.349	0.000	0.000	0.000	8.246	3.265

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	646	646	661	1638	0	0	0	5439	2500
N.S.	1	1.00	1.02	2.54	0.00	0.00	0.00	8.42	3.87
time (sec)	N/A	2.481	2.569	0.321	0.000	0.000	0.000	9.381	4.558

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	739	1165	0	0	0	7044	2500
N.S.	1	1.00	1.09	1.72	0.00	0.00	0.00	10.37	3.68
time (sec)	N/A	2.860	4.133	0.142	0.000	0.000	0.000	8.501	5.347

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	728	728	821	1295	0	0	0	27123	2500
N.S.	1	1.00	1.13	1.78	0.00	0.00	0.00	37.26	3.43
time (sec)	N/A	1.864	4.959	0.173	0.000	0.000	0.000	9.936	7.160

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1144	1590	1987	0	0	0	22437	2500
N.S.	1	0.99	1.38	1.73	0.00	0.00	0.00	19.51	2.17
time (sec)	N/A	5.100	6.905	1.543	0.000	0.000	0.000	9.727	20.572

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	645	775	1072	0	0	0	16720	2500
N.S.	1	1.00	1.20	1.66	0.00	0.00	0.00	25.92	3.88
time (sec)	N/A	2.222	2.687	0.589	0.000	0.000	0.000	8.159	8.852

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1177	1179	1649	2058	0	0	0	30016	2500
N.S.	1	1.00	1.40	1.75	0.00	0.00	0.00	25.50	2.12
time (sec)	N/A	5.278	6.850	0.819	0.000	0.000	0.000	12.627	17.175

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	416	829	440	418	503	478	398
N.S.	1	1.00	1.00	1.99	1.06	1.00	1.21	1.15	0.96
time (sec)	N/A	0.420	0.076	16.243	0.278	0.576	0.043	4.318	0.383

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	259	354	265	251	309	295	246
N.S.	1	1.00	1.00	1.37	1.02	0.97	1.19	1.14	0.95
time (sec)	N/A	0.224	0.033	15.152	0.278	0.736	0.034	4.416	0.948

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	161	146	138	165	157	138
N.S.	1	1.00	1.00	1.05	0.95	0.90	1.07	1.02	0.90
time (sec)	N/A	0.097	0.020	0.079	0.291	0.386	0.017	3.657	0.089

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	17	16	15	17	16
N.S.	1	1.00	1.00	0.85	0.85	0.80	0.75	0.85	0.80
time (sec)	N/A	0.021	0.001	0.013	0.274	0.393	0.017	4.046	0.027

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	240	0	723401	0	1620	2500
N.S.	1	1.00	1.11	1.14	0.00	3428.44	0.00	7.68	11.85
time (sec)	N/A	0.185	0.144	0.041	0.000	18.478	0.000	6.845	1.174

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	398	579	0	0	0	5164	2500
N.S.	1	1.00	1.08	1.57	0.00	0.00	0.00	14.03	6.79
time (sec)	N/A	0.601	0.758	0.146	0.000	0.000	0.000	8.956	1.523

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	625	1311	0	0	0	5288	2500
N.S.	1	1.00	1.01	2.11	0.00	0.00	0.00	8.52	4.03
time (sec)	N/A	3.288	2.219	0.332	0.000	0.000	0.000	6.091	3.161

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.25	1.00
time (sec)	N/A	0.008	0.001	0.010	0.270	0.398	0.010	4.599	0.018

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	15	16	14	12	17	14
N.S.	1	1.00	1.14	1.07	1.14	1.00	0.86	1.21	1.00
time (sec)	N/A	0.016	0.003	0.031	0.284	0.411	0.044	4.594	0.728

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	28	32	27	26	30	27
N.S.	1	1.00	0.97	0.90	1.03	0.87	0.84	0.97	0.87
time (sec)	N/A	0.035	0.007	0.018	0.284	0.392	0.053	4.534	0.037

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	47	48	43	41	49	44
N.S.	1	1.00	0.88	0.92	0.94	0.84	0.80	0.96	0.86
time (sec)	N/A	0.057	0.016	0.019	0.292	0.397	0.072	5.367	0.038

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	72	67	62	63	74	64
N.S.	1	1.00	1.00	1.06	0.99	0.91	0.93	1.09	0.94
time (sec)	N/A	0.082	0.014	0.029	0.298	0.398	0.088	3.446	0.034

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	103	78	84	88	98	87
N.S.	1	1.00	1.00	1.12	0.85	0.91	0.96	1.07	0.95
time (sec)	N/A	0.109	0.021	0.026	0.278	0.411	0.109	3.704	0.038

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	13	8
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	0.73
time (sec)	N/A	0.007	0.002	0.012	0.286	0.416	0.031	3.739	0.083

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	24	24	22	29	26	22
N.S.	1	1.00	1.05	1.09	1.09	1.00	1.32	1.18	1.00
time (sec)	N/A	0.015	0.005	0.034	0.291	0.388	0.140	3.853	0.799

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	30	31	31	29	44	33	29
N.S.	1	1.00	1.03	1.07	1.07	1.00	1.52	1.14	1.00
time (sec)	N/A	0.035	0.009	0.026	0.282	0.388	0.279	3.602	0.071

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	47	47	45	66	49	45
N.S.	1	1.00	0.94	1.00	1.00	0.96	1.40	1.04	0.96
time (sec)	N/A	0.045	0.014	0.029	0.300	0.384	0.545	4.968	0.764

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	67	64	62	94	69	63
N.S.	1	1.00	1.02	1.02	0.97	0.94	1.42	1.05	0.95
time (sec)	N/A	0.057	0.016	0.028	0.280	0.462	0.887	2.861	0.074

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	91	95	75	84	122	89	86
N.S.	1	1.00	1.01	1.06	0.83	0.93	1.36	0.99	0.96
time (sec)	N/A	0.071	0.023	0.039	0.281	0.576	1.600	4.418	0.084

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	19	19	19	22	19
N.S.	1	1.00	1.00	0.69	0.66	0.66	0.66	0.76	0.66
time (sec)	N/A	0.017	0.004	0.018	0.286	0.505	0.050	6.264	0.078

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	39	38	35	32	304	38	38
N.S.	1	1.00	0.93	0.90	0.83	0.76	7.24	0.90	0.90
time (sec)	N/A	0.034	0.012	0.023	0.283	0.424	1.202	4.625	0.843

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	47	40	37	716	43	47
N.S.	1	1.00	0.94	1.00	0.85	0.79	15.23	0.91	1.00
time (sec)	N/A	0.041	0.015	0.026	0.286	0.408	8.280	3.464	0.111

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	59	50	47	1389	53	59
N.S.	1	1.00	0.96	1.04	0.88	0.82	24.37	0.93	1.04
time (sec)	N/A	0.051	0.019	0.040	0.306	0.412	59.413	4.444	0.820

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	78	65	62	0	68	78
N.S.	1	1.00	0.96	1.05	0.88	0.84	0.00	0.92	1.05
time (sec)	N/A	0.070	0.023	0.040	0.278	0.416	0.000	3.187	0.880

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	91	102	76	82	0	84	99
N.S.	1	1.00	0.95	1.06	0.79	0.85	0.00	0.88	1.03
time (sec)	N/A	0.094	0.031	0.044	0.278	0.448	0.000	3.563	0.883

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	33	32	45	34	36	32
N.S.	1	1.00	0.91	0.72	0.70	0.98	0.74	0.78	0.70
time (sec)	N/A	0.033	0.016	0.025	0.285	0.390	0.114	3.032	0.049

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	64	62	93	1188	66	64
N.S.	1	1.00	0.93	0.90	0.87	1.31	16.73	0.93	0.90
time (sec)	N/A	0.119	0.034	0.038	0.291	0.410	7.548	3.775	0.807

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	79	73	116	0	77	79
N.S.	1	1.00	0.94	0.96	0.89	1.41	0.00	0.94	0.96
time (sec)	N/A	0.128	0.042	0.039	0.278	0.472	0.000	3.609	0.842

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	94	86	141	0	90	94
N.S.	1	1.00	0.95	0.99	0.91	1.48	0.00	0.95	0.99
time (sec)	N/A	0.149	0.032	0.047	0.281	0.828	0.000	3.700	0.876

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	109	97	164	0	101	108
N.S.	1	1.00	0.96	1.03	0.92	1.55	0.00	0.95	1.02
time (sec)	N/A	0.175	0.040	0.052	0.296	3.236	0.000	5.318	1.364

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	118	127	105	200	0	109	127
N.S.	1	1.00	0.97	1.04	0.86	1.64	0.00	0.89	1.04
time (sec)	N/A	0.205	0.041	0.063	0.284	19.297	0.000	4.776	1.673

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	48	40	42	72	46	46	42
N.S.	1	1.00	0.86	0.71	0.75	1.29	0.82	0.82	0.75
time (sec)	N/A	0.037	0.018	0.040	0.278	0.401	0.162	4.214	0.046

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	78	81	153	1255	85	79
N.S.	1	1.00	0.90	0.88	0.91	1.72	14.10	0.96	0.89
time (sec)	N/A	0.178	0.036	0.036	0.278	0.408	7.605	3.958	0.101

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	96	97	191	0	101	97
N.S.	1	1.00	0.92	0.91	0.92	1.82	0.00	0.96	0.92
time (sec)	N/A	0.200	0.051	0.041	0.288	0.467	0.000	3.344	0.828

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	114	113	229	0	117	115
N.S.	1	1.00	0.97	0.97	0.97	1.96	0.00	1.00	0.98
time (sec)	N/A	0.165	0.037	0.049	0.288	0.859	0.000	3.014	0.905

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	136	132	129	267	0	133	133
N.S.	1	1.00	1.04	1.01	0.98	2.04	0.00	1.02	1.02
time (sec)	N/A	0.184	0.043	0.058	0.294	5.114	0.000	3.645	1.332

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	153	150	135	305	0	139	151
N.S.	1	1.00	1.04	1.02	0.92	2.07	0.00	0.95	1.03
time (sec)	N/A	0.210	0.055	0.067	0.280	20.435	0.000	3.493	1.680

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	47	52	103	53	56	52
N.S.	1	1.00	0.88	0.69	0.76	1.51	0.78	0.82	0.76
time (sec)	N/A	0.039	0.022	0.031	0.292	0.390	0.137	3.987	0.046

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	92	94	211	1034	98	90
N.S.	1	1.00	0.92	0.88	0.90	2.01	9.85	0.93	0.86
time (sec)	N/A	0.131	0.060	0.039	0.292	0.417	6.025	3.394	0.093

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	113	114	267	0	118	113
N.S.	1	1.00	0.99	0.93	0.93	2.19	0.00	0.97	0.93
time (sec)	N/A	0.144	0.036	0.043	0.280	0.484	0.000	3.573	0.126

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	144	134	132	321	0	136	131
N.S.	1	1.00	1.02	0.95	0.94	2.28	0.00	0.96	0.93
time (sec)	N/A	0.168	0.049	0.049	0.285	0.878	0.000	3.526	0.881

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	169	155	151	376	0	155	152
N.S.	1	1.00	1.07	0.98	0.96	2.38	0.00	0.98	0.96
time (sec)	N/A	0.194	0.061	0.056	0.280	3.601	0.000	3.463	1.392

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	176	157	430	0	161	170
N.S.	1	1.00	1.10	0.99	0.89	2.43	0.00	0.91	0.96
time (sec)	N/A	0.229	0.074	0.077	0.285	21.481	0.000	3.787	1.755

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	717	717	2588	1580	0	911	0	0	-1
N.S.	1	1.00	3.61	2.20	0.00	1.27	0.00	0.00	-0.00
time (sec)	N/A	0.393	11.641	0.083	0.000	0.305	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	661	1042	0	574	0	0	-1
N.S.	1	1.00	1.31	2.06	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.179	12.546	0.066	0.000	0.256	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	525	454	0	376	0	0	-1
N.S.	1	1.00	1.46	1.26	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.101	10.836	0.056	0.000	0.207	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	513	1005	0	723	0	0	-1
N.S.	1	1.00	1.15	2.25	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.174	10.985	0.050	0.000	0.116	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	680	598	1395	0	1948	0	0	-1
N.S.	1	1.00	0.88	2.05	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.329	11.614	0.091	0.000	0.115	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	938	17	17	0	60	17
N.S.	1	1.00	1.00	49.37	0.89	0.89	0.00	3.16	0.89
time (sec)	N/A	0.011	10.042	0.041	0.320	0.416	0.000	6.006	0.987

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	974	53	82	0	142	51
N.S.	1	1.00	0.89	17.09	0.93	1.44	0.00	2.49	0.89
time (sec)	N/A	0.043	10.095	0.046	0.331	0.401	0.000	4.648	0.928

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	976	49	80	0	136	51
N.S.	1	1.00	0.84	17.12	0.86	1.40	0.00	2.39	0.89
time (sec)	N/A	0.049	10.102	0.047	0.329	0.397	0.000	3.941	0.957

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	1012	94	92	0	166	62
N.S.	1	1.00	0.88	14.67	1.36	1.33	0.00	2.41	0.90
time (sec)	N/A	0.059	10.123	0.048	0.342	0.400	0.000	4.485	0.977

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [60] had the largest ratio of [63]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	18	0.056
2	A	2	1	1.00	23	0.043
3	A	2	1	1.00	28	0.036
4	A	2	1	1.00	33	0.030
5	A	2	1	1.00	38	0.026
6	A	2	1	1.00	20	0.050
7	A	2	1	1.00	25	0.040
8	A	2	1	1.00	30	0.033
9	A	2	1	1.00	35	0.029
10	A	10	7	1.00	18	0.389
11	A	9	7	1.00	23	0.304
12	A	8	6	1.00	28	0.214
13	A	10	7	1.00	33	0.212
14	A	12	8	1.00	38	0.210
15	A	15	8	1.00	16	0.500
16	A	14	8	1.00	21	0.381
17	A	15	7	1.00	26	0.269
18	A	17	8	1.00	31	0.258
19	A	19	9	1.00	36	0.250
20	A	9	7	1.00	20	0.350
21	A	8	7	1.00	25	0.280
22	A	9	8	1.00	30	0.267
23	A	11	9	1.00	35	0.257
24	A	13	10	1.00	40	0.250
25	A	13	10	1.00	55	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	12	9	1.00	18	0.500
27	A	11	9	1.00	23	0.391
28	A	10	8	1.00	28	0.286
29	A	10	8	1.00	33	0.242
30	A	11	9	1.00	38	0.237
31	A	17	10	1.00	16	0.625
32	A	16	10	1.00	21	0.476
33	A	15	9	1.00	26	0.346
34	A	15	9	1.00	31	0.290
35	A	16	10	1.00	36	0.278
36	A	11	9	1.00	20	0.450
37	A	10	9	1.00	25	0.360
38	A	9	8	1.00	30	0.267
39	A	9	8	1.00	35	0.229
40	A	10	9	1.00	40	0.225
41	A	13	11	1.00	55	0.200
42	A	14	10	1.00	18	0.556
43	A	13	9	1.00	23	0.391
44	A	12	9	1.00	28	0.321
45	A	12	10	1.00	33	0.303
46	A	13	11	1.00	38	0.290
47	A	19	11	1.00	16	0.688
48	A	18	10	1.00	21	0.476
49	A	17	10	1.00	26	0.385
50	A	17	11	1.00	31	0.355
51	A	18	12	1.00	36	0.333
52	A	13	10	1.00	20	0.500
53	A	12	9	1.00	25	0.360
54	A	11	9	1.00	30	0.300
55	A	11	10	1.00	35	0.286
56	A	12	11	1.00	40	0.275
57	A	11	9	0.99	55	0.164
58	A	11	10	1.00	50	0.200
59	A	13	10	1.00	50	0.200
60	A	2	1	1.00	63	0.016

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	1	1.00	63	0.016
62	A	2	1	1.00	61	0.016
63	A	2	1	1.00	63	0.016
64	A	9	8	1.00	63	0.127
65	A	11	10	1.00	63	0.159
66	A	13	10	1.00	63	0.159
67	A	2	2	1.00	26	0.077
68	A	3	2	1.00	31	0.065
69	A	3	2	1.00	36	0.056
70	A	3	2	1.00	41	0.049
71	A	3	2	1.00	46	0.043
72	A	3	2	1.00	51	0.039
73	A	4	3	1.00	21	0.143
74	A	4	3	1.00	26	0.115
75	A	6	4	1.00	31	0.129
76	A	6	4	1.00	36	0.111
77	A	6	4	1.00	41	0.098
78	A	6	4	1.00	46	0.087
79	A	3	2	1.00	16	0.125
80	A	3	2	1.00	21	0.095
81	A	3	2	1.00	26	0.077
82	A	3	2	1.00	31	0.065
83	A	3	2	1.00	36	0.056
84	A	3	2	1.00	41	0.049
85	A	3	2	1.00	26	0.077
86	A	3	2	1.00	31	0.065
87	A	3	2	1.00	36	0.056
88	A	3	2	1.00	41	0.049
89	A	3	2	1.00	46	0.043
90	A	3	2	1.00	51	0.039
91	A	9	5	1.00	21	0.238
92	A	9	5	1.00	26	0.192
93	A	9	5	1.00	31	0.161
94	A	3	2	1.00	36	0.056
95	A	3	2	1.00	41	0.049

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	46	0.043
97	A	3	2	1.00	16	0.125
98	A	3	2	1.00	21	0.095
99	A	3	2	1.00	26	0.077
100	A	3	2	1.00	31	0.065
101	A	3	2	1.00	36	0.056
102	A	3	2	1.00	41	0.049
103	A	12	10	1.00	32	0.312
104	A	10	10	1.00	32	0.312
105	A	8	8	1.00	32	0.250
106	A	7	7	1.00	32	0.219
107	A	9	8	1.00	32	0.250
108	A	1	1	1.00	28	0.036
109	A	5	5	1.00	31	0.161
110	A	5	5	1.00	33	0.152
111	A	4	4	1.00	36	0.111

Chapter 3

Listing of integrals

Local contents

3.1 $\int (d+ex)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=50

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*b*d*x^3+1/4*b*e*x^4+1/5*c*d*x^5+1/6*c*e*x^6
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1685}

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+bx^2+cx^4\right)\,dx &= \int \left(ad+aex+bdx^2+bex^3+cdx^4+cex^5\right)\,dx\\ &= adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6\end{aligned}$$

Mathematica [A]
time = 0.00, size = 50, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Maple [A]
time = 0.01, size = 41, normalized size = 0.82

method	result	size
gosper	$adx+\frac{1}{2}ae\,x^2+\frac{1}{3}bd\,x^3+\frac{1}{4}be\,x^4+\frac{1}{5}cd\,x^5+\frac{1}{6}ce\,x^6$	41
default	$adx+\frac{1}{2}ae\,x^2+\frac{1}{3}bd\,x^3+\frac{1}{4}be\,x^4+\frac{1}{5}cd\,x^5+\frac{1}{6}ce\,x^6$	41
norman	$adx+\frac{1}{2}ae\,x^2+\frac{1}{3}bd\,x^3+\frac{1}{4}be\,x^4+\frac{1}{5}cd\,x^5+\frac{1}{6}ce\,x^6$	41
risch	$adx+\frac{1}{2}ae\,x^2+\frac{1}{3}bd\,x^3+\frac{1}{4}be\,x^4+\frac{1}{5}cd\,x^5+\frac{1}{6}ce\,x^6$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] a*d*x+1/2*a*e*x^2+1/3*b*d*x^3+1/4*b*e*x^4+1/5*c*d*x^5+1/6*c*e*x^6
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.86

$$\frac{1}{6}\,cx^6e+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bx^4e+\frac{1}{3}\,bdx^3+\frac{1}{2}\,ax^2e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*a*x^2*e + a*d*x
```

Fricas [A]
time = 0.79, size = 40, normalized size = 0.80

$$\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bex^4+\frac{1}{3}\,bdx^3+\frac{1}{2}\,aex^2+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x^2 + a*d*x
```

Sympy [A]
time = 0.01, size = 46, normalized size = 0.92

$$adx+\frac{aex^2}{2}+\frac{bdx^3}{3}+\frac{bex^4}{4}+\frac{cdx^5}{5}+\frac{cex^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*d*x**3/3 + b*e*x**4/4 + c*d*x**5/5 + c*e*x**6/6
```

Giac [A]
time = 4.12, size = 43, normalized size = 0.86

$$\frac{1}{6}\,cx^6e + \frac{1}{5}\,cdx^5 + \frac{1}{4}\,bx^4e + \frac{1}{3}\,bdx^3 + \frac{1}{2}\,ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/2*a*x^2*e + a*d*x
```

Mupad [B]
time = 0.03, size = 40, normalized size = 0.80

$$\frac{c\,e\,x^6}{6} + \frac{c\,d\,x^5}{5} + \frac{b\,e\,x^4}{4} + \frac{b\,d\,x^3}{3} + \frac{a\,e\,x^2}{2} + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(a + b*x^2 + c*x^4),x)
[Out] a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/6
```

3.2 $\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=69

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5+1/6*c*e*x
^6+1/7*c*f*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1671}

$$\frac{1}{3}x^3(af+bd)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{4}bex^4+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5)
/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex+fx^2)(a+bx^2+cx^4)\,dx = \int (ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6)\,d$$
$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}$$

Mathematica [A]
time = 0.01, size = 69, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}bex^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}cex^6+\frac{1}{7}cfx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + (b*e*x^4)/4 + ((c*d + b*f)*x^5)
/5 + (c*e*x^6)/6 + (c*f*x^7)/7
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.84

method	result	size
default	$adx+\frac{ae\,x^2}{2}+\frac{(fa+bd)x^3}{3}+\frac{be\,x^4}{4}+\frac{(bf+cd)x^5}{5}+\frac{ce\,x^6}{6}+\frac{cf\,x^7}{7}$	58
norman	$\frac{cf\,x^7}{7}+\frac{ce\,x^6}{6}+\left(\frac{bf}{5}+\frac{cd}{5}\right)x^5+\frac{be\,x^4}{4}+\left(\frac{fa}{3}+\frac{bd}{3}\right)x^3+\frac{ae\,x^2}{2}+adx$	60
gosper	$\frac{1}{7}cf\,x^7+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}be\,x^4+\frac{1}{3}x^3fa+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2+adx$	62
risch	$\frac{1}{7}cf\,x^7+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}be\,x^4+\frac{1}{3}x^3fa+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2+adx$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*b*e*x^4+1/5*(b*f+c*d)*x^5+1/6*c*e*x
^6+1/7*c*f*x^7
```

Maxima [A]
time = 0.28, size = 60, normalized size = 0.87

$$\frac{1}{7}\,cfx^7+\frac{1}{6}\,cx^6e+\frac{1}{5}\,(cd+bf)x^5+\frac{1}{4}\,bx^4e+\frac{1}{3}\,(bd+af)x^3+\frac{1}{2}\,ax^2e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*x^6*e + 1/5*(c*d + b*f)*x^5 + 1/4*b*x^4*e + 1/3*(b*d +
a*f)*x^3 + 1/2*a*x^2*e + a*d*x
```

Fricas [A]
time = 0.39, size = 57, normalized size = 0.83

$$\frac{1}{7}\,cfx^7+\frac{1}{6}\,cex^6+\frac{1}{4}\,bex^4+\frac{1}{5}\,(cd+bf)x^5+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bd+af)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*e*x^6 + 1/4*b*e*x^4 + 1/5*(c*d + b*f)*x^5 + 1/2*a*e*x^2
 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Sympy [A]
time = 0.01, size = 65, normalized size = 0.94

$$adx+\frac{aex^2}{2}+\frac{bex^4}{4}+\frac{cex^6}{6}+\frac{cfx^7}{7}+x^5\left(\frac{bf}{5}+\frac{cd}{5}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + b*e*x**4/4 + c*e*x**6/6 + c*f*x**7/7 + x**5*(b*f/5 + c
*d/5) + x**3*(a*f/3 + b*d/3)
```

Giac [A]
time = 3.97, size = 64, normalized size = 0.93

$$\frac{1}{7}cfx^7 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*f*x^7 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/4*b*x^4*e + 1/3*b
*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

Mupad [B]
time = 0.03, size = 59, normalized size = 0.86

$$\frac{c\,f\,x^7}{7} + \frac{c\,e\,x^6}{6} + \left(\frac{c\,d}{5} + \frac{b\,f}{5}\right)\,x^5 + \frac{b\,e\,x^4}{4} + \left(\frac{b\,d}{3} + \frac{a\,f}{3}\right)\,x^3 + \frac{a\,e\,x^2}{2} + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d)/3 + (a*f)/3) + x^5*((c*d)/5 + (b*f)/5) + a*d*x + (a*e*x^2)/2 + (
b*e*x^4)/4 + (c*e*x^6)/6 + (c*f*x^7)/7
```

3.3 $\int (d+ex+fx^2+gx^3)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=88

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}(ce+bg)x^6+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(b*f+c*d)*x^5+1/6
*(b*g+c*e)*x^6+1/7*c*f*x^7+1/8*c*g*x^8
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1685}

$$\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{5}x^5(bf+cd)+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x^8)/8
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (d+ex+fx^2+gx^3)(a+bx^2+cx^4)\,dx = \int (ad+aex+(bd+af)x^2+(be+ag)x^3+(cd+bf)x^4+$$
$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf)$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf)x^5+\frac{1}{6}(ce+bg)x^6+\frac{1}{7}cfx^7+\frac{1}{8}cgx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f)*x^5)/5 + ((c*e + b*g)*x^6)/6 + (c*f*x^7)/7 + (c*g*x^8)/8
```

Maple [A]
time = 0.08, size = 75, normalized size = 0.85

method	result
default	$adx+\frac{ae\,x^2}{2}+\frac{(fa+bd)x^3}{3}+\frac{(ag+eb)x^4}{4}+\frac{(bf+cd)x^5}{5}+\frac{(bg+ce)x^6}{6}+\frac{cf\,x^7}{7}+\frac{cg\,x^8}{8}$
norman	$\frac{cg\,x^8}{8}+\frac{cf\,x^7}{7}+\left(\frac{bg}{6}+\frac{ce}{6}\right)x^6+\left(\frac{bf}{5}+\frac{cd}{5}\right)x^5+\left(\frac{ag}{4}+\frac{eb}{4}\right)x^4+\left(\frac{fa}{3}+\frac{bd}{3}\right)x^3+\frac{ae\,x^2}{2}+adx$
gosper	$\frac{1}{8}cg\,x^8+\frac{1}{7}cf\,x^7+\frac{1}{6}x^6bg+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}x^4ag+\frac{1}{4}be\,x^4+\frac{1}{3}x^3fa+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2$
risch	$\frac{1}{8}cg\,x^8+\frac{1}{7}cf\,x^7+\frac{1}{6}x^6bg+\frac{1}{6}ce\,x^6+\frac{1}{5}x^5bf+\frac{1}{5}cd\,x^5+\frac{1}{4}x^4ag+\frac{1}{4}be\,x^4+\frac{1}{3}x^3fa+\frac{1}{3}bd\,x^3+\frac{1}{2}ae\,x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(b*f+c*d)*x^5+1/6
*(b*g+c*e)*x^6+1/7*c*f*x^7+1/8*c*g*x^8
```

Maxima [A]
time = 0.27, size = 77, normalized size = 0.88

$$\frac{1}{8}\,cgx^8+\frac{1}{7}\,cfx^7+\frac{1}{6}\,(bg+ce)x^6+\frac{1}{5}\,(cd+bf)x^5+\frac{1}{4}\,(ag+be)x^4+\frac{1}{3}\,(bd+af)x^3+\frac{1}{2}\,ax^2e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*(b*g + c*e)*x^6 + 1/5*(c*d + b*f)*x^5 + 1/4
*(a*g + b*e)*x^4 + 1/3*(b*d + a*f)*x^3 + 1/2*a*x^2*e + a*d*x
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.84

$$\frac{1}{8}\,cgx^8+\frac{1}{7}\,cfx^7+\frac{1}{6}\,(ce+bg)x^6+\frac{1}{5}\,(cd+bf)x^5+\frac{1}{4}\,(be+ag)x^4+\frac{1}{2}\,aex^2+\frac{1}{3}\,(bd+af)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*(c*e + b*g)*x^6 + 1/5*(c*d + b*f)*x^5 + 1/4
*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Sympy [A]
time = 0.01, size = 83, normalized size = 0.94

$$adx+\frac{aex^2}{2}+\frac{cfx^7}{7}+\frac{cgx^8}{8}+x^6\left(\frac{bg}{6}+\frac{ce}{6}\right)+x^5\left(\frac{bf}{5}+\frac{cd}{5}\right)+x^4\left(\frac{ag}{4}+\frac{be}{4}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*f*x**7/7 + c*g*x**8/8 + x**6*(b*g/6 + c*e/6) + x**5*
(b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Giac [A]
time = 5.21, size = 85, normalized size = 0.97

$$\frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/6*b*g*x^6 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b
*f*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*
e + a*d*x
```

Mupad [B]
time = 0.66, size = 78, normalized size = 0.89

$$\frac{c\,g\,x^8}{8} + \frac{c\,f\,x^7}{7} + \left(\frac{c\,e}{6} + \frac{b\,g}{6}\right)\,x^6 + \left(\frac{c\,d}{5} + \frac{b\,f}{5}\right)\,x^5 + \left(\frac{b\,e}{4} + \frac{a\,g}{4}\right)\,x^4 + \left(\frac{b\,d}{3} + \frac{a\,f}{3}\right)\,x^3 + \frac{a\,e\,x^2}{2} + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3),x)
```

```
[Out] x^3*((b*d)/3 + (a*f)/3) + x^4*((b*e)/4 + (a*g)/4) + x^5*((c*d)/5 + (b*f)/5)
 + x^6*((c*e)/6 + (b*g)/6) + (c*g*x^8)/8 + a*d*x + (a*e*x^2)/2 + (c*f*x^7)/
7
```

3.4 $\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4)\,dx$

Optimal. Leaf size=105

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{6}(ce+bg)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*c*g*x^8+1/9*c*h*x^9
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {1685}

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{6}x^6(bg+ce)+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x^7)/7 + (c*g*x^8)/8
+ (c*h*x^9)/9
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\left(d+ex+fx^2+gx^3+hx^4\right)\,dx = \int \left(ad+aex+(bd+af)x^2+(be+ag)x^3+(cd+bf\right.$$
$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd$$

Mathematica [A]
time = 0.02, size = 105, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{6}(ce+bg)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}cgx^8+\frac{1}{9}chx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g)*x^6)/6 + ((c*f + b*h)*x^7)/7 + (c*g*x^8)/8
+ (c*h*x^9)/9
```

Maple [A]
time = 0.13, size = 90, normalized size = 0.86

method	result
default	$adx + \frac{ae\,x^2}{2} + \frac{(fa+bd)x^3}{3} + \frac{(ag+eb)x^4}{4} + \frac{(ah+bf+cd)x^5}{5} + \frac{(bg+ce)x^6}{6} + \frac{(bh+cf)x^7}{7} + \frac{cg\,x^8}{8} + \frac{ch\,x^9}{9}$
norman	$\frac{ch\,x^9}{9} + \frac{cg\,x^8}{8} + \left(\frac{bh}{7} + \frac{cf}{7}\right)x^7 + \left(\frac{bg}{6} + \frac{ce}{6}\right)x^6 + \left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right)x^5 + \left(\frac{ag}{4} + \frac{eb}{4}\right)x^4 + \left(\frac{fa}{3} + \frac{bd}{3}\right)x^3 + a$
gosper	$\frac{1}{9}ch\,x^9 + \frac{1}{8}cg\,x^8 + \frac{1}{7}x^7bh + \frac{1}{7}cf\,x^7 + \frac{1}{6}x^6bg + \frac{1}{6}ce\,x^6 + \frac{1}{5}x^5ah + \frac{1}{5}x^5bf + \frac{1}{5}cd\,x^5 + \frac{1}{4}x^4ag + \frac{1}{4}be\,x$
risch	$\frac{1}{9}ch\,x^9 + \frac{1}{8}cg\,x^8 + \frac{1}{7}x^7bh + \frac{1}{7}cf\,x^7 + \frac{1}{6}x^6bg + \frac{1}{6}ce\,x^6 + \frac{1}{5}x^5ah + \frac{1}{5}x^5bf + \frac{1}{5}cd\,x^5 + \frac{1}{4}x^4ag + \frac{1}{4}be\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*c*g*x^8+1/9*c*h*x^9
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.88

$$\frac{1}{9}chx^9 + \frac{1}{8}cgx^8 + \frac{1}{7}(cf + bh)x^7 + \frac{1}{6}(bg + ce)x^6 + \frac{1}{5}(cd + bf + ah)x^5 + \frac{1}{4}(ag + be)x^4 + \frac{1}{3}(bd + af)x^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*(c*f + b*h)*x^7 + 1/6*(b*g + c*e)*x^6 + 1/5
*(c*d + b*f + a*h)*x^5 + 1/4*(a*g + b*e)*x^4 + 1/3*(b*d + a*f)*x^3 + 1/2*a*
x^2*e + a*d*x
```

Fricas [A]
time = 0.40, size = 89, normalized size = 0.85

$$\frac{1}{9}chx^9 + \frac{1}{8}cgx^8 + \frac{1}{7}(cf + bh)x^7 + \frac{1}{6}(ce + bg)x^6 + \frac{1}{5}(cd + bf + ah)x^5 + \frac{1}{4}(be + ag)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}(bd + af)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*(c*f + b*h)*x^7 + 1/6*(c*e + b*g)*x^6 + 1/5
*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^4 + 1/2*a*e*x^2 + 1/3*(b*d + a*f
)*x^3 + a*d*x
```

Sympy [A]
time = 0.01, size = 102, normalized size = 0.97

$$adx+\frac{aex^2}{2}+\frac{cgx^8}{8}+\frac{chx^9}{9}+x^7\left(\frac{bh}{7}+\frac{cf}{7}\right)+x^6\left(\frac{bg}{6}+\frac{ce}{6}\right)+x^5\left(\frac{ah}{5}+\frac{bf}{5}+\frac{cd}{5}\right)+x^4\left(\frac{ag}{4}+\frac{be}{4}\right)+x^3\left(\frac{af}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*g*x**8/8 + c*h*x**9/9 + x**7*(b*h/7 + c*f/7) + x**6*
(b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*d/5) + x**4*(a*g/4 + b*e/4) + x**
3*(a*f/3 + b*d/3)
```

Giac [A]
time = 4.71, size = 106, normalized size = 1.01

$$\frac{1}{9}chx^9+\frac{1}{8}cgx^8+\frac{1}{7}cfx^7+\frac{1}{7}bhx^7+\frac{1}{6}bgx^6+\frac{1}{6}cx^6e+\frac{1}{5}cdx^5+\frac{1}{5}bfx^5+\frac{1}{5}ahx^5+\frac{1}{4}agx^4+\frac{1}{4}bx^4e+\frac{1}{3}bdx^3+\frac{1}{3}afx^3+\frac{1}{2}ax^2e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/9*c*h*x^9 + 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/7*b*h*x^7 + 1/6*b*g*x^6 + 1/6*c
*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/5*a*h*x^5 + 1/4*a*g*x^4 + 1/4*b*x^4*
e + 1/3*b*d*x^3 + 1/3*a*f*x^3 + 1/2*a*x^2*e + a*d*x
```

Mupad [B]
time = 0.66, size = 95, normalized size = 0.90

$$\frac{c\,h\,x^9}{9}+\frac{c\,g\,x^8}{8}+\left(\frac{c\,f}{7}+\frac{b\,h}{7}\right)\,x^7+\left(\frac{c\,e}{6}+\frac{b\,g}{6}\right)\,x^6+\left(\frac{c\,d}{5}+\frac{b\,f}{5}+\frac{a\,h}{5}\right)\,x^5+\left(\frac{b\,e}{4}+\frac{a\,g}{4}\right)\,x^4+\left(\frac{b\,d}{3}+\frac{a\,f}{3}\right)\,x^3+\frac{a\,e\,x^2}{2}+a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4),x)
```

```
[Out] x^5*((c*d)/5 + (b*f)/5 + (a*h)/5) + x^3*((b*d)/3 + (a*f)/3) + x^4*((b*e)/4
+ (a*g)/4) + x^6*((c*e)/6 + (b*g)/6) + x^7*((c*f)/7 + (b*h)/7) + (c*g*x^8)/
8 + (c*h*x^9)/9 + a*d*x + (a*e*x^2)/2
```

3.5 $\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+ix^5)\,dx$

Optimal. Leaf size=122

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{6}(ce+bg+ai)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}(cg+bi)x^8+\frac{1}{9}$$

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*x^8+1/9*c*h*x^9+1/10
*c*i*x^10
```

Rubi [A]
time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {1685}

$$\frac{1}{5}x^5(ah+bf+cd)+\frac{1}{6}x^6(ai+bg+ce)+\frac{1}{3}x^3(af+bd)+\frac{1}{4}x^4(ag+be)+adx+\frac{1}{2}aex^2+\frac{1}{7}x^7(bh+cf)+\frac{1}{8}x^8(bi+cg)+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*g
+ b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx^2+cx^4)(d+ex+fx^2+gx^3+hx^4+5x^5)\,dx = \int (ad+aex+(bd+af)x^2+(be+ag)x^3+(c$$

$$= adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4$$

Mathematica [A]
time = 0.03, size = 122, normalized size = 1.00

$$adx+\frac{1}{2}aex^2+\frac{1}{3}(bd+af)x^3+\frac{1}{4}(be+ag)x^4+\frac{1}{5}(cd+bf+ah)x^5+\frac{1}{6}(ce+bg+ai)x^6+\frac{1}{7}(cf+bh)x^7+\frac{1}{8}(cg+bi)x^8+\frac{1}{9}chx^9+\frac{1}{10}cix^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + ((b*d + a*f)*x^3)/3 + ((b*e + a*g)*x^4)/4 + ((c*d + b
*f + a*h)*x^5)/5 + ((c*e + b*g + a*i)*x^6)/6 + ((c*f + b*h)*x^7)/7 + ((c*g
+ b*i)*x^8)/8 + (c*h*x^9)/9 + (c*i*x^10)/10
```

Maple [A]
time = 1.03, size = 105, normalized size = 0.86

method	result
default	$adx + \frac{ae\,x^2}{2} + \frac{(fa+bd)x^3}{3} + \frac{(ag+eb)x^4}{4} + \frac{(ah+bf+cd)x^5}{5} + \frac{(ai+bg+ce)x^6}{6} + \frac{(bh+cf)x^7}{7} + \frac{(bi+cg)x^8}{8} + \frac{ch\,x^9}{9} + \frac{ci\,x^1}{10}$
norman	$\frac{ci\,x^{10}}{10} + \frac{ch\,x^9}{9} + \left(\frac{bi}{8} + \frac{cg}{8}\right)x^8 + \left(\frac{bh}{7} + \frac{cf}{7}\right)x^7 + \left(\frac{ai}{6} + \frac{bg}{6} + \frac{ce}{6}\right)x^6 + \left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right)x^5 + \left(\frac{ag}{4} + \frac{eb}{4}\right)x^4$
gosper	$\frac{1}{10}ci\,x^{10} + \frac{1}{9}ch\,x^9 + \frac{1}{8}x^8bi + \frac{1}{8}cg\,x^8 + \frac{1}{7}x^7bh + \frac{1}{7}cf\,x^7 + \frac{1}{6}x^6ai + \frac{1}{6}x^6bg + \frac{1}{6}ce\,x^6 + \frac{1}{5}x^5ah + \frac{1}{5}x^5bf$
risch	$\frac{1}{10}ci\,x^{10} + \frac{1}{9}ch\,x^9 + \frac{1}{8}x^8bi + \frac{1}{8}cg\,x^8 + \frac{1}{7}x^7bh + \frac{1}{7}cf\,x^7 + \frac{1}{6}x^6ai + \frac{1}{6}x^6bg + \frac{1}{6}ce\,x^6 + \frac{1}{5}x^5ah + \frac{1}{5}x^5bf$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE
)
```

```
[Out] a*d*x+1/2*a*e*x^2+1/3*(a*f+b*d)*x^3+1/4*(a*g+b*e)*x^4+1/5*(a*h+b*f+c*d)*x^5
+1/6*(a*i+b*g+c*e)*x^6+1/7*(b*h+c*f)*x^7+1/8*(b*i+c*g)*x^8+1/9*c*h*x^9+1/10
*c*i*x^10
```

Maxima [A]
time = 0.29, size = 106, normalized size = 0.87

$$\frac{1}{9}chx^9 + \frac{1}{10}i\,cx^{10} + \frac{1}{8}(cg + i\,b)x^8 + \frac{1}{7}(cf + bh)x^7 + \frac{1}{6}(bg + ce + i\,a)x^6 + \frac{1}{5}(cd + bf + ah)x^5 + \frac{1}{4}(ag + be)x^4 + \frac{1}{3}(bd + af)x^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/9*c*h*x^9 + 1/10*I*c*x^10 + 1/8*(c*g + I*b)*x^8 + 1/7*(c*f + b*h)*x^7 + 1
/6*(b*g + c*e + I*a)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(a*g + b*e)*x^4
+ 1/3*(b*d + a*f)*x^3 + 1/2*a*x^2*e + a*d*x
```

Fricas [A]
time = 0.39, size = 104, normalized size = 0.85

$$\frac{1}{10}cix^{10} + \frac{1}{9}chx^9 + \frac{1}{8}(cg + bi)x^8 + \frac{1}{7}(cf + bh)x^7 + \frac{1}{6}(ce + bg + ai)x^6 + \frac{1}{5}(cd + bf + ah)x^5 + \frac{1}{4}(be + ag)x^4 + \frac{1}{2}aex^2 + \frac{1}{3}(bd + af)x^3 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/10*c*i*x^10 + 1/9*c*h*x^9 + 1/8*(c*g + b*i)*x^8 + 1/7*(c*f + b*h)*x^7 + 1
/6*(c*e + b*g + a*i)*x^6 + 1/5*(c*d + b*f + a*h)*x^5 + 1/4*(b*e + a*g)*x^4
+ 1/2*a*e*x^2 + 1/3*(b*d + a*f)*x^3 + a*d*x
```

Sympy [A]
time = 0.01, size = 121, normalized size = 0.99

$$adx + \frac{aex^2}{2} + \frac{chx^9}{9} + \frac{cix^{10}}{10} + x^8\left(\frac{bi}{8} + \frac{cg}{8}\right) + x^7\left(\frac{bh}{7} + \frac{cf}{7}\right) + x^6\left(\frac{ai}{6} + \frac{bg}{6} + \frac{ce}{6}\right) + x^5\left(\frac{ah}{5} + \frac{bf}{5} + \frac{cd}{5}\right) + x^4\left(\frac{ag}{4} + \frac{be}{4}\right) + x^3\left(\frac{af}{3} + \frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a*d*x + a*e*x**2/2 + c*h*x**9/9 + c*i*x**10/10 + x**8*(b*i/8 + c*g/8) + x**
7*(b*h/7 + c*f/7) + x**6*(a*i/6 + b*g/6 + c*e/6) + x**5*(a*h/5 + b*f/5 + c*
d/5) + x**4*(a*g/4 + b*e/4) + x**3*(a*f/3 + b*d/3)
```

Giac [A]
time = 3.56, size = 124, normalized size = 1.02

$$\frac{1}{9}chx^9 + \frac{1}{10}icx^{10} + \frac{1}{8}cgx^8 + \frac{1}{7}cfx^7 + \frac{1}{7}bhx^7 + \frac{1}{8}ibx^8 + \frac{1}{6}bgx^6 + \frac{1}{6}cx^6e + \frac{1}{5}cdx^5 + \frac{1}{5}bfx^5 + \frac{1}{5}ahx^5 + \frac{1}{6}iax^6 + \frac{1}{4}agx^4 + \frac{1}{4}bx^4e + \frac{1}{3}bdx^3 + \frac{1}{3}afx^3 + \frac{1}{2}ax^2e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="gia
c")
```

```
[Out] 1/9*c*h*x^9 + 1/10*I*c*x^10 + 1/8*c*g*x^8 + 1/7*c*f*x^7 + 1/7*b*h*x^7 + 1/8
*I*b*x^8 + 1/6*b*g*x^6 + 1/6*c*x^6*e + 1/5*c*d*x^5 + 1/5*b*f*x^5 + 1/5*a*h*
x^5 + 1/6*I*a*x^6 + 1/4*a*g*x^4 + 1/4*b*x^4*e + 1/3*b*d*x^3 + 1/3*a*f*x^3 +
 1/2*a*x^2*e + a*d*x
```

Mupad [B]
time = 0.06, size = 112, normalized size = 0.92

$$\frac{c\,i\,x^{10}}{10} + \frac{c\,h\,x^9}{9} + \left(\frac{c\,g}{8} + \frac{b\,i}{8}\right)\,x^8 + \left(\frac{c\,f}{7} + \frac{b\,h}{7}\right)\,x^7 + \left(\frac{c\,e}{6} + \frac{b\,g}{6} + \frac{a\,i}{6}\right)\,x^6 + \left(\frac{c\,d}{5} + \frac{b\,f}{5} + \frac{a\,h}{5}\right)\,x^5 + \left(\frac{b\,e}{4} + \frac{a\,g}{4}\right)\,x^4 + \left(\frac{b\,d}{3} + \frac{a\,f}{3}\right)\,x^3 + \frac{a\,e\,x^2}{2} + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5),x)
```

```
[Out] x^5*((c*d)/5 + (b*f)/5 + (a*h)/5) + x^6*((c*e)/6 + (b*g)/6 + (a*i)/6) + x^3
*((b*d)/3 + (a*f)/3) + x^4*((b*e)/4 + (a*g)/4) + x^7*((c*f)/7 + (b*h)/7) +
x^8*((c*g)/8 + (b*i)/8) + (c*h*x^9)/9 + (c*i*x^10)/10 + a*d*x + (a*e*x^2)/2
```

3.6 $\int (d+ex)\left(a+bx^2+cx^4\right)^2\, dx$

Optimal. Leaf size=112

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2+2ac\right)dx^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)*d*x^5+1/6
*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*d*x^9+1/10*c^2*e*x^1
0
```

Rubi [A]
time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1685\}$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2 + 2*a*c)*
d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 + (b*c*e*x^8)/4 + (c^2
*d*x^9)/9 + (c^2*e*x^10)/10
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(a+bx^2+cx^4\right)^2\, dx &= \int \left(a^2d+a^2ex+2abdx^2+2abex^3+\left(b^2+2ac\right)dx^4+\left(b^2+2ac\right)ex^5+2b\right.\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2+2ac\right)dx^5+\frac{1}{6}\left(b^2+2ac\right)ex^6\end{aligned}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 0.87

$$\frac{630a^2x(2d+ex)+42b^2x^5(6d+5ex)+45bcx^7(8d+7ex)+14c^2x^9(10d+9ex)+42a\left(5bx^3(4d+3ex)+2cx^5(6d+5ex)\right)}{1260}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (630*a^2*x*(2*d + e*x) + 42*b^2*x^5*(6*d + 5*e*x) + 45*b*c*x^7*(8*d + 7*e*x
) + 14*c^2*x^9*(10*d + 9*e*x) + 42*a*(5*b*x^3*(4*d + 3*e*x) + 2*c*x^5*(6*d
+ 5*e*x)))/1260
```

Maple [A]
time = 0.06, size = 95, normalized size = 0.85

method	result
default	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abd\,x^3}{3}+\frac{abe\,x^4}{2}+\frac{(2ac+b^2)\,d\,x^5}{5}+\frac{(2ac+b^2)\,e\,x^6}{6}+\frac{2bcd\,x^7}{7}+\frac{bce\,x^8}{4}+\frac{c^2d\,x^9}{9}+\frac{c^2e\,x^{10}}{10}$
norman	$\frac{c^2e\,x^{10}}{10}+\frac{c^2d\,x^9}{9}+\frac{bce\,x^8}{4}+\frac{2bcd\,x^7}{7}+\left(\frac{1}{3}ace+\frac{1}{6}b^2e\right)x^6+\left(\frac{2}{5}acd+\frac{1}{5}b^2d\right)x^5+\frac{abe\,x^4}{2}+\frac{2abd\,x^3}{3}+\frac{a^2e\,x^2}{2}$ -
gosper	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}bcd\,x^7+\frac{1}{3}x^6ace+\frac{1}{6}x^6b^2e+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5b^2d+\frac{1}{2}abe\,x^4+\frac{2}{3}ab$
risch	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}bcd\,x^7+\frac{1}{3}x^6ace+\frac{1}{6}x^6b^2e+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5b^2d+\frac{1}{2}abe\,x^4+\frac{2}{3}ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)*d*x^5+1/6
*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*d*x^9+1/10*c^2*e*x^1
0
```

Maxima [A]
time = 0.28, size = 102, normalized size = 0.91

$$\frac{1}{10}\,c^2x^{10}e+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcx^8e+\frac{2}{7}\,bcdx^7+\frac{1}{5}\,(b^2+2\,ac)dx^5+\frac{1}{6}\,(b^2e+2\,ace)x^6+\frac{1}{2}\,abx^4e+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/5*(b^2
+ 2*a*c)*d*x^5 + 1/6*(b^2*e + 2*a*c*e)*x^6 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3
+ 1/2*a^2*x^2*e + a^2*d*x
```

Fricas [A]
time = 0.40, size = 94, normalized size = 0.84

$$\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{6}\,(b^2+2\,ac)ex^6+\frac{1}{2}\,abex^4+\frac{1}{5}\,(b^2+2\,ac)dx^5+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 + 1/6*(b^2
+ 2*a*c)*e*x^6 + 1/2*a*b*e*x^4 + 1/5*(b^2 + 2*a*c)*d*x^5 + 2/3*a*b*d*x^3 +
1/2*a^2*e*x^2 + a^2*d*x
```

Sympy [A]
time = 0.01, size = 116, normalized size = 1.04

$$a^2dx+\frac{a^2ex^2}{2}+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{2bcdx^7}{7}+\frac{bcex^8}{4}+\frac{c^2dx^9}{9}+\frac{c^2ex^{10}}{10}+x^6\left(\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\cdot\left(\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x**3/3 + a*b*e*x**4/2 + 2*b*c*d*x**7/7 +
 b*c*e*x**8/4 + c**2*d*x**9/9 + c**2*e*x**10/10 + x**6*(a*c*e/3 + b**2*e/6)
 + x**5*(2*a*c*d/5 + b**2*d/5)
```

Giac [A]
time = 3.68, size = 106, normalized size = 0.95

$$\frac{1}{10}c^2x^{10}e+\frac{1}{9}c^2dx^9+\frac{1}{4}bcx^8e+\frac{2}{7}bcdx^7+\frac{1}{6}b^2x^6e+\frac{1}{3}acx^6e+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5+\frac{1}{2}abx^4e+\frac{2}{3}abdx^3+\frac{1}{2}a^2x^2e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 1/4*b*c*x^8*e + 2/7*b*c*d*x^7 + 1/6*b^2*x
^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 1/2*a*b*x^4*e + 2/3*
a*b*d*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

Mupad [B]
time = 0.06, size = 94, normalized size = 0.84

$$\frac{a^2\,e\,x^2}{2}+\frac{c^2\,d\,x^9}{9}+\frac{c^2\,e\,x^{10}}{10}+\frac{d\,x^5\,\left(b^2+2\,a\,c\right)}{5}+\frac{e\,x^6\,\left(b^2+2\,a\,c\right)}{6}+a^2\,d\,x+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}+\frac{2\,b\,c\,d\,x^7}{7}+\frac{b\,c\,e\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (a^2*e*x^2)/2 + (c^2*d*x^9)/9 + (c^2*e*x^10)/10 + (d*x^5*(2*a*c + b^2))/5 +
 (e*x^6*(2*a*c + b^2))/6 + a^2*d*x + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (2*b
*c*d*x^7)/7 + (b*c*e*x^8)/4
```

3.7 $\int (d+ex+fx^2)(a+bx^2+cx^4)^2 \, dx$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right.$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/2*a*b*e*x^4+1/5*(2*a*b*f+2*a*
c*d+b^2*d)*x^5+1/6*(2*a*c+b^2)*e*x^6+1/7*(2*a*c*f+b^2*f+2*b*c*d)*x^7+1/4*b*
c*e*x^8+1/9*c*(2*b*f+c*d)*x^9+1/10*c^2*e*x^10+1/11*c^2*f*x^11
```

Rubi [A]
time = 0.09, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1671}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)\left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2d+a^2ex+a(2bd+af)x^2+2abex^3+\left(b^2d+2acd+2abf\right)\right.$$
$$= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2a\right.$$

Mathematica [A]
time = 0.03, size = 154, normalized size = 1.00

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right)x^7+\frac{1}{4}bcex^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Maple [A]
time = 0.09, size = 139, normalized size = 0.90

method	result
default	$\frac{c^{2} f \,x^{11}}{11}+\frac{c^{2} e \,x^{10}}{10}+\frac{\left(2 f b c +c^{2} d \right) x^{9}}{9}+\frac{b c e \,x^{8}}{4}+\frac{\left(2 b c d +f \left(2 a c +b^{2}\right)\right) x^{7}}{7}+\frac{\left(2 a c +b^{2}\right) e \,x^{6}}{6}+\frac{\left(d \left(2 a c +b^{2}\right)+2 a b f \right) x^{5}}{5}+\frac{a b e \,x^{4}}{2}+$
norman	$\frac{c^{2} f \,x^{11}}{11}+\frac{c^{2} e \,x^{10}}{10}+\left(\frac{2}{9} f b c +\frac{1}{9} c^{2} d \right) x^{9}+\frac{b c e \,x^{8}}{4}+\left(\frac{2}{7} a c f +\frac{1}{7} b^{2} f +\frac{2}{7} b c d \right) x^{7}+\left(\frac{1}{3} a c e +\frac{1}{6} b^{2} e \right) x^{6}+\left(\frac{2}{5} a b\right.$
gosper	$\frac{1}{11} c^{2} f \,x^{11}+\frac{1}{10} c^{2} e \,x^{10}+\frac{2}{9} x^{9} f b c +\frac{1}{9} c^{2} d \,x^{9}+\frac{1}{4} b c e \,x^{8}+\frac{2}{7} x^{7} a c f +\frac{1}{7} x^{7} b^{2} f +\frac{2}{7} b c d \,x^{7}+\frac{1}{3} x^{6} a c e +\frac{1}{6} x^{6}$
risch	$\frac{1}{11} c^{2} f \,x^{11}+\frac{1}{10} c^{2} e \,x^{10}+\frac{2}{9} x^{9} f b c +\frac{1}{9} c^{2} d \,x^{9}+\frac{1}{4} b c e \,x^{8}+\frac{2}{7} x^{7} a c f +\frac{1}{7} x^{7} b^{2} f +\frac{2}{7} b c d \,x^{7}+\frac{1}{3} x^{6} a c e +\frac{1}{6} x^{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(2*b*c*f+c^2*d)*x^9+1/4*b*c*e*x^8+1/7*(
2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*(2*a*c+b^2)*e*x^6+1/5*(d*(2*a*c+b^2)+2*a*b*f
)*x^5+1/2*a*b*e*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A]
time = 0.28, size = 146, normalized size = 0.95

$$\frac{1}{11} \, c^{2} f x^{11} + \frac{1}{10} \, c^{2} x^{10} e + \frac{1}{4} \, b c x^{8} e + \frac{1}{9} \, {\left(c^{2} d + 2 \, b c f\right)} x^{9} + \frac{1}{7} \, {\left(2 \, b c d + {\left(b^{2} + 2 \, a c\right)} f\right)} x^{7} + \frac{1}{6} \, {\left(b^{2} e + 2 \, a c e\right)} x^{6} + \frac{1}{2} \, a b x^{4} e + \frac{1}{5} \, {\left(2 \, a b f + {\left(b^{2} + 2 \, a c\right)} d\right)} x^{5} + \frac{1}{2} \, a^{2} x^{2} e + a^{2} d x + \frac{1}{3} \, {\left(2 \, a b d + a^{2} f\right)} x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/4*b*c*x^8*e + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/6*(b^2*e + 2*a*c*e)*x^6 + 1/2*
a*b*x^4*e + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*x^2*e + a^2*d*x +
 1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A]
time = 0.35, size = 138, normalized size = 0.90

$$\frac{1}{11} \, c^{2} f x^{11} + \frac{1}{10} \, c^{2} e x^{10} + \frac{1}{4} \, b c e x^{8} + \frac{1}{9} \, {\left(c^{2} d + 2 \, b c f\right)} x^{9} + \frac{1}{6} \, {\left(b^{2} + 2 \, a c\right)} e x^{6} + \frac{1}{7} \, {\left(2 \, b c d + {\left(b^{2} + 2 \, a c\right)} f\right)} x^{7} + \frac{1}{2} \, a b e x^{4} + \frac{1}{5} \, {\left(2 \, a b f + {\left(b^{2} + 2 \, a c\right)} d\right)} x^{5} + \frac{1}{2} \, a^{2} e x^{2} + a^{2} d x + \frac{1}{3} \, {\left(2 \, a b d + a^{2} f\right)} x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*a*
b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x + 1
/3*(2*a*b*d + a^2*f)*x^3
```

Sympy [**A**]
time = 0.02, size = 165, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{abex^4}{2} + \frac{bcex^8}{4} + \frac{c^2ex^{10}}{10} + \frac{c^2fx^{11}}{11} + x^9 \cdot \left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^7 \cdot \left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x^5 \cdot \left(\frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/7
+ 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b*
*2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [**A**]
time = 4.24, size = 157, normalized size = 1.02

$$\frac{1}{11}c^2fx^{11} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abfx^5 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*c
*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*x^6*e + 1/
3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e
 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

Mupad [**B**]
time = 0.70, size = 138, normalized size = 0.90

$$x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,f\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{f\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,f}{7}\right)+x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,f\,c}{9}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,e\,x^{10}}{10}+\frac{c^2\,f\,x^{11}}{11}+\frac{e\,x^6\,\left(b^2+2\,a\,c\right)}{6}+a^2\,d\,x+\frac{a\,b\,e\,x^4}{2}+\frac{b\,c\,e\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^7*((b^2*f)/7 + (2*b*c*d)/7
+ (2*a*c*f)/7) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*f)
/9) + (a^2*e*x^2)/2 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11 + (e*x^6*(2*a*c + b
^2))/6 + a^2*d*x + (a*b*e*x^4)/2 + (b*c*e*x^8)/4
```

3.8 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=196

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2e+2ace+2abg\right)x^6+\frac{1}{7}(2bcd$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/4*a*(a*g+2*b*e)*x^4+1/5*(2*a*
b*f+2*a*c*d+b^2*d)*x^5+1/6*(2*a*b*g+2*a*c*e+b^2*e)*x^6+1/7*(2*a*c*f+b^2*f+2
*b*c*d)*x^7+1/8*(2*a*c*g+b^2*g+2*b*c*e)*x^8+1/9*c*(2*b*f+c*d)*x^9+1/10*c*(2
*b*g+c*e)*x^10+1/11*c^2*f*x^11+1/12*c^2*g*x^12
```

Rubi [A]
time = 0.12, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {1685}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}c^2fx^{11}+\frac{1}{12}c^2gx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*g)*x^6)/
6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)
/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11
+ (c^2*g*x^12)/12
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d+a^2ex+a(2bd+af)x^2+a(2be+ag)x^3+\left(b^2d+2\right.\right.\\ &= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 196, normalized size = 1.00

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2e+2ace+2abg\right)x^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right)x^7+\frac{1}{8}\left(2bce+b^2g+2acg\right)x^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c(ce+2bg)x^{10}+\frac{1}{11}c^2fx^{11}+\frac{1}{12}c^2gx^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*g)*x^6)/
6 + ((2*b*c*d + b^2*f + 2*a*c*f)*x^7)/7 + ((2*b*c*e + b^2*g + 2*a*c*g)*x^8)
/8 + (c*(c*d + 2*b*f)*x^9)/9 + (c*(c*e + 2*b*g)*x^10)/10 + (c^2*f*x^11)/11
+ (c^2*g*x^12)/12
```

Maple [A]
time = 0.09, size = 183, normalized size = 0.93

method	result
default	$\frac{c^2 g\,x^{12}}{12}+\frac{c^2 f\,x^{11}}{11}+\frac{(2bcg+c^2e)x^{10}}{10}+\frac{(2fbc+c^2d)x^9}{9}+\frac{(2bce+g(2ac+b^2))x^8}{8}+\frac{(2bcd+f(2ac+b^2))x^7}{7}+\frac{(e(2ac+b^2)+2a}{6}$
norman	$\frac{c^2 g\,x^{12}}{12}+\frac{c^2 f\,x^{11}}{11}+\left(\frac{1}{5}bcg+\frac{1}{10}c^2e\right)x^{10}+\left(\frac{2}{9}fbc+\frac{1}{9}c^2d\right)x^9+\left(\frac{1}{4}acg+\frac{1}{8}b^2g+\frac{1}{4}bce\right)x^8+\left(\frac{2}{7}acf+\right.$
gosper	$\frac{1}{12}c^2g\,x^{12}+\frac{1}{11}c^2f\,x^{11}+\frac{1}{5}x^{10}bcg+\frac{1}{10}c^2e\,x^{10}+\frac{2}{9}x^9fbc+\frac{1}{9}c^2d\,x^9+\frac{1}{4}x^8acg+\frac{1}{8}x^8b^2g+\frac{1}{4}bce\,x^8+$
risch	$\frac{1}{12}c^2g\,x^{12}+\frac{1}{11}c^2f\,x^{11}+\frac{1}{5}x^{10}bcg+\frac{1}{10}c^2e\,x^{10}+\frac{2}{9}x^9fbc+\frac{1}{9}c^2d\,x^9+\frac{1}{4}x^8acg+\frac{1}{8}x^8b^2g+\frac{1}{4}bce\,x^8+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c^2*g*x^12+1/11*c^2*f*x^11+1/10*(2*b*c*g+c^2*e)*x^10+1/9*(2*b*c*f+c^2*
d)*x^9+1/8*(2*b*c*e+g*(2*a*c+b^2))*x^8+1/7*(2*b*c*d+f*(2*a*c+b^2))*x^7+1/6*
(e*(2*a*c+b^2)+2*a*b*g)*x^6+1/5*(d*(2*a*c+b^2)+2*a*b*f)*x^5+1/4*(a^2*g+2*a*
b*e)*x^4+1/3*(a^2*f+2*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A]
time = 0.28, size = 188, normalized size = 0.96

$$\frac{1}{12}\,c^2gx^{12}+\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,(2\,bcg+c^2e)x^{10}+\frac{1}{9}\,(c^2d+2\,bcf)x^9+\frac{1}{8}\,(2\,bce+(b^2+2\,ac)g)x^8+\frac{1}{7}\,(2\,bcd+(b^2+2\,ac)f)x^7+\frac{1}{6}\,(2\,abg+b^2e+2\,ace)x^6+\frac{1}{5}\,(2\,abf+(b^2+2\,ac)d)x^5+\frac{1}{4}\,(a^2g+2\,abe)x^4+\frac{1}{2}\,a^2x^2e+a^2dx+\frac{1}{3}\,(2\,abd+a^2f)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/10*(2*b*c*g + c^2*e)*x^10 + 1/9*(c^2*
d + 2*b*c*f)*x^9 + 1/8*(2*b*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + (b^
2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b*g + b^2*e + 2*a*c*e)*x^6 + 1/5*(2*a*b*f + (b
^2 + 2*a*c)*d)*x^5 + 1/4*(a^2*g + 2*a*b*e)*x^4 + 1/2*a^2*x^2*e + a^2*d*x +
1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A]
time = 0.38, size = 182, normalized size = 0.93

$$\frac{1}{12}\,c^2gx^{12}+\frac{1}{11}\,c^2fx^{11}+\frac{1}{10}\,(c^2e+2\,bcg)x^{10}+\frac{1}{9}\,(c^2d+2\,bcf)x^9+\frac{1}{8}\,(2\,bce+(b^2+2\,ac)g)x^8+\frac{1}{7}\,(2\,bcd+(b^2+2\,ac)f)x^7+\frac{1}{6}\,(2\,abg+(b^2+2\,ac)e)x^6+\frac{1}{5}\,(2\,abf+(b^2+2\,ac)d)x^5+\frac{1}{2}\,a^2ex^2+\frac{1}{4}\,(2\,abe+a^2g)x^4+a^2dx+\frac{1}{3}\,(2\,abd+a^2f)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/10*(c^2*e + 2*b*c*g)*x^10 + 1/9*(c^2*
d + 2*b*c*f)*x^9 + 1/8*(2*b*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + (b^
2 + 2*a*c)*f)*x^7 + 1/6*(2*a*b*g + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + (b
^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x +
1/3*(2*a*b*d + a^2*f)*x^3
```

Sympy [A]
time = 0.02, size = 209, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2fx^{11}}{11} + \frac{c^2gx^{12}}{12} + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\cdot\left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\cdot\left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{abg}{3} + \frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\cdot\left(\frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^4\left(\frac{a^2g}{4} + \frac{abe}{2}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*f*x**11/11 + c**2*g*x**12/12 + x**10*(b*c*g
/5 + c**2*e/10) + x**9*(2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 + b**2*g/8 +
b*c*e/4) + x**7*(2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*b*g/3 + a*c*e/
3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/4 +
a*b*e/2) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A]
time = 3.82, size = 208, normalized size = 1.06

$$\frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{1}{5}bcgx^{10} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{4}acgx^8 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{3}abgx^6 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abfx^5 + \frac{1}{4}a^2gx^4 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 1/5*b*c*g*x^10 + 1/10*c^2*x^10*e + 1/9*
c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x^8 + 1/4*b*c*x^8*e +
 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/3*a*b*g*x^6 + 1/6*b^2*x^
6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/4*a
^2*g*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e +
a^2*d*x
```

Mupad [B]
time = 0.72, size = 182, normalized size = 0.93

$$x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,f\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^6\left(\frac{e\,b^2}{6}+\frac{a\,g\,b}{3}+\frac{a\,c\,e}{3}\right)+x^7\left(\frac{f\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,f}{7}\right)+x^8\left(\frac{g\,b^2}{8}+\frac{c\,e\,b}{4}+\frac{a\,c\,g}{4}\right)+x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,f\,c}{9}\right)+x^{10}\left(\frac{e\,c^2}{10}+\frac{b\,g\,c}{5}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,f\,x^{11}}{11}+\frac{c^2\,g\,x^{12}}{12}+a^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3),x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^6*((b^2*e)/6 + (a*c*e)/3 +
(a*b*g)/3) + x^7*((b^2*f)/7 + (2*b*c*d)/7 + (2*a*c*f)/7) + x^8*((b^2*g)/8 +
 (b*c*e)/4 + (a*c*g)/4) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^4*((a^2*g)/4 +
(a*b*e)/2) + x^9*((c^2*d)/9 + (2*b*c*f)/9) + x^10*((c^2*e)/10 + (b*c*g)/5)
+ (a^2*e*x^2)/2 + (c^2*f*x^11)/11 + (c^2*g*x^12)/12 + a^2*d*x
```

3.9 $\int (a+bx^2+cx^4)^2 (d+ex+fx^2+gx^3+hx^4) \, dx$

Optimal. Leaf size=234

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2abf+a(2cd+ah)\right)x^5+\frac{1}{6}\left(b^2e+2ace+2abg\right)x^6+$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/4*a*(a*g+2*b*e)*x^4+1/5*(b^2*
d+2*a*b*f+a*(a*h+2*c*d))*x^5+1/6*(2*a*b*g+2*a*c*e+b^2*e)*x^6+1/7*(b^2*f+2*a
*c*f+2*b*(a*h+c*d))*x^7+1/8*(2*a*c*g+b^2*g+2*b*c*e)*x^8+1/9*(c^2*d+b^2*h+2*
c*(a*h+b*f))*x^9+1/10*c*(2*b*g+c*e)*x^10+1/11*c*(2*b*h+c*f)*x^11+1/12*c^2*g
*x^12+1/13*c^2*h*x^13
```

Rubi [A]
time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1685}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{9}x^9\left(2c(ah+bf)+b^2h+c^2d\right)+\frac{1}{7}x^7\left(2b(ah+cd)+2acf+b^2f\right)+\frac{1}{5}x^5\left(2abf+a(ah+2cd)+b^2d\right)+\frac{1}{8}x^8\left(2acg+b^2g+2bce\right)+\frac{1}{6}x^6\left(2abg+2ace+b^2e\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{4}ax^4(ag+2be)+\frac{1}{10}cx^{10}(2bg+ce)+\frac{1}{11}cx^{11}(2bh+cf)+\frac{1}{12}c^2gx^{12}+\frac{1}{13}c^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*b*f + a*(2*c*d + a*h))*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*
g)*x^6)/6 + ((b^2*f + 2*a*c*f + 2*b*(c*d + a*h))*x^7)/7 + ((2*b*c*e + b^2*g
 + 2*a*c*g)*x^8)/8 + ((c^2*d + b^2*h + 2*c*(b*f + a*h))*x^9)/9 + (c*(c*e +
2*b*g)*x^10)/10 + (c*(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*x^13
)/13
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^2 \left(d+ex+fx^2+gx^3+hx^4\right) \, dx = \int \left(a^2d+a^2ex+a(2bd+af)x^2+a(2be+ag)x^3+(b\right.$$
$$= a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+$$

Mathematica [A]
time = 0.06, size = 234, normalized size = 1.00

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{4}a(2be+ag)x^4+\frac{1}{5}\left(b^2d+2acd+2abf+a^2h\right)x^5+\frac{1}{6}\left(b^2e+2ace+2abg\right)x^6+\frac{1}{7}\left(2bcd+b^2f+2acf+2abh\right)x^7+\frac{1}{8}\left(2bce+b^2g+2acg\right)x^8+\frac{1}{9}\left(c^2d+2bcf+b^2h+2ach\right)x^9+\frac{1}{10}c(ce+2bg)x^{10}+\frac{1}{11}c(cf+2bh)x^{11}+\frac{1}{12}c^2gx^{12}+\frac{1}{13}c^2hx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*(2*b*e + a*g)*x^4)/4
 + ((b^2*d + 2*a*c*d + 2*a*b*f + a^2*h)*x^5)/5 + ((b^2*e + 2*a*c*e + 2*a*b*
g)*x^6)/6 + ((2*b*c*d + b^2*f + 2*a*c*f + 2*a*b*h)*x^7)/7 + ((2*b*c*e + b^2
*g + 2*a*c*g)*x^8)/8 + ((c^2*d + 2*b*c*f + b^2*h + 2*a*c*h)*x^9)/9 + (c*(c*
e + 2*b*g)*x^10)/10 + (c*(c*f + 2*b*h)*x^11)/11 + (c^2*g*x^12)/12 + (c^2*h*
x^13)/13
```

Maple [A]
time = 0.14, size = 219, normalized size = 0.94

method	result
default	$\frac{c^2 h\,x^{13}}{13}+\frac{c^2 g\,x^{12}}{12}+\frac{(2bch+c^2 f)x^{11}}{11}+\frac{(2bcg+c^2 e)x^{10}}{10}+\frac{((2ac+b^2)h+2fbc+c^2 d)x^9}{9}+\frac{(2bce+g(2ac+b^2))x^8}{8}+\frac{(2abh+f($
norman	$\frac{c^2 h\,x^{13}}{13}+\frac{c^2 g\,x^{12}}{12}+\left(\frac{2}{11}bch+\frac{1}{11}c^2 f\right)x^{11}+\left(\frac{1}{5}bcg+\frac{1}{10}c^2 e\right)x^{10}+\left(\frac{2}{9}ach+\frac{1}{9}b^2 h+\frac{2}{9}fbc+\frac{1}{9}c^2 d\right)x^9+$
gosper	$\frac{1}{6}x^6b^2e+\frac{1}{5}x^5b^2d+\frac{1}{7}x^7b^2f+\frac{1}{3}x^3a^2f+\frac{1}{8}x^8b^2g+\frac{1}{4}x^4g\,a^2+\frac{1}{5}x^{10}bcg+\frac{1}{4}x^8acg+\frac{2}{5}x^5abf+\frac{2}{7}x^7ac$
risch	$\frac{1}{6}x^6b^2e+\frac{1}{5}x^5b^2d+\frac{1}{7}x^7b^2f+\frac{1}{3}x^3a^2f+\frac{1}{8}x^8b^2g+\frac{1}{4}x^4g\,a^2+\frac{1}{5}x^{10}bcg+\frac{1}{4}x^8acg+\frac{2}{5}x^5abf+\frac{2}{7}x^7ac$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*h*x^13+1/12*c^2*g*x^12+1/11*(2*b*c*h+c^2*f)*x^11+1/10*(2*b*c*g+c^2
*e)*x^10+1/9*((2*a*c+b^2)*h+2*f*b*c+c^2*d)*x^9+1/8*(2*b*c*e+g*(2*a*c+b^2))*
x^8+1/7*(2*a*b*h+f*(2*a*c+b^2)+2*b*c*d)*x^7+1/6*(e*(2*a*c+b^2)+2*a*b*g)*x^6
+1/5*(a^2*h+2*a*b*f+d*(2*a*c+b^2))*x^5+1/4*(a^2*g+2*a*b*e)*x^4+1/3*(a^2*f+2
*a*b*d)*x^3+1/2*a^2*e*x^2+a^2*d*x
```

Maxima [A]
time = 0.27, size = 224, normalized size = 0.96

$$\frac{1}{13}c^2hx^{13}+\frac{1}{12}c^2gx^{12}+\frac{1}{11}\left(c^2f+2\,bch\right)x^{11}+\frac{1}{10}\left(2\,bcg+c^2e\right)x^{10}+\frac{1}{9}\left(c^2d+2\,bcf+\left(b^2+2\,ac\right)h\right)x^9+\frac{1}{8}\left(2\,bce+\left(b^2+2\,ac\right)g\right)x^8+\frac{1}{7}\left(2\,bcd+2\,abh+\left(b^2+2\,ac\right)f\right)x^7+\frac{1}{6}\left(2\,abg+b^2e+2\,ace\right)x^6+\frac{1}{5}\left(2\,abf+a^2h+\left(b^2+2\,ac\right)d\right)x^5+\frac{1}{4}\left(a^2g+2\,abe\right)x^4+\frac{1}{2}a^2x^2e+a^2dx+\frac{1}{3}\left(2\,abd+a^2f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*(c^2*f + 2*b*c*h)*x^11 + 1/10*(2*b
*c*g + c^2*e)*x^10 + 1/9*(c^2*d + 2*b*c*f + (b^2 + 2*a*c)*h)*x^9 + 1/8*(2*b
```

```
*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + 2*a*b*h + (b^2 + 2*a*c)*f)*x^7
 + 1/6*(2*a*b*g + b^2*e + 2*a*c*e)*x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*
c)*d)*x^5 + 1/4*(a^2*g + 2*a*b*e)*x^4 + 1/2*a^2*x^2*e + a^2*d*x + 1/3*(2*a*
b*d + a^2*f)*x^3
```

Fricas [A]
time = 0.39, size = 218, normalized size = 0.93

$$\frac{1}{13}c^2hx^{13} + \frac{1}{12}c^2gx^{12} + \frac{1}{11}(c^2f + 2bch)x^{11} + \frac{1}{10}(c^2e + 2bcg)x^{10} + \frac{1}{9}(c^2d + 2bcf + (b^2 + 2ac)h)x^9 + \frac{1}{8}(2bce + (b^2 + 2ac)g)x^8 + \frac{1}{7}(2bcd + 2abh + (b^2 + 2ac)f)x^7 + \frac{1}{6}(2abg + (b^2 + 2ac)e)x^6 + \frac{1}{5}(2abf + a^2h + (b^2 + 2ac)d)x^5 + \frac{1}{2}a^2ex^2 + \frac{1}{4}(2abe + a^2g)x^4 + a^2dx + \frac{1}{3}(2abd + a^2f)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*(c^2*f + 2*b*c*h)*x^11 + 1/10*(c^2
*e + 2*b*c*g)*x^10 + 1/9*(c^2*d + 2*b*c*f + (b^2 + 2*a*c)*h)*x^9 + 1/8*(2*b
*c*e + (b^2 + 2*a*c)*g)*x^8 + 1/7*(2*b*c*d + 2*a*b*h + (b^2 + 2*a*c)*f)*x^7
 + 1/6*(2*a*b*g + (b^2 + 2*a*c)*e)*x^6 + 1/5*(2*a*b*f + a^2*h + (b^2 + 2*a*
c)*d)*x^5 + 1/2*a^2*e*x^2 + 1/4*(2*a*b*e + a^2*g)*x^4 + a^2*d*x + 1/3*(2*a*
b*d + a^2*f)*x^3
```

Sympy [A]
time = 0.02, size = 258, normalized size = 1.10

$$a^2dx + \frac{a^2ex^2}{2} + \frac{c^2gx^{12}}{12} + \frac{c^2hx^{13}}{13} + x^{11}\cdot\left(\frac{2bch}{11} + \frac{c^2f}{11}\right) + x^{10}\left(\frac{bcg}{5} + \frac{c^2e}{10}\right) + x^9\cdot\left(\frac{2ach}{9} + \frac{b^2h}{9} + \frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^8\left(\frac{acg}{4} + \frac{b^2g}{8} + \frac{bce}{4}\right) + x^7\cdot\left(\frac{2abh}{7} + \frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{abg}{3} + \frac{ace}{3} + \frac{b^2e}{6}\right) + x^5\left(\frac{a^2h}{5} + \frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^4\left(\frac{a^2g}{4} + \frac{abe}{2}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2*(h*x**4+g*x**3+f*x**2+e*x+d),x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + c**2*g*x**12/12 + c**2*h*x**13/13 + x**11*(2*b*c
*h/11 + c**2*f/11) + x**10*(b*c*g/5 + c**2*e/10) + x**9*(2*a*c*h/9 + b**2*h
/9 + 2*b*c*f/9 + c**2*d/9) + x**8*(a*c*g/4 + b**2*g/8 + b*c*e/4) + x**7*(2*
a*b*h/7 + 2*a*c*f/7 + b**2*f/7 + 2*b*c*d/7) + x**6*(a*b*g/3 + a*c*e/3 + b**
2*e/6) + x**5*(a**2*h/5 + 2*a*b*f/5 + 2*a*c*d/5 + b**2*d/5) + x**4*(a**2*g/
4 + a*b*e/2) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A]
time = 4.48, size = 259, normalized size = 1.11

$$\frac{1}{13}c^2hx^{13} + \frac{1}{12}c^2gx^{12} + \frac{1}{11}c^2fx^{11} + \frac{2}{11}bchx^{11} + \frac{1}{5}bcgx^{10} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{9}b^2hx^9 + \frac{2}{9}achx^9 + \frac{1}{8}b^2gx^8 + \frac{1}{4}acgx^8 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{2}{7}abhx^7 + \frac{1}{3}abgx^6 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abfx^5 + \frac{1}{5}a^2hx^5 + \frac{1}{4}a^2gx^4 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13*c^2*h*x^13 + 1/12*c^2*g*x^12 + 1/11*c^2*f*x^11 + 2/11*b*c*h*x^11 + 1/5
*b*c*g*x^10 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/9*b^2*h*x
```

```
^9 + 2/9*a*c*h*x^9 + 1/8*b^2*g*x^8 + 1/4*a*c*g*x^8 + 1/4*b*c*x^8*e + 2/7*b*
c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 2/7*a*b*h*x^7 + 1/3*a*b*g*x^6 + 1
/6*b^2*x^6*e + 1/3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^
5 + 1/5*a^2*h*x^5 + 1/4*a^2*g*x^4 + 1/2*a*b*x^4*e + 2/3*a*b*d*x^3 + 1/3*a^2
*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

Mupad [B]
time = 0.11, size = 220, normalized size = 0.94

$$x^6\left(\frac{e\,b^2}{6}+\frac{a\,g\,b}{3}+\frac{a\,c\,e}{3}\right)+x^8\left(\frac{g\,b^2}{8}+\frac{c\,e\,b}{4}+\frac{a\,c\,g}{4}\right)+x^3\left(\frac{f\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^4\left(\frac{g\,a^2}{4}+\frac{b\,e\,a}{2}\right)+x^{10}\left(\frac{e\,c^2}{10}+\frac{b\,g\,c}{5}\right)+x^{11}\left(\frac{f\,c^2}{11}+\frac{2\,b\,h\,c}{11}\right)+x^5\left(\frac{h\,a^2}{5}+\frac{2\,f\,a\,b}{5}+\frac{2\,c\,d\,a}{5}+\frac{d\,b^2}{5}\right)+x^7\left(\frac{b^2\,f}{7}+\frac{2\,b\,c\,d}{7}+\frac{2\,a\,c\,f}{7}+\frac{2\,a\,b\,h}{7}\right)+x^9\left(\frac{h\,b^2}{9}+\frac{2\,f\,b\,c}{9}+\frac{d\,c^2}{9}+\frac{2\,a\,h\,c}{9}\right)+\frac{a^2\,e\,x^2}{2}+\frac{c^2\,g\,x^{12}}{12}+\frac{c^2\,h\,x^{13}}{13}+a^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2*(d + e*x + f*x^2 + g*x^3 + h*x^4),x)
```

```
[Out] x^6*((b^2*e)/6 + (a*c*e)/3 + (a*b*g)/3) + x^8*((b^2*g)/8 + (b*c*e)/4 + (a*c
*g)/4) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^4*((a^2*g)/4 + (a*b*e)/2) + x^10
*((c^2*e)/10 + (b*c*g)/5) + x^11*((c^2*f)/11 + (2*b*c*h)/11) + x^5*((b^2*d)
/5 + (a^2*h)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^7*((b^2*f)/7 + (2*b*c*d)/7
+ (2*a*c*f)/7 + (2*a*b*h)/7) + x^9*((c^2*d)/9 + (b^2*h)/9 + (2*b*c*f)/9 + (
2*a*c*h)/9) + (a^2*e*x^2)/2 + (c^2*g*x^12)/12 + (c^2*h*x^13)/13 + a^2*d*x
```

3.10 $\int \frac{d+ex}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=45

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -1/6*d*arctanh(1/2*x)+1/3*d*arctanh(x)-1/6*e*ln(-x^2+1)+1/6*e*ln(-x^2+4)
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1687, 12, 1107, 213, 1121, 630, 31}

$$-\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/6*(d*ArcTanh[x/2]) + (d*ArcTanh[x])/3 - (e*Log[1 - x^2])/6 + (e*Log[4 -
x^2])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{4-5x^2+x^4}\,dx &= \int \frac{d}{4-5x^2+x^4}\,dx+\int \frac{ex}{4-5x^2+x^4}\,dx\\ &= d\int \frac{1}{4-5x^2+x^4}\,dx+e\int \frac{x}{4-5x^2+x^4}\,dx\\ &= \frac{1}{3}d\int \frac{1}{-4+x^2}\,dx-\frac{1}{3}d\int \frac{1}{-1+x^2}\,dx+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx,x,x^2\right)\\ &= -\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)+\frac{1}{6}e\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)-\frac{1}{6}e\text{Subst}\left(\int \frac{}{}\right.\\ &= -\frac{1}{6}d\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}d\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.11

$$\frac{1}{12}(-2(d+e)\log(1-x)+(d+2e)\log(2-x)+2(d-e)\log(1+x)-(d-2e)\log(2+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e)*Log[1 - x] + (d + 2*e)*Log[2 - x] + 2*(d - e)*Log[1 + x] - (d -
 2*e)*Log[2 + x])/12
```

Maple [A]
time = 0.02, size = 50, normalized size = 1.11

method	result	size
default	$\left(-\frac{d}{12}+\frac{e}{6}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)$	50
norman	$\left(-\frac{d}{12}+\frac{e}{6}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)$	50
risch	$-\frac{\ln(1-x)d}{6}-\frac{\ln(1-x)e}{6}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}-\frac{\ln(x+2)d}{12}+\frac{\ln(x+2)e}{6}+\frac{\ln(2-x)d}{12}+\frac{\ln(2-x)e}{6}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12*d+1/6*e)*ln(x+2)+(1/12*d+1/6*e)*ln(x-2)+(-1/6*d-1/6*e)*ln(-1+x)+(1/6
*d-1/6*e)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.04

$$-\frac{1}{12}\,{\left(d-2\,e\right)}\log\left(x+2\right)+\frac{1}{6}\,{\left(d-e\right)}\log\left(x+1\right)-\frac{1}{6}\,{\left(d+e\right)}\log\left(x-1\right)+\frac{1}{12}\,{\left(d+2\,e\right)}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)*log(x - 1
) + 1/12*(d + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.39, size = 43, normalized size = 0.96

$$-\frac{1}{12}\,{\left(d-2\,e\right)}\log\left(x+2\right)+\frac{1}{6}\,{\left(d-e\right)}\log\left(x+1\right)-\frac{1}{6}\,{\left(d+e\right)}\log\left(x-1\right)+\frac{1}{12}\,{\left(d+2\,e\right)}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/6*(d + e)*log(x - 1
) + 1/12*(d + 2*e)*log(x - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. 2(34) = 68.
time = 1.80, size = 515, normalized size = 11.44

$$-\frac{(d-2e)\log\left(x+\frac{-35d^4e+\frac{51d^4(d-2e)}{2}-180d^2e^3-90d^2e^2(d-2e)+41d^2e(d-2e)^2-\frac{15d^2(d-2e)^3}{2}+320e^5-96e^4(d-2e)-80e^3(d-2e)^2+24e^2(d-2e)^3}{9d^5-160d^3e^2+256de^4}\right)}{12}+\frac{(d-e)\log\left(x+\frac{-35d^4e-51d^4(d-e)-180d^2e^3+180d^2e^2(d-e)+164d^2e(d-e)^2+60d^2(d-e)^3+320e^5+192e^4(d-e)-320e^3(d-e)^2-192e^2(d-e)^3}{9d^5-160d^3e^2+256de^4}\right)}{6}-\frac{(d+e)\log\left(x+\frac{-35d^4e+51d^4(d+e)-180d^2e^3-180d^2e^2(d+e)+164d^2e(d+e)^2-60d^2(d+e)^3+320e^5-192e^4(d+e)-320e^3(d+e)^2+192e^2(d+e)^3}{9d^5-160d^3e^2+256de^4}\right)}{6}+\frac{(d+2e)\log\left(x+\frac{-35d^4e-\frac{51d^4(d+2e)}{2}-180d^2e^3+90d^2e^2(d+2e)+41d^2e(d+2e)^2+\frac{15d^2(d+2e)^3}{2}+320e^5+96e^4(d+2e)-80e^3(d+2e)^2-24e^2(d+2e)^3}{9d^5-160d^3e^2+256de^4}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e)*log(x + (-35*d**4*e + 51*d**4*(d - 2*e)/2 - 180*d**2*e**3 - 90*d
**2*e**2*(d - 2*e) + 41*d**2*e*(d - 2*e)**2 - 15*d**2*(d - 2*e)**3/2 + 320*
e**5 - 96*e**4*(d - 2*e) - 80*e**3*(d - 2*e)**2 + 24*e**2*(d - 2*e)**3)/(9*
d**5 - 160*d**3*e**2 + 256*d*e**4))/12 + (d - e)*log(x + (-35*d**4*e - 51*d
**4*(d - e) - 180*d**2*e**3 + 180*d**2*e**2*(d - e) + 164*d**2*e*(d - e)**2
 + 60*d**2*(d - e)**3 + 320*e**5 + 192*e**4*(d - e) - 320*e**3*(d - e)**2 -
 192*e**2*(d - e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/6 - (d + e)*lo
g(x + (-35*d**4*e + 51*d**4*(d + e) - 180*d**2*e**3 - 180*d**2*e**2*(d + e)
 + 164*d**2*e*(d + e)**2 - 60*d**2*(d + e)**3 + 320*e**5 - 192*e**4*(d + e)
 - 320*e**3*(d + e)**2 + 192*e**2*(d + e)**3)/(9*d**5 - 160*d**3*e**2 + 256
*d*e**4))/6 + (d + 2*e)*log(x + (-35*d**4*e - 51*d**4*(d + 2*e)/2 - 180*d**
2*e**3 + 90*d**2*e**2*(d + 2*e) + 41*d**2*e*(d + 2*e)**2 + 15*d**2*(d + 2*e
)**3/2 + 320*e**5 + 96*e**4*(d + 2*e) - 80*e**3*(d + 2*e)**2 - 24*e**2*(d +
 2*e)**3)/(9*d**5 - 160*d**3*e**2 + 256*d*e**4))/12
```

Giac [A]
time = 4.85, size = 51, normalized size = 1.13

$$-\frac{1}{12}\left(d-2\,e\right)\log\left(\left|x+2\right|\right)+\frac{1}{6}\left(d-e\right)\log\left(\left|x+1\right|\right)-\frac{1}{6}\left(d+e\right)\log\left(\left|x-1\right|\right)+\frac{1}{12}\left(d+2\,e\right)\log\left(\left|x-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d - 2*e)*log(abs(x + 2)) + 1/6*(d - e)*log(abs(x + 1)) - 1/6*(d + e)
*log(abs(x - 1)) + 1/12*(d + 2*e)*log(abs(x - 2))
```

Mupad [B]
time = 0.71, size = 51, normalized size = 1.13

$$\ln\left(x+1\right)\,\left(\frac{d}{6}-\frac{e}{6}\right)-\ln\left(x-1\right)\,\left(\frac{d}{6}+\frac{e}{6}\right)+\ln\left(x-2\right)\,\left(\frac{d}{12}+\frac{e}{6}\right)-\ln\left(x+2\right)\,\left(\frac{d}{12}-\frac{e}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)*(d/6 - e/6) - log(x - 1)*(d/6 + e/6) + log(x - 2)*(d/12 + e/6) -
 log(x + 2)*(d/12 - e/6)
```

3.11 $\int \frac{d+ex+fx^2}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

```
[Out] -1/6*(d+4*f)*arctanh(1/2*x)+1/3*(d+f)*arctanh(x)-1/6*e*ln(-x^2+1)+1/6*e*ln(
-x^2+4)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1687, 1180, 213, 12, 1121, 630, 31}

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}e\log\left(1-x^2\right)+\frac{1}{6}e\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/6*((d + 4*f)*ArcTanh[x/2]) + ((d + f)*ArcTanh[x])/3 - (e*Log[1 - x^2])/6
 + (e*Log[4 - x^2])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{4-5x^2+x^4}\,dx &= \int \frac{ex}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2}{4-5x^2+x^4}\,dx \\
&= e\int \frac{x}{4-5x^2+x^4}\,dx - \frac{1}{3}(d+f)\int \frac{1}{-1+x^2}\,dx + \frac{1}{3}(d+4f)\int \frac{1}{-4+x^2}\,dx \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx, x, x^2\right. \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{6}e\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right) - \\
&= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) - \frac{1}{6}e\log\left(1-x^2\right) + \frac{1}{6}e\log\left(4-x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 58, normalized size = 1.14

$$\frac{1}{12}(-2(d+e+f)\log(1-x)+(d+2e+4f)\log(2-x)+2(d-e+f)\log(1+x)-(d-2e+4f)\log(2+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] + 2*(d - e + f)*Log
[1 + x] - (d - 2*e + 4*f)*Log[2 + x])/12
```

Maple [A]
time = 0.03, size = 62, normalized size = 1.22

method	result
default	$\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln$
norman	$\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln$
risch	$\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(x+2)d}{12}+\frac{\ln(x+2)e}{6}-\frac{\ln(x+2)f}{3}-\frac{\ln(1-x)d}{6}-\frac{\ln(1-x)e}{6}-\frac{\ln(1-x)f}{6}+\frac{\ln(2-x}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12*d+1/6*e-1/3*f)*ln(x+2)+(1/12*d+1/6*e+1/3*f)*ln(x-2)+(-1/6*d-1/6*e-1/
6*f)*ln(-1+x)+(1/6*d-1/6*e+1/6*f)*ln(1+x)
```

Maxima [A]
time = 0.27, size = 55, normalized size = 1.08

$$-\frac{1}{12}\left(d+4\,f-2\,e\right)\log\left(x+2\right)+\frac{1}{6}\left(d+f-e\right)\log\left(x+1\right)-\frac{1}{6}\left(d+f+e\right)\log\left(x-1\right)+\frac{1}{12}\left(d+4\,f+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d + 4*f - 2*e)*log(x + 2) + 1/6*(d + f - e)*log(x + 1) - 1/6*(d + f
+ e)*log(x - 1) + 1/12*(d + 4*f + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.42, size = 51, normalized size = 1.00

$$-\frac{1}{12}\left(d-2\,e+4\,f\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{6}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2\,e+4\,f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f)*log(x + 2) + 1/6*(d - e + f)*log(x + 1) - 1/6*(d + e
+ f)*log(x - 1) + 1/12*(d + 2*e + 4*f)*log(x - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. $2(44) = 88$.
time = 97.91, size = 2195, normalized size = 43.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] -(d - 2*e + 4*f)*log(x + (-35*d**5*e + 51*d**5*(d - 2*e + 4*f)/2 - 820*d**4
*e*f + 90*d**4*f*(d - 2*e + 4*f) - 180*d**3*e**3 - 90*d**3*e**2*(d - 2*e +
4*f) - 4100*d**3*e*f**2 + 41*d**3*e*(d - 2*e + 4*f)**2 + 42*d**3*f**2*(d -
2*e + 4*f) - 15*d**3*(d - 2*e + 4*f)**3/2 - 432*d**2*e**2*f*(d - 2*e + 4*f)
 - 8000*d**2*e*f**3 + 240*d**2*e*f*(d - 2*e + 4*f)**2 - 240*d**2*f**3*(d -
2*e + 4*f) - 12*d**2*f*(d - 2*e + 4*f)**3 + 320*d*e**5 - 96*d*e**4*(d - 2*e
 + 4*f) + 720*d*e**3*f**2 - 80*d*e**3*(d - 2*e + 4*f)**2 - 1080*d*e**2*f**2
*(d - 2*e + 4*f) + 24*d*e**2*(d - 2*e + 4*f)**3 - 6400*d*e*f**4 + 492*d*e*f
**2*(d - 2*e + 4*f)**2 - 576*d*f**4*(d - 2*e + 4*f) + 30*d*f**2*(d - 2*e +
4*f)**3 + 512*e**5*f - 128*e**3*f*(d - 2*e + 4*f)**2 - 576*e**2*f**3*(d - 2
*e + 4*f) - 1472*e*f**5 + 320*e*f**3*(d - 2*e + 4*f)**2 - 480*f**5*(d - 2*e
 + 4*f) + 48*f**3*(d - 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 -
 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**
2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5
 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/12 + (d - e + f)*log(x + (-
35*d**5*e - 51*d**5*(d - e + f) - 820*d**4*e*f - 180*d**4*f*(d - e + f) - 1
80*d**3*e**3 + 180*d**3*e**2*(d - e + f) - 4100*d**3*e*f**2 + 164*d**3*e*(d
 - e + f)**2 - 84*d**3*f**2*(d - e + f) + 60*d**3*(d - e + f)**3 + 864*d**2
*e**2*f*(d - e + f) - 8000*d**2*e*f**3 + 960*d**2*e*f*(d - e + f)**2 + 480*
d**2*f**3*(d - e + f) + 96*d**2*f*(d - e + f)**3 + 320*d*e**5 + 192*d*e**4*
(d - e + f) + 720*d*e**3*f**2 - 320*d*e**3*(d - e + f)**2 + 2160*d*e**2*f**
2*(d - e + f) - 192*d*e**2*(d - e + f)**3 - 6400*d*e*f**4 + 1968*d*e*f**2*(
d - e + f)**2 + 1152*d*f**4*(d - e + f) - 240*d*f**2*(d - e + f)**3 + 512*e
**5*f - 512*e**3*f*(d - e + f)**2 + 1152*e**2*f**3*(d - e + f) - 1472*e*f**
5 + 1280*e*f**3*(d - e + f)**2 + 960*f**5*(d - e + f) - 384*f**3*(d - e + f
)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f
 - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 12
80*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f*
*4 + 576*f**6))/6 - (d + e + f)*log(x + (-35*d**5*e + 51*d**5*(d + e + f) -
 820*d**4*e*f + 180*d**4*f*(d + e + f) - 180*d**3*e**3 - 180*d**3*e**2*(d +
 e + f) - 4100*d**3*e*f**2 + 164*d**3*e*(d + e + f)**2 + 84*d**3*f**2*(d +
e + f) - 60*d**3*(d + e + f)**3 - 864*d**2*e**2*f*(d + e + f) - 8000*d**2*e
*f**3 + 960*d**2*e*f*(d + e + f)**2 - 480*d**2*f**3*(d + e + f) - 96*d**2*f
*(d + e + f)**3 + 320*d*e**5 - 192*d*e**4*(d + e + f) + 720*d*e**3*f**2 - 3
20*d*e**3*(d + e + f)**2 - 2160*d*e**2*f**2*(d + e + f) + 192*d*e**2*(d + e
 + f)**3 - 6400*d*e*f**4 + 1968*d*e*f**2*(d + e + f)**2 - 1152*d*f**4*(d +
e + f) + 240*d*f**2*(d + e + f)**3 + 512*e**5*f - 512*e**3*f*(d + e + f)**2
 - 1152*e**2*f**3*(d + e + f) - 1472*e*f**5 + 1280*e*f**3*(d + e + f)**2 -
960*f**5*(d + e + f) + 384*f**3*(d + e + f)**3)/(9*d**6 + 45*d**5*f - 160*d
**4*e**2 - 36*d**4*f**2 - 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4
 - 3840*d**2*e**2*f**2 - 144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 +
```

```
720*d*f**5 + 1024*e**4*f**2 - 2560*e**2*f**4 + 576*f**6))/6 + (d + 2*e + 4*
f)*log(x + (-35*d**5*e - 51*d**5*(d + 2*e + 4*f)/2 - 820*d**4*e*f - 90*d**4
*f*(d + 2*e + 4*f) - 180*d**3*e**3 + 90*d**3*e**2*(d + 2*e + 4*f) - 4100*d*
*3*e*f**2 + 41*d**3*e*(d + 2*e + 4*f)**2 - 42*d**3*f**2*(d + 2*e + 4*f) + 1
5*d**3*(d + 2*e + 4*f)**3/2 + 432*d**2*e**2*f*(d + 2*e + 4*f) - 8000*d**2*e
*f**3 + 240*d**2*e*f*(d + 2*e + 4*f)**2 + 240*d**2*f**3*(d + 2*e + 4*f) + 1
2*d**2*f*(d + 2*e + 4*f)**3 + 320*d*e**5 + 96*d*e**4*(d + 2*e + 4*f) + 720*
d*e**3*f**2 - 80*d*e**3*(d + 2*e + 4*f)**2 + 1080*d*e**2*f**2*(d + 2*e + 4*
f) - 24*d*e**2*(d + 2*e + 4*f)**3 - 6400*d*e*f**4 + 492*d*e*f**2*(d + 2*e +
 4*f)**2 + 576*d*f**4*(d + 2*e + 4*f) - 30*d*f**2*(d + 2*e + 4*f)**3 + 512*
e**5*f - 128*e**3*f*(d + 2*e + 4*f)**2 + 576*e**2*f**3*(d + 2*e + 4*f) - 14
72*e*f**5 + 320*e*f**3*(d + 2*e + 4*f)**2 + 480*f**5*(d + 2*e + 4*f) - 48*f
**3*(d + 2*e + 4*f)**3)/(9*d**6 + 45*d**5*f - 160*d**4*e**2 - 36*d**4*f**2
- 1312*d**3*e**2*f - 360*d**3*f**3 + 256*d**2*e**4 - 3840*d**2*e**2*f**2 -
144*d**2*f**4 + 1280*d*e**4*f - 5248*d*e**2*f**3 + 720*d*f**5 + 1024*e**4*f
**2 - 2560*e**2*f**4 + 576*f**6))/12
```

Giac [**A**]
time = 3.27, size = 59, normalized size = 1.16

$$-\frac{1}{12}(d+4f-2e)\log(|x+2|)+\frac{1}{6}(d+f-e)\log(|x+1|)-\frac{1}{6}(d+f+e)\log(|x-1|)+\frac{1}{12}(d+4f+2e)\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 2*e)*log(abs(x + 2)) + 1/6*(d + f - e)*log(abs(x + 1)) - 1
/6*(d + f + e)*log(abs(x - 1)) + 1/12*(d + 4*f + 2*e)*log(abs(x - 2))
```

Mupad [**B**]
time = 0.71, size = 63, normalized size = 1.24

$$\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)-\ln(x-1)\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}\right)+\ln(x-2)\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}\right)-\ln(x+2)\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)*(d/6 - e/6 + f/6) - log(x - 1)*(d/6 + e/6 + f/6) + log(x - 2)*(d
/12 + e/6 + f/3) - log(x + 2)*(d/12 - e/6 + f/3)
```

3.12 $\int \frac{d+ex+fx^2+gx^3}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

```
[Out] -1/6*(d+4*f)*arctanh(1/2*x)+1/3*(d+f)*arctanh(x)-1/6*(e+g)*ln(-x^2+1)+1/6*(
e+4*g)*ln(-x^2+4)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1687, 1180, 213, 1261, 646, 31}

$$-\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/6*((d + 4*f)*ArcTanh[x/2]) + ((d + f)*ArcTanh[x])/3 - ((e + g)*Log[1 - x
^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{4-5x^2+x^4}\,dx = \int \frac{d+fx^2}{4-5x^2+x^4}\,dx + \int \frac{x(e+gx^2)}{4-5x^2+x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{4-5x+x^2}\,dx, x, x^2\right) - \frac{1}{3}(d+f)\int \frac{1}{-1+x^2}\,dx + \frac{1}{3}(d+4f)\int \frac{}{-}$$

$$= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) + \frac{1}{6}(-e-g)\text{Subst}\left(\int \frac{1}{-1+x}\right.$$

$$= -\frac{1}{6}(d+4f)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f)\tanh^{-1}(x) - \frac{1}{6}(e+g)\log\left(1-x^2\right) + \frac{1}{6}(e+$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.19

$$\frac{1}{12}(-2(d+e+f+g)\log(1-x)+(d+2e+4f+8g)\log(2-x)+2(d-e+f-g)\log(1+x)-(d-2e+4f-8g)\log(2+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-2*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x] + 2*(d -
e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)*Log[2 + x])/12
```

Maple [A]
time = 0.03, size = 74, normalized size = 1.30

method	result
default	$\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}-\frac{g}{6}\right)\ln\left(-1+x\right)$ -
norman	$\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}\right)\ln\left(x+2\right)+\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{6}-\frac{e}{6}-\frac{f}{6}-\frac{g}{6}\right)\ln\left(-1+x\right)$ -
risch	$-\frac{\ln(1-x)d}{6}-\frac{\ln(1-x)e}{6}-\frac{\ln(1-x)f}{6}-\frac{\ln(1-x)g}{6}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(2-x)d}{12}+$ ln(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/12*d+1/6*e-1/3*f+2/3*g)*ln(x+2)+(1/12*d+1/6*e+1/3*f+2/3*g)*ln(x-2)+(-1/
6*d-1/6*e-1/6*f-1/6*g)*ln(-1+x)+(1/6*d-1/6*e+1/6*f-1/6*g)*ln(1+x)
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.14

$$-\frac{1}{12}\left(d+4\,f-8\,g-2\,e\right)\log\left(x+2\right)+\frac{1}{6}\left(d+f-g-e\right)\log\left(x+1\right)-\frac{1}{6}\left(d+f+g+e\right)\log\left(x-1\right)+\frac{1}{12}\left(d+4\,f+8\,g+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -1/12*(d + 4*f - 8*g - 2*e)*log(x + 2) + 1/6*(d + f - g - e)*log(x + 1) - 1
/6*(d + f + g + e)*log(x - 1) + 1/12*(d + 4*f + 8*g + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.57, size = 61, normalized size = 1.07

$$-\frac{1}{12}\left(d-2\,e+4\,f-8\,g\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e+f-g\right)\log\left(x+1\right)-\frac{1}{6}\left(d+e+f+g\right)\log\left(x-1\right)+\frac{1}{12}\left(d+2\,e+4\,f+8\,g\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/12*(d - 2*e + 4*f - 8*g)*log(x + 2) + 1/6*(d - e + f - g)*log(x + 1) - 1
/6*(d + e + f + g)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.22, size = 69, normalized size = 1.21

$$-\frac{1}{12}\left(d+4f-8g-2e\right)\log\left(|x+2|\right)+\frac{1}{6}\left(d+f-g-e\right)\log\left(|x+1|\right)-\frac{1}{6}\left(d+f+g+e\right)\log\left(|x-1|\right)+\frac{1}{12}\left(d+4f+8g+2e\right)\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*(d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + 1/6*(d + f - g - e)*log(abs(x
 + 1)) - 1/6*(d + f + g + e)*log(abs(x - 1)) + 1/12*(d + 4*f + 8*g + 2*e)*l
og(abs(x - 2))
```

Mupad [B]
time = 0.74, size = 75, normalized size = 1.32

$$\ln\left(x+1\right)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)-\ln\left(x-1\right)\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}+\frac{g}{6}\right)+\ln\left(x-2\right)\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}\right)-\ln\left(x+2\right)\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)*(d/6 - e/6 + f/6 - g/6) - log(x - 1)*(d/6 + e/6 + f/6 + g/6) + l
og(x - 2)*(d/12 + e/6 + f/3 + (2*g)/3) - log(x + 2)*(d/12 - e/6 + f/3 - (2*
g)/3)
```

3.13 $\int \frac{d+ex+fx^2+gx^3+hx^4}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=64

$$hx-\frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f+h)\tanh^{-1}(x)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)$$

```
[Out] h*x-1/6*(d+4*f+16*h)*arctanh(1/2*x)+1/3*(d+f+h)*arctanh(x)-1/6*(e+g)*ln(-x^
2+1)+1/6*(e+4*g)*ln(-x^2+4)
```

Rubi [A]
time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1687, 1690, 1180, 213, 1261, 646, 31}

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}(e+g)\log\left(1-x^2\right)+\frac{1}{6}(e+4g)\log\left(4-x^2\right)+hx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[x])/3 - ((e
+ g)*Log[1 - x^2])/6 + ((e + 4*g)*Log[4 - x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{4-5x^2+x^4}\,dx &= \int \frac{x(e+gx^2)}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{4-5x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-4h+(f+5h)x^2}{4-5x^2+x^4}\right) dx \\
&= hx + \frac{1}{6}(-e-g)\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{1}{6}(e+4g)\text{Subst}\left(\int \frac{1}{-4+\cdots}\right. \\
&= hx - \frac{1}{6}(e+g)\log\left(1-x^2\right) + \frac{1}{6}(e+4g)\log\left(4-x^2\right) - \frac{1}{3}(d+f+h)\int \frac{\cdots}{-\cdots} \\
&= hx - \frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f+h)\tanh^{-1}(x) - \frac{1}{6}(e+g)\cdots
\end{aligned}
$$

Mathematica [A]

time = 0.03, size = 81, normalized size = 1.27

$$\frac{1}{12}(12hx-2(d+e+f+g+h)\log(1-x)+(d+2(e+2f+4g+8h))\log(2-x)+2(d-e+f-g+h)\log(1+x)-(d-2e+4f-8g+16h)\log(2+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x - 2*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2*f + 4*g + 8*h))*
Log[2 - x] + 2*(d - e + f - g + h)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h
)*Log[2 + x])/12
```

Maple [A]
time = 0.04, size = 89, normalized size = 1.39

method	result
default	$hx + \left(-\frac{d}{12} + \frac{e}{6} - \frac{f}{3} + \frac{2g}{3} - \frac{4h}{3}\right)\ln(x+2) + \left(\frac{d}{12} + \frac{e}{6} + \frac{f}{3} + \frac{2g}{3} + \frac{4h}{3}\right)\ln(x-2) + \left(-\frac{d}{6} - \frac{e}{6} - \frac{f}{6} -\right.$
norman	$hx + \left(-\frac{d}{12} + \frac{e}{6} - \frac{f}{3} + \frac{2g}{3} - \frac{4h}{3}\right)\ln(x+2) + \left(\frac{d}{12} + \frac{e}{6} + \frac{f}{3} + \frac{2g}{3} + \frac{4h}{3}\right)\ln(x-2) + \left(-\frac{d}{6} - \frac{e}{6} - \frac{f}{6} -\right.$
risch	$hx - \frac{\ln(x+2)d}{12} + \frac{\ln(x+2)e}{6} - \frac{\ln(x+2)f}{3} + \frac{2\ln(x+2)g}{3} - \frac{4\ln(x+2)h}{3} - \frac{\ln(1-x)d}{6} - \frac{\ln(1-x)e}{6} - \frac{\ln(1-x)f}{6} - \frac{\ln(1-x)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] h*x+(-1/12*d+1/6*e-1/3*f+2/3*g-4/3*h)*ln(x+2)+(1/12*d+1/6*e+1/3*f+2/3*g+4/3
*h)*ln(x-2)+(-1/6*d-1/6*e-1/6*f-1/6*g-1/6*h)*ln(-1+x)+(1/6*d-1/6*e+1/6*f-1/
6*g+1/6*h)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 1.19

$$hx - \frac{1}{12}(d + 4f - 8g + 16h - 2e)\log(x+2) + \frac{1}{6}(d + f - g + h - e)\log(x+1) - \frac{1}{6}(d + f + g + h + e)\log(x-1) + \frac{1}{12}(d + 4f + 8g + 16h + 2e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] h*x - 1/12*(d + 4*f - 8*g + 16*h - 2*e)*log(x + 2) + 1/6*(d + f - g + h - e
)*log(x + 1) - 1/6*(d + f + g + h + e)*log(x - 1) + 1/12*(d + 4*f + 8*g + 1
6*h + 2*e)*log(x - 2)
```

Fricas [A]
time = 1.30, size = 72, normalized size = 1.12

$$hx - \frac{1}{12}(d - 2e + 4f - 8g + 16h)\log(x+2) + \frac{1}{6}(d - e + f - g + h)\log(x+1) - \frac{1}{6}(d + e + f + g + h)\log(x-1) + \frac{1}{12}(d + 2e + 4f + 8g + 16h)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h)*log(x + 2) + 1/6*(d - e + f - g + h
)*log(x + 1) - 1/6*(d + e + f + g + h)*log(x - 1) + 1/12*(d + 2*e + 4*f + 8
*g + 16*h)*log(x - 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
[Out] Timed out
```

Giac [A]
time = 4.72, size = 80, normalized size = 1.25

$$hx - \frac{1}{12}(d+4f-8g+16h-2e)\log(|x+2|) + \frac{1}{6}(d+f-g+h-e)\log(|x+1|) - \frac{1}{6}(d+f+g+h+e)\log(|x-1|) + \frac{1}{12}(d+4f+8g+16h+2e)\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] h*x - 1/12*(d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2)) + 1/6*(d + f - g +
h - e)*log(abs(x + 1)) - 1/6*(d + f + g + h + e)*log(abs(x - 1)) + 1/12*(d
+ 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2))
```

Mupad [B]
time = 0.81, size = 90, normalized size = 1.41

$$hx - \ln(x-1)\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}+\frac{g}{6}+\frac{h}{6}\right) + \ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right) + \ln(x-2)\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}+\frac{4h}{3}\right) - \ln(x+2)\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^4 - 5*x^2 + 4),x)
[Out] h*x - log(x - 1)*(d/6 + e/6 + f/6 + g/6 + h/6) + log(x + 1)*(d/6 - e/6 + f/
6 - g/6 + h/6) + log(x - 2)*(d/12 + e/6 + f/3 + (2*g)/3 + (4*h)/3) - log(x
+ 2)*(d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/3)
```

3.14 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=76

$$hx+\frac{ix^2}{2}-\frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}(d+f+h)\tanh^{-1}(x)-\frac{1}{6}(e+g+i)\log\left(1-x^2\right)+\frac{1}{6}(e+4g+16i)\log\left(4\right.$$

```
[Out] h*x+1/2*i*x^2-1/6*(d+4*f+16*h)*arctanh(1/2*x)+1/3*(d+f+h)*arctanh(x)-1/6*(e
+g+i)*ln(-x^2+1)+1/6*(e+4*g+16*i)*ln(-x^2+4)
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1687, 1690, 1180, 213, 1677, 1671, 646, 31}

$$-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)(d+4f+16h)+\frac{1}{3}\tanh^{-1}(x)(d+f+h)-\frac{1}{6}\log\left(1-x^2\right)(e+g+i)+\frac{1}{6}\log\left(4-x^2\right)(e+4g+16i)+hx+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] h*x + (i*x^2)/2 - ((d + 4*f + 16*h)*ArcTanh[x/2])/6 + ((d + f + h)*ArcTanh[
x])/3 - ((e + g + i)*Log[1 - x^2])/6 + ((e + 4*g + 16*i)*Log[4 - x^2])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+14x^5}{4-5x^2+x^4}\,dx = \int \frac{x(e+gx^2+14x^4)}{4-5x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{4-5x^2+x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+14x^2}{4-5x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-4h+(f+5}{4-5x^2+x}\right.$$

$$= hx + \frac{1}{2}\text{Subst}\left(\int \left(14 - \frac{56-e-(70+g)x}{4-5x+x^2}\right)dx, x, x^2\right) + \int \frac{d}{}$$

$$= hx + 7x^2 - \frac{1}{2}\text{Subst}\left(\int \frac{56-e-(70+g)x}{4-5x+x^2}\,dx, x, x^2\right) - \frac{1}{3}(d+$$

$$= hx + 7x^2 - \frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f+h)\tanh^{-}$$

$$= hx + 7x^2 - \frac{1}{6}(d+4f+16h)\tanh^{-1}\left(\frac{x}{2}\right) + \frac{1}{3}(d+f+h)\tanh^{-}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.29

$$\frac{1}{12}(12hx+6ix^2-2(d+e+f+g+h+i)\log(1-x)+(d+2e+4(f+2g+4h+8i))\log(2-x)+2(d-e+f-g+h-i)\log(1+x)-(d-2(e-2f+4g-8h+16i))\log(2+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4),x]
```

```
[Out] (12*h*x + 6*i*x^2 - 2*(d + e + f + g + h + i)*Log[1 - x] + (d + 2*e + 4*(f
+ 2*g + 4*h + 8*i))*Log[2 - x] + 2*(d - e + f - g + h - i)*Log[1 + x] - (d
- 2*(e - 2*f + 4*g - 8*h + 16*i))*Log[2 + x])/12
```

Maple [A]
time = 0.05, size = 107, normalized size = 1.41

method	result
default	$\frac{i x^2}{2} + hx + \left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}+\frac{8i}{3}\right)\ln(x+2) + \left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}+\frac{4h}{3}+\frac{8i}{3}\right)\ln(x-2) +$
norman	$\frac{i x^2}{2} + hx + \left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}+\frac{8i}{3}\right)\ln(x+2) + \left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2g}{3}+\frac{4h}{3}+\frac{8i}{3}\right)\ln(x-2) +$
risch	$hx + \frac{2\ln(x+2)g}{3} + \frac{2\ln(2-x)g}{3} - \frac{\ln(1+x)g}{6} - \frac{\ln(1-x)g}{6} - \frac{\ln(1-x)f}{6} + \frac{\ln(2-x)f}{3} - \frac{\ln(x+2)f}{3} + \frac{\ln(1+x)f}{6} + \frac{\ln(2-x)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*i*x^2+h*x+(-1/12*d+1/6*e-1/3*f+2/3*g-4/3*h+8/3*i)*ln(x+2)+(1/12*d+1/6*e
+1/3*f+2/3*g+4/3*h+8/3*i)*ln(x-2)+(-1/6*d-1/6*e-1/6*f-1/6*g-1/6*h-1/6*i)*ln
(-1+x)+(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h-1/6*i)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 85, normalized size = 1.12

$$hx+\frac{1}{2}ix^2-\frac{1}{12}(d+4f-8g+16h-2e-32i)\log(x+2)+\frac{1}{6}(d+f-g+h-e-i)\log(x+1)-\frac{1}{6}(d+f+g+h+e+i)\log(x-1)+\frac{1}{12}(d+4f+8g+16h+2e+32i)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] h*x + 1/2*I*x^2 - 1/12*(d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(x + 2) + 1/6
*(d + f - g + h - e - I)*log(x + 1) - 1/6*(d + f + g + h + e + I)*log(x - 1
) + 1/12*(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(x - 2)
```

Fricas [A]
time = 6.52, size = 88, normalized size = 1.16

$$\frac{1}{2}ix^2+hx-\frac{1}{12}(d-2e+4f-8g+16h-32i)\log(x+2)+\frac{1}{6}(d-e+f-g+h-i)\log(x+1)-\frac{1}{6}(d+e+f+g+h+i)\log(x-1)+\frac{1}{12}(d+2e+4f+8g+16h+32i)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*i*x^2 + h*x - 1/12*(d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + 1/6
*(d - e + f - g + h - i)*log(x + 1) - 1/6*(d + e + f + g + h + i)*log(x - 1
) + 1/12*(d + 2*e + 4*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.50, size = 89, normalized size = 1.17

$$hx+\frac{1}{2}ix^2-\frac{1}{12}(d+4f-8g+16h-2e-32i)\log(|x+2|)+\frac{1}{6}(d+f-g+h-e-i)\log(|x+1|)-\frac{1}{6}(d+f+g+h+e+i)\log(|x-1|)+\frac{1}{12}(d+4f+8g+16h+2e+32i)\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] h*x + 1/2*I*x^2 - 1/12*(d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(abs(x + 2))
+ 1/6*(d + f - g + h - e - I)*log(abs(x + 1)) - 1/6*(d + f + g + h + e + I)
*log(abs(x - 1)) + 1/12*(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(abs(x - 2))
```

Mupad [B]
time = 1.19, size = 108, normalized size = 1.42

$$h\,x+\frac{i\,x^2}{2}-\ln\left(x-1\right)\,\left(\frac{d}{6}+\frac{e}{6}+\frac{f}{6}+\frac{g}{6}+\frac{h}{6}+\frac{i}{6}\right)+\ln\left(x+1\right)\,\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)+\ln\left(x-2\right)\,\left(\frac{d}{12}+\frac{e}{6}+\frac{f}{3}+\frac{2\,g}{3}+\frac{4\,h}{3}+\frac{8\,i}{3}\right)-\ln\left(x+2\right)\,\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}-\frac{8\,i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] h*x + (i*x^2)/2 - log(x - 1)*(d/6 + e/6 + f/6 + g/6 + h/6 + i/6) + log(x +
1)*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/12 + e/6 + f/3 + (2*
g)/3 + (4*h)/3 + (8*i)/3) - log(x + 2)*(d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/
3 - (8*i)/3)
```

3.15 $\int \frac{d+ex}{1+x^2+x^4} dx$

Optimal. Leaf size=92

$$-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{4}d\log\left(1-x+x^2\right)+\frac{1}{4}d\log\left(1+x+x^2\right)$$

```
[Out] -1/4*d*ln(x^2-x+1)+1/4*d*ln(x^2+x+1)-1/6*d*arctan(1/3*(1-2*x)*3^(1/2))*3^(1
/2)+1/6*d*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*e*arctan(1/3*(2*x^2+1)*3^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1687, 12, 1108, 648, 632, 210, 642, 1121}

$$-\frac{d\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{d\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*(d*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - (d*Log[1 - x + x^2]
)/4 + (d*Log[1 + x + x^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex}{1+x^2+x^4}\,dx = \int \frac{d}{1+x^2+x^4}\,dx + \int \frac{ex}{1+x^2+x^4}\,dx$$
$$= d\int \frac{1}{1+x^2+x^4}\,dx + e\int \frac{x}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}d\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}d\int \frac{1+x}{1+x+x^2}\,dx + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{4}d\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}d\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}d\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}d\int \frac{1+2x}{1+x+x^2}\,d$$
$$= \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}d\log\left(1-x+x^2\right) + \frac{1}{4}d\log\left(1+x+x^2\right) - \frac{1}{2}d\text{Subst}\left(\int \frac{1}{-3-x^2}\right.$$
$$= -\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}d\log\left(1-x+x^2\right) + \frac{1}{4}d\,\text{lo}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 98, normalized size = 1.07

$$\frac{1}{6}i\left(\sqrt{6-6i\sqrt{3}}\,d\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right) - \sqrt{6+6i\sqrt{3}}\,d\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right) + 2i\sqrt{3}\,e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4),x]
[Out] (I/6)*(Sqrt[6 - (6*I)*Sqrt[3]]*d*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[6 + (6
*I)*Sqrt[3]]*d*ArcTan[((I + Sqrt[3])*x)/2] + (2*I)*Sqrt[3]*e*ArcTan[Sqrt[3]
/(1 + 2*x^2)])
```

Maple [A]
time = 0.06, size = 68, normalized size = 0.74

method	result
default	$\frac{d\ln(x^2+x+1)}{4} + \frac{\left(\frac{d}{2}-e\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3} - \frac{d\ln(x^2-x+1)}{4} + \frac{\left(\frac{d}{2}+e\right)\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$\frac{d\ln(36d^2x^2+48e^2x^2+36d^2x+48e^2x+36d^2+48e^2)}{4} - \frac{d\ln(36d^2x^2+48e^2x^2-36d^2x-48e^2x+36d^2+48e^2)}{4} + \frac{\sqrt{3}\,d\arctan\left(\frac{8e^2\sqrt{3}}{3(3d^2+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*d*ln(x^2+x+1)+1/3*(1/2*d-e)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*d*l
n(x^2-x+1)+1/3*(1/2*d+e)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 67, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

Fricas [A]
time = 0.41, size = 65, normalized size = 0.71

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.81, size = 923, normalized size = 10.03

$$\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)\log\left(x+\frac{-7d^4e+6d^4\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)-15d^2e^3-18d^2e^2\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)+60d^2e\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)^2+72d^2\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)^3+4e^5+24e^4\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)+48e^3\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)^2+288e^2\left(-\frac{d}{4}-\frac{\sqrt{3}i(d+2e)}{12}\right)^3}{3d^5-8d^3e^2-16de^4}\right)+\left(-\frac{d}{4}+\frac{\sqrt{3}i(d+2e)}{12}\right)\log\left(x+\frac{-7d^4e+6d^4\left(-\frac{d}{4}+\frac{\sqrt{3}i(d+2e)}{12}\right)-15d^2e^3-18d^2e^2\left(-\frac{d}{4}+\frac{\sqrt{3}i(d+2e)}{12}\right)+\ldots}{\ldots}\right)+\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(-d/4 - sqrt(3)
*I*(d + 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(-d/4 - sqrt(3)*I*(d + 2*e)/
12) + 60*d**2*e*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**2 + 72*d**2*(-d/4 - sqrt(3
)*I*(d + 2*e)/12)**3 + 4*e**5 + 24*e**4*(-d/4 - sqrt(3)*I*(d + 2*e)/12) + 4
8*e**3*(-d/4 - sqrt(3)*I*(d + 2*e)/12)**2 + 288*e**2*(-d/4 - sqrt(3)*I*(d +
 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (-d/4 + sqrt(3)*I*(d +
2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/12) - 15*d
**2*e**3 - 18*d**2*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12) + 60*d**2*e*(-d/4 +
 sqrt(3)*I*(d + 2*e)/12)**2 + 72*d**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**3 +
4*e**5 + 24*e**4*(-d/4 + sqrt(3)*I*(d + 2*e)/12) + 48*e**3*(-d/4 + sqrt(3)*
```

```
I*(d + 2*e)/12)**2 + 288*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/12)**3)/(3*d**5 -
 8*d**3*e**2 - 16*d*e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/12)*log(x + (-7*d**
4*e + 6*d**4*(d/4 - sqrt(3)*I*(d - 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(
d/4 - sqrt(3)*I*(d - 2*e)/12) + 60*d**2*e*(d/4 - sqrt(3)*I*(d - 2*e)/12)**2
 + 72*d**2*(d/4 - sqrt(3)*I*(d - 2*e)/12)**3 + 4*e**5 + 24*e**4*(d/4 - sqrt
(3)*I*(d - 2*e)/12) + 48*e**3*(d/4 - sqrt(3)*I*(d - 2*e)/12)**2 + 288*e**2*
(d/4 - sqrt(3)*I*(d - 2*e)/12)**3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4)) + (d
/4 + sqrt(3)*I*(d - 2*e)/12)*log(x + (-7*d**4*e + 6*d**4*(d/4 + sqrt(3)*I*(
d - 2*e)/12) - 15*d**2*e**3 - 18*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/12) +
 60*d**2*e*(d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 72*d**2*(d/4 + sqrt(3)*I*(d
- 2*e)/12)**3 + 4*e**5 + 24*e**4*(d/4 + sqrt(3)*I*(d - 2*e)/12) + 48*e**3*(
d/4 + sqrt(3)*I*(d - 2*e)/12)**2 + 288*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/12)*
*3)/(3*d**5 - 8*d**3*e**2 - 16*d*e**4))
```

Giac [A]
time = 3.36, size = 67, normalized size = 0.73

$$\frac{1}{6}\sqrt{3}(d-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}d\log\left(x^2+x+1\right)-\frac{1}{4}d\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1)
```

Mupad [B]
time = 0.24, size = 118, normalized size = 1.28

$$-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x - (3^(1/2)*1i)/2 + 1/2)*(d/4 - (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6)
- log(x - (3^(1/2)*1i)/2 - 1/2)*(d/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6
) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*d*1i)/12 - d/4 + (3^(1/2)*e*1i)
/6) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 + (3^(1/2)*d*1i)/12 - (3^(1/2)*e*1
i)/6)
```

3.16 $\int \frac{d+ex+fx^2}{1+x^2+x^4} dx$

Optimal. Leaf size=104

$$-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{4}(d-f)\log\left(1-x+x^2\right)+\frac{1}{4}(d-f)\log$$

```
[Out] -1/4*(d-f)*ln(x^2-x+1)+1/4*(d-f)*ln(x^2+x+1)-1/6*(d+f)*arctan(1/3*(1-2*x)*3
^(1/2))*3^(1/2)+1/6*(d+f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*e*arctan(
1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1687, 1183, 648, 632, 210, 642, 12, 1121}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f)}{2\sqrt{3}}+\frac{e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + (e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/Sqrt[3] - ((d - f
)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{1+x^2+x^4}\,dx = \int \frac{ex}{1+x^2+x^4}\,dx + \int \frac{d+fx^2}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}\int \frac{d-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d+(d-f)x}{1+x+x^2}\,dx + e\int \frac{x}{1+x^2+x^4}\,dx$$
$$= \frac{1}{2}e\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-d+f)\int \frac{-1}{1-$$
$$= -\frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+x^2\right) - e\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x$$
$$= -\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{4}(d-f)\log$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 121, normalized size = 1.16

$$\frac{\left(2id+\left(-i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}} + \frac{\left(-2id+\left(i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}} - \frac{e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]] - (e*ArcTan[Sqrt[3]/(1 + 2*x^2)])/Sqrt[3]
```

Maple [A]
time = 0.10, size = 82, normalized size = 0.79

method	result
default	$\frac{(d-f)\ln(x^2+x+1)}{4} + \frac{\left(\frac{d}{2}-e+\frac{f}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3} + \frac{(f-d)\ln(x^2-x+1)}{4} + \frac{\left(\frac{d}{2}+e+\frac{f}{2}\right)\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right.}{3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(d-f)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/
2)+1/4*(f-d)*ln(x^2-x+1)+1/3*(1/2*d+e+1/2*f)*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

Maxima [A]
time = 0.49, size = 77, normalized size = 0.74

$$\frac{1}{6}\sqrt{3}(d+f-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d + f - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
f + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

Fricas [A]
time = 0.42, size = 75, normalized size = 0.72

$$\frac{1}{6}\sqrt{3}(d-2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e+f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
2*e + f)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 72.27, size = 3589, normalized size = 34.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] (-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(-d/4
 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/4
- sqrt(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4 - s
qrt(3)*I*(d + 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 - sqrt(
3)*I*(d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e +
 f)/12) + 72*d**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*d**2*e
**2*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2
*e*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*f**3*(-d/4 + f/
4 - sqrt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2
*e + f)/12)**3 + 4*d*e**5 + 24*d*e**4*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)
/12) + 15*d*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)
**2 - 54*d*e**2*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 288*d*e**2
*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*
```

```
(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/4 + f/4 - sqrt
(3)*I*(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/1
2)**3 - 8*e**5*f - 96*e**3*f*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 +
 36*e**2*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*
f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4 - sq
rt(3)*I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 - sqrt(3)*I*(d + 2*e + f)/
12)**3)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6
*d**3*f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f +
 40*d*e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2*f**4 + 3*f**6)) + (-d/4
+ f/4 + sqrt(3)*I*(d + 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(-d/4 + f/4
 + sqrt(3)*I*(d + 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(-d/4 + f/4 + sqrt
(3)*I*(d + 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(-d/4 + f/4 + sqrt(3)
*I*(d + 2*e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(-d/4 + f/4 + sqrt(3)*I*(
d + 2*e + f)/12)**2 - 42*d**3*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12
) + 72*d**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 + 108*d**2*e**2*f*
(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2*e*f*(
-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 - 12*d**2*f**3*(-d/4 + f/4 + sq
rt(3)*I*(d + 2*e + f)/12) - 144*d**2*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f
)/12)**3 + 4*d*e**5 + 24*d*e**4*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) +
 15*d*e**3*f**2 + 48*d*e**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 -
54*d*e**2*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 288*d*e**2*(-d/4
 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*(-d/4
+ f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*d*f**4*(-d/4 + f/4 + sqrt(3)*I*
(d + 2*e + f)/12) - 72*d*f**2*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3
- 8*e**5*f - 96*e**3*f*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 + 36*e*
*2*f**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(
-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**2 - 6*f**5*(-d/4 + f/4 + sqrt(3)*
I*(d + 2*e + f)/12) + 144*f**3*(-d/4 + f/4 + sqrt(3)*I*(d + 2*e + f)/12)**3
)/(3*d**6 - 3*d**5*f - 8*d**4*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6*d**3*
f**3 - 16*d**2*e**4 - 48*d**2*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*
e**2*f**3 - 3*d*f**5 - 16*e**4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 -
 sqrt(3)*I*(d - 2*e + f)/12)*log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 - sqrt(
3)*I*(d - 2*e + f)/12) + 25*d**4*e*f + 18*d**4*f*(d/4 - f/4 - sqrt(3)*I*(d
- 2*e + f)/12) - 15*d**3*e**3 - 18*d**3*e**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*
e + f)/12) - 25*d**3*e*f**2 + 60*d**3*e*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f
)/12)**2 - 42*d**3*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 72*d**3*
(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 + 108*d**2*e**2*f*(d/4 - f/4 -
sqrt(3)*I*(d - 2*e + f)/12) + 20*d**2*e*f**3 - 144*d**2*e*f*(d/4 - f/4 - sq
rt(3)*I*(d - 2*e + f)/12)**2 - 12*d**2*f**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e
 + f)/12) - 144*d**2*f*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 + 4*d*e*
*5 + 24*d*e**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 15*d*e**3*f**2 +
48*d*e**3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 - 54*d*e**2*f**2*(d/4
 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 288*d*e**2*(d/4 - f/4 - sqrt(3)*I*(d
 - 2*e + f)/12)**3 - 20*d*e*f**4 + 180*d*e*f**2*(d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)**2 + 36*d*f**4*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) - 72*
```

```
d*f**2*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3 - 8*e**5*f - 96*e**3*f*(
d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**2 + 36*e**2*f**3*(d/4 - f/4 - sqrt
(3)*I*(d - 2*e + f)/12) + 11*e*f**5 - 48*e*f**3*(d/4 - f/4 - sqrt(3)*I*(d -
 2*e + f)/12)**2 - 6*f**5*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12) + 144*f*
*3*(d/4 - f/4 - sqrt(3)*I*(d - 2*e + f)/12)**3)/(3*d**6 - 3*d**5*f - 8*d**4
*e**2 - 3*d**4*f**2 + 40*d**3*e**2*f + 6*d**3*f**3 - 16*d**2*e**4 - 48*d**2
*e**2*f**2 - 3*d**2*f**4 + 16*d*e**4*f + 40*d*e**2*f**3 - 3*d*f**5 - 16*e**
4*f**2 - 8*e**2*f**4 + 3*f**6)) + (d/4 - f/4 + sqrt(3)*I*(d - 2*e + f)/12)*
log(x + (-7*d**5*e + 6*d**5*(d/4 - f/4 + sqrt(3...
```

Giac [A]
time = 5.52, size = 77, normalized size = 0.74

$$\frac{1}{6}\sqrt{3}\,(d+f-2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,(d+f+2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\,(d-f)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(d +
f + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f)*log(x^2 + x + 1) - 1/4
*(d - f)*log(x^2 - x + 1)
```

Mupad [B]
time = 0.95, size = 159, normalized size = 1.53

$$-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{12}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{12}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*(f/4 - d/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1
i)/6 + (3^(1/2)*f*1i)/12) - log(x - (3^(1/2)*1i)/2 + 1/2)*(f/4 - d/4 + (3^(
1/2)*d*1i)/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12) - log(x - (3^(1/2)*1i
)/2 - 1/2)*(d/4 - f/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1
i)/12) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/4 + (3^(1/2)*d*1i)/12 - (3^
(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12)
```

3.17 $\int \frac{d+ex+fx^2+gx^3}{1+x^2+x^4}\,dx$

Optimal. Leaf size=127

$$-\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(1-x+x^2\right)+\frac{1}{4}(d-$$

```
[Out] -1/4*(d-f)*ln(x^2-x+1)+1/4*(d-f)*ln(x^2+x+1)+1/4*g*ln(x^4+x^2+1)-1/6*(d+f)*
arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*(d+f)*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+1/6*(2*e-g)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1687, 1183, 648, 632, 210, 642, 1261}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{1}{4}g\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*((d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + ((d + f)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3
]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x^2])/4 + (g*Log[1
 + x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{1+x^2+x^4}\,dx &= \int \frac{d+fx^2}{1+x^2+x^4}\,dx+\int \frac{x(e+gx^2)}{1+x^2+x^4}\,dx\\
&= \frac{1}{2}\int \frac{d-(d-f)x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{d+(d-f)x}{1+x+x^2}\,dx+\frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{1+x+x^2}\,dx,x,x^2\right.\\
&= \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{4}(-d+f)\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}(d+f)\int \frac{1}{1-x-}\\
&= -\frac{1}{4}(d-f)\log\left(1-x+x^2\right)+\frac{1}{4}(d-f)\log\left(1+x+x^2\right)+\frac{1}{4}g\log\left(1+x^2+x^4\right)\\
&= -\frac{(d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 150, normalized size = 1.18

$$\frac{2\sqrt{2-2i\sqrt{3}}\left(2id+\left(-i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+2\left(\sqrt{2+2i\sqrt{3}}\left(-2id+\left(i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)+(-4e+2g)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)+\sqrt{3}\,g\log\left(1+x^2+x^4\right)\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2 - (2*I)*Sqrt[3]]*((2*I)*d + (-I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[
3])*x)/2] + 2*(Sqrt[2 + (2*I)*Sqrt[3]]*((-2*I)*d + (I + Sqrt[3])*f)*ArcTan[
((I + Sqrt[3])*x)/2] + (-4*e + 2*g)*ArcTan[Sqrt[3]/(1 + 2*x^2)] + Sqrt[3]*g
*Log[1 + x^2 + x^4]))/(8*Sqrt[3])
```

Maple [A]
time = 0.12, size = 90, normalized size = 0.71

method	result
default	$\frac{(d-f+g)\ln\left(x^2+x+1\right)}{4}+\frac{\left(\frac{d}{2}-e+\frac{f}{2}+\frac{g}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(f-d+g)\ln\left(x^2-x+1\right)}{4}+\frac{\left(\frac{d}{2}+e+\frac{f}{2}-\frac{g}{2}\right)\sqrt{3}\,\arctan\left(\right.}{3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(d-f+g)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f+1/2*g)*arctan(1/3*(2*x+1)*3^(1/2
))*3^(1/2)+1/4*(f-d+g)*ln(x^2-x+1)+1/3*(1/2*d+e+1/2*f-1/2*g)*3^(1/2)*arctan
(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 85, normalized size = 0.67

$$\frac{1}{6}\sqrt{3}\left(d+f+g-2e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\frac{1}{6}\sqrt{3}\left(d+f-g+2e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{4}\left(d-f+g\right)\log\left(x^2+x+1\right)-\frac{1}{4}\left(d-f-g\right)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + f - g + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Fricas [A]
time = 0.55, size = 83, normalized size = 0.65

$$\frac{1}{6}\sqrt{3}\left(d-2e+f+g\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\frac{1}{6}\sqrt{3}\left(d+2e+f-g\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{4}\left(d-f+g\right)\log\left(x^2+x+1\right)-\frac{1}{4}\left(d-f-g\right)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + 2*e + f - g)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.78, size = 85, normalized size = 0.67

$$\frac{1}{6}\sqrt{3}\,(d+f+g-2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,(d+f-g+2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\,(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}\,(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(
d + f - g + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(d - f + g)*log(x^2 +
x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Mupad [**B**]
time = 1.13, size = 199, normalized size = 1.57

$$-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}-\frac{g}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,g\,1\mathrm{i}}{12}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}-\frac{g}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,g\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{4}-\frac{d}{4}+\frac{g}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,g\,1\mathrm{i}}{12}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{4}+\frac{g}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{12}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{6}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{12}+\frac{\sqrt{3}\,g\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x + (3^(1/2)*1i)/2 - 1/2)*(f/4 - d/4 + g/4 + (3^(1/2)*d*1i)/12 + (3^(1/
2)*e*1i)/6 + (3^(1/2)*f*1i)/12 - (3^(1/2)*g*1i)/12) - log(x - (3^(1/2)*1i)/
2 + 1/2)*(f/4 - d/4 - g/4 + (3^(1/2)*d*1i)/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)
*f*1i)/12 + (3^(1/2)*g*1i)/12) - log(x - (3^(1/2)*1i)/2 - 1/2)*(d/4 - f/4 -
 g/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 - (3^(1/2)*
g*1i)/12) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/4 + g/4 + (3^(1/2)*d*1i)
/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 + (3^(1/2)*g*1i)/12)
```

3.18 $\int \frac{d+ex+fx^2+gx^3+hx^4}{1+x^2+x^4}\,dx$

Optimal. Leaf size=136

$$hx-\frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f-2h)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(1-\right.$$

```
[Out] h*x-1/4*(d-f)*ln(x^2-x+1)+1/4*(d-f)*ln(x^2+x+1)+1/4*g*ln(x^4+x^2+1)-1/6*(d+
f-2*h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*(d+f-2*h)*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)+1/6*(2*e-g)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {1687, 1690, 1183, 648, 632, 210, 642, 1261}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{1}{4}g\log\left(x^4+x^2+1\right)+hx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```

```
[Out] h*x - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((d + f - 2*h
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sq
rt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d - f)*Log[1 + x + x
^2])/4 + (g*Log[1 + x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{1+x^2+x^4}\,dx &= \int \frac{x(e+gx^2)}{1+x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{1+x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-h+(f-h)x^2}{1+x^2+x^4}\right)\,dx \\
&= hx + \frac{1}{4}(2e-g)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{4}g\text{Subst}\left(\int \frac{1+2x}{1+x+\cdots}\right. \\
&= hx + \frac{1}{4}g\log\left(1+x^2+x^4\right) + \frac{1}{2}\int \frac{d-h-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d-h+(\cdots}{1+x\cdots} \\
&= hx + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{4}g\log\left(1+x^2+x^4\right) + \frac{1}{4}(d-f)\int \frac{1}{1+\cdots} \\
&= hx + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\,\text{l}\cdots \\
&= hx - \frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f-2h)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(2e\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.38, size = 165, normalized size = 1.21

$$
\frac{1}{24}\left(24hx+4\left(\left(3i+\sqrt{3}\right)d+\left(-3i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+4\left(\left(-3i+\sqrt{3}\right)d+\left(3i+\sqrt{3}\right)f-2\sqrt{3}\,h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)-8\sqrt{3}\,e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)+4\sqrt{3}\,g\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)+6g\log\left(1+x^2+x^4\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4),x]
```

```
[Out] (24*h*x + 4*((3*I + Sqrt[3])*d + (-3*I + Sqrt[3])*f - 2*Sqrt[3]*h)*ArcTan[(
(-I + Sqrt[3])*x)/2] + 4*((-3*I + Sqrt[3])*d + (3*I + Sqrt[3])*f - 2*Sqrt[3
]*h)*ArcTan[((I + Sqrt[3])*x)/2] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2*x^2)]
+ 4*Sqrt[3]*g*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 6*g*Log[1 + x^2 + x^4])/24
```

Maple [A]
time = 0.18, size = 99, normalized size = 0.73

method	result
default	$hx+\frac{(d-f+g)\ln(x^2+x+1)}{4}+\frac{\left(\frac{d}{2}-e+\frac{f}{2}+\frac{g}{2}-h\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(f-d+g)\ln(x^2-x+1)}{4}+\frac{\left(\frac{d}{2}+e+\frac{f}{2}-\frac{g}{2}-h\right)\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] h*x+1/4*(d-f+g)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f+1/2*g-h)*arctan(1/3*(2*x+1)*
3^(1/2))*3^(1/2)+1/4*(f-d+g)*ln(x^2-x+1)+1/3*(1/2*d+e+1/2*f-1/2*g-h)*3^(1/2
)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 94, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}(d+f+g-2h-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f-g-2h+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + f - g - 2*h + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Fricas [A]
time = 1.27, size = 92, normalized size = 0.68

$$\frac{1}{6}\sqrt{3}(d-2e+f+g-2h)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+2e+f-g-2h)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(d - 2*e + f + g - 2*h)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + 2*e + f - g - 2*h)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.73, size = 94, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}(d+f+g-2h-2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}(d+f-g-2h+2e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+hx+\frac{1}{4}(d-f+g)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqr
t(3)*(d + f - g - 2*h + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x + 1/4*(d -
 f + g)*log(x^2 + x + 1) - 1/4*(d - f - g)*log(x^2 - x + 1)
```

Mupad [B]
time = 6.11, size = 1209, normalized size = 8.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^2 + x^4 + 1),x)
```

```
[Out] log(d*f*9i - d*e*6i + d*g*3i - d*h*3i + e*h*6i + f*h*3i - g*h*3i - 3*3^(1/2
)*d^2 - d^2*x*6i - f^2*x*3i - d^2*3i - f^2*6i + 2*3^(1/2)*d*e + 3*3^(1/2)*d
*f - 3^(1/2)*d*g - 4*3^(1/2)*e*f + 3*3^(1/2)*d*h + 2*3^(1/2)*e*h + 2*3^(1/2
)*f*g - 3*3^(1/2)*f*h - 3^(1/2)*g*h + d*f*x*9i - e*f*x*6i + d*h*x*3i + e*h*
x*6i + f*g*x*3i - f*h*x*3i - g*h*x*3i + 3*3^(1/2)*f^2*x - 3*3^(1/2)*d*f*x -
 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x + 3*3^(1/2)*d*h*x - 2*3^(1/2)*e*h*x + 3^
(1/2)*f*g*x - 3*3^(1/2)*f*h*x + 3^(1/2)*g*h*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4
 + g/4 - (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 - (3^(1/2)*f*1i)/12 - (3^(1/2
)*g*1i)/12 + (3^(1/2)*h*1i)/6) - log(d*g*3i - d*f*9i - d*e*6i + d*h*3i + e*
h*6i - f*h*3i - g*h*3i - 3*3^(1/2)*d^2 - d^2*x*6i - f^2*x*3i + d^2*3i + f^2
*6i - 2*3^(1/2)*d*e + 3*3^(1/2)*d*f + 3^(1/2)*d*g + 4*3^(1/2)*e*f + 3*3^(1/
2)*d*h - 2*3^(1/2)*e*h - 2*3^(1/2)*f*g - 3*3^(1/2)*f*h + 3^(1/2)*g*h + d*f*
x*9i + e*f*x*6i + d*h*x*3i - e*h*x*6i - f*g*x*3i - f*h*x*3i + g*h*x*3i - 3*
3^(1/2)*f^2*x + 3*3^(1/2)*d*f*x - 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x - 3*3^(
1/2)*d*h*x - 2*3^(1/2)*e*h*x + 3^(1/2)*f*g*x + 3*3^(1/2)*f*h*x + 3^(1/2)*g*
h*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4 - g/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i
)/6 + (3^(1/2)*f*1i)/12 - (3^(1/2)*g*1i)/12 - (3^(1/2)*h*1i)/6) + log(d*f*9
i - d*e*6i + d*g*3i - d*h*3i + e*h*6i + f*h*3i - g*h*3i + 3*3^(1/2)*d^2 - d
^2*x*6i - f^2*x*3i - d^2*3i - f^2*6i - 2*3^(1/2)*d*e - 3*3^(1/2)*d*f + 3^(1
/2)*d*g + 4*3^(1/2)*e*f - 3*3^(1/2)*d*h - 2*3^(1/2)*e*h - 2*3^(1/2)*f*g + 3
*3^(1/2)*f*h + 3^(1/2)*g*h + d*f*x*9i - e*f*x*6i + d*h*x*3i + e*h*x*6i + f*
g*x*3i - f*h*x*3i - g*h*x*3i - 3*3^(1/2)*f^2*x + 3*3^(1/2)*d*f*x + 2*3^(1/2
)*d*g*x + 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x + 2*3^(1/2)*e*h*x - 3^(1/2)*f*g
*x + 3*3^(1/2)*f*h*x - 3^(1/2)*g*h*x - 4*3^(1/2)*d*e*x)*(d/4 - f/4 + g/4 +
(3^(1/2)*d*1i)/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 + (3^(1/2)*g*1i)/1
2 - (3^(1/2)*h*1i)/6) + log(d*g*3i - d*f*9i - d*e*6i + d*h*3i + e*h*6i - f*
h*3i - g*h*3i + 3*3^(1/2)*d^2 - d^2*x*6i - f^2*x*3i + d^2*3i + f^2*6i + 2*3
^(1/2)*d*e - 3*3^(1/2)*d*f - 3^(1/2)*d*g - 4*3^(1/2)*e*f - 3*3^(1/2)*d*h +
2*3^(1/2)*e*h + 2*3^(1/2)*f*g + 3*3^(1/2)*f*h - 3^(1/2)*g*h + d*f*x*9i + e*
f*x*6i + d*h*x*3i - e*h*x*6i - f*g*x*3i - f*h*x*3i + g*h*x*3i + 3*3^(1/2)*f
```

```
^2*x - 3*3^(1/2)*d*f*x + 2*3^(1/2)*d*g*x + 2*3^(1/2)*e*f*x + 3*3^(1/2)*d*h*
x + 2*3^(1/2)*e*h*x - 3^(1/2)*f*g*x - 3*3^(1/2)*f*h*x - 3^(1/2)*g*h*x - 4*3
^(1/2)*d*e*x)*(f/4 - d/4 + g/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^
(1/2)*f*1i)/12 - (3^(1/2)*g*1i)/12 - (3^(1/2)*h*1i)/6) + h*x
```

3.19 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{1+x^2+x^4}\,dx$

Optimal. Leaf size=151

$$hx+\frac{ix^2}{2}-\frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(d+f-2h)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(2e-g-i)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(d-f)$$

```
[Out] h*x+1/2*i*x^2-1/4*(d-f)*ln(x^2-x+1)+1/4*(d-f)*ln(x^2+x+1)+1/4*(g-i)*ln(x^4+
x^2+1)-1/6*(d+f-2*h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*(d+f-2*h)*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/6*(2*e-g-i)*arctan(1/3*(2*x^2+1)*3^(1/2))
*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1687, 1690, 1183, 648, 632, 210, 642, 1677, 1671}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(d+f-2h)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g-i)}{2\sqrt{3}}-\frac{1}{4}(d-f)\log\left(x^2-x+1\right)+\frac{1}{4}(d-f)\log\left(x^2+x+1\right)+\frac{1}{4}(g-i)\log\left(x^4+x^2+1\right)+hx+\frac{ix^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

```
[Out] h*x + (i*x^2)/2 - ((d + f - 2*h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(2*Sqrt[3]) + (
(d + f - 2*h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(2*Sqrt[3]) + ((2*e - g - i)*ArcTa
n[(1 + 2*x^2)/Sqrt[3]])/(2*Sqrt[3]) - ((d - f)*Log[1 - x + x^2])/4 + ((d -
f)*Log[1 + x + x^2])/4 + ((g - i)*Log[1 + x^2 + x^4])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+19x^5}{1+x^2+x^4}\,dx = \int \frac{x(e+gx^2+19x^4)}{1+x^2+x^4}\,dx + \int \frac{d+fx^2+hx^4}{1+x^2+x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+19x^2}{1+x+x^2}\,dx, x, x^2\right) + \int \left(h + \frac{d-h+(f-h}{1+x^2+x^4}\right.$$

$$= hx + \frac{1}{2}\text{Subst}\left(\int \left(19 - \frac{19-e+(19-g)x}{1+x+x^2}\right)dx, x, x^2\right) + \int \frac{d}{}$$

$$= hx + \frac{19x^2}{2} + \frac{1}{2}\int \frac{d-h-(d-f)x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{d-h+(d-f)x}{1+x+x^2}$$

$$= hx + \frac{19x^2}{2} + \frac{1}{4}(d-f)\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{4}(-d+f)\int \frac{-1+}{1-x+}$$

$$= hx + \frac{19x^2}{2} - \frac{1}{4}(d-f)\log\left(1-x+x^2\right) + \frac{1}{4}(d-f)\log\left(1+x+\right.$$

$$= hx + \frac{19x^2}{2} - \frac{(d+f-2h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{(d+f-2h)\tan^{-1}}{2\sqrt{3}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 0.36, size = 187, normalized size = 1.24

$$\frac{1}{12}\left(6x(2h+ix)+\left(1+i\sqrt{3}\right)\left(2\sqrt{3}d-\left(3i+\sqrt{3}\right)f-\left(-3i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)+\left(i+\sqrt{3}\right)\left(-2i\sqrt{3}d+\left(3+i\sqrt{3}\right)f+i\left(3i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)-2\sqrt{3}(2e-g-i)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)+3(g-i)\log\left(1+x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4),x]
```

```
[Out] (6*x*(2*h + i*x) + (1 + I*Sqrt[3])*(2*Sqrt[3]*d - (3*I + Sqrt[3])*f - (-3*I
 + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2] + (I + Sqrt[3])*((-2*I)*Sqrt[3]
*d + (3 + I*Sqrt[3])*f + I*(3*I + Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2] -
 2*Sqrt[3]*(2*e - g - i)*ArcTan[Sqrt[3]/(1 + 2*x^2)] + 3*(g - i)*Log[1 + x^
2 + x^4])/12
```

<u>Maple</u> [A]
time = 0.21, size = 117, normalized size = 0.77

method	result
default	$\frac{i\,x^2}{2}+hx+\frac{(d-f+g-i)\ln(x^2+x+1)}{4}+\frac{\left(\frac{d}{2}-e+\frac{f}{2}+\frac{g}{2}-h+\frac{i}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(g-i+f-d)\ln(x^2-x+1)}{4}+$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*i*x^2+h*x+1/4*(d-f+g-i)*ln(x^2+x+1)+1/3*(1/2*d-e+1/2*f+1/2*g-h+1/2*i)*a
rctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/4*(g-i+f-d)*ln(x^2-x+1)+1/3*(1/2*d+e+1
/2*f-1/2*g-h-1/2*i)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 103, normalized size = 0.68

$$\frac{1}{6}\sqrt{3}\,(d+f+g-2\,h-2\,e+i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,(d+f-g-2\,h+2\,e-i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+hx+\frac{1}{2}i\,x^2+\frac{1}{4}(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g+i)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="maxima"
)
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e + I)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6
*sqrt(3)*(d + f - g - 2*h + 2*e - I)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x +
1/2*I*x^2 + 1/4*(d - f + g - I)*log(x^2 + x + 1) - 1/4*(d - f - g + I)*log(
x^2 - x + 1)
```

Fricas [A]
time = 4.47, size = 106, normalized size = 0.70

$$\frac{1}{2}ix^2+\frac{1}{6}\sqrt{3}\,(d-2\,e+f+g-2\,h+i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,(d+2\,e+f-g-2\,h-i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+hx+\frac{1}{4}(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g+i)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="fricas"
)
```

```
[Out] 1/2*i*x^2 + 1/6*sqrt(3)*(d - 2*e + f + g - 2*h + i)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/6*sqrt(3)*(d + 2*e + f - g - 2*h - i)*arctan(1/3*sqrt(3)*(2*x -
1)) + h*x + 1/4*(d - f + g - i)*log(x^2 + x + 1) - 1/4*(d - f - g + i)*log(
x^2 - x + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.75, size = 103, normalized size = 0.68

$$\frac{1}{6}\sqrt{3}\,(d+f+g-2\,h-2\,e+i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,(d+f-g-2\,h+2\,e-i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+hx+\frac{1}{2}i\,x^2+\frac{1}{4}(d-f+g-i)\log\left(x^2+x+1\right)-\frac{1}{4}(d-f-g+i)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(d + f + g - 2*h - 2*e + I)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6
*sqrt(3)*(d + f - g - 2*h + 2*e - I)*arctan(1/3*sqrt(3)*(2*x - 1)) + h*x +
1/2*I*x^2 + 1/4*(d - f + g - I)*log(x^2 + x + 1) - 1/4*(d - f - g + I)*log(
x^2 - x + 1)
```

Mupad [B]
time = 7.80, size = 1509, normalized size = 9.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1),x)
```

```
[Out] h*x - log(d*g*3i - d*f*9i - d*e*6i + d*h*3i + d*i*3i + e*h*6i - f*h*3i - g*
h*3i - h*i*3i - 3*3^(1/2)*d^2 - d^2*x*6i - f^2*x*3i + d^2*3i + f^2*6i - 2*3
^(1/2)*d*e + 3*3^(1/2)*d*f + 3^(1/2)*d*g + 4*3^(1/2)*e*f + 3*3^(1/2)*d*h +
3^(1/2)*d*i - 2*3^(1/2)*e*h - 2*3^(1/2)*f*g - 3*3^(1/2)*f*h - 2*3^(1/2)*f*i
 + 3^(1/2)*g*h + 3^(1/2)*h*i + d*f*x*9i + e*f*x*6i + d*h*x*3i - e*h*x*6i -
f*g*x*3i - f*h*x*3i - f*i*x*3i + g*h*x*3i + h*i*x*3i - 3*3^(1/2)*f^2*x + 3*
3^(1/2)*d*f*x - 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x - 2*3^(
1/2)*d*i*x - 2*3^(1/2)*e*h*x + 3^(1/2)*f*g*x + 3*3^(1/2)*f*h*x + 3^(1/2)*f*
i*x + 3^(1/2)*g*h*x + 3^(1/2)*h*i*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4 - g/4 + i
/4 + (3^(1/2)*d*1i)/12 + (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 - (3^(1/2)*g*
1i)/12 - (3^(1/2)*h*1i)/6 - (3^(1/2)*i*1i)/12) - log(d*e*6i + d*f*9i - d*g*
3i - d*h*3i - d*i*3i - e*h*6i + f*h*3i + g*h*3i + h*i*3i - 3*3^(1/2)*d^2 +
d^2*x*6i + f^2*x*3i - d^2*3i - f^2*6i - 2*3^(1/2)*d*e + 3*3^(1/2)*d*f + 3^(
1/2)*d*g + 4*3^(1/2)*e*f + 3*3^(1/2)*d*h + 3^(1/2)*d*i - 2*3^(1/2)*e*h - 2*
3^(1/2)*f*g - 3*3^(1/2)*f*h - 2*3^(1/2)*f*i + 3^(1/2)*g*h + 3^(1/2)*h*i - d
*f*x*9i - e*f*x*6i - d*h*x*3i + e*h*x*6i + f*g*x*3i + f*h*x*3i + f*i*x*3i -
 g*h*x*3i - h*i*x*3i - 3*3^(1/2)*f^2*x + 3*3^(1/2)*d*f*x - 2*3^(1/2)*d*g*x
- 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x - 2*3^(1/2)*d*i*x - 2*3^(1/2)*e*h*x + 3
^(1/2)*f*g*x + 3*3^(1/2)*f*h*x + 3^(1/2)*f*i*x + 3^(1/2)*g*h*x + 3^(1/2)*h*
i*x + 4*3^(1/2)*d*e*x)*(d/4 - f/4 - g/4 + i/4 - (3^(1/2)*d*1i)/12 - (3^(1/2
)*e*1i)/6 - (3^(1/2)*f*1i)/12 + (3^(1/2)*g*1i)/12 + (3^(1/2)*h*1i)/6 + (3^(
1/2)*i*1i)/12) - log(d*f*9i - d*e*6i + d*g*3i - d*h*3i + d*i*3i + e*h*6i +
f*h*3i - g*h*3i - h*i*3i - 3*3^(1/2)*d^2 - d^2*x*6i - f^2*x*3i - d^2*3i - f
^2*6i + 2*3^(1/2)*d*e + 3*3^(1/2)*d*f - 3^(1/2)*d*g - 4*3^(1/2)*e*f + 3*3^(
1/2)*d*h - 3^(1/2)*d*i + 2*3^(1/2)*e*h + 2*3^(1/2)*f*g - 3*3^(1/2)*f*h + 2*
3^(1/2)*f*i - 3^(1/2)*g*h - 3^(1/2)*h*i + d*f*x*9i - e*f*x*6i + d*h*x*3i +
e*h*x*6i + f*g*x*3i - f*h*x*3i + f*i*x*3i - g*h*x*3i - h*i*x*3i + 3*3^(1/2)
*f^2*x - 3*3^(1/2)*d*f*x - 2*3^(1/2)*d*g*x - 2*3^(1/2)*e*f*x + 3*3^(1/2)*d*
h*x - 2*3^(1/2)*d*i*x - 2*3^(1/2)*e*h*x + 3^(1/2)*f*g*x - 3*3^(1/2)*f*h*x +
```

```
 3^(1/2)*f*i*x + 3^(1/2)*g*h*x + 3^(1/2)*h*i*x + 4*3^(1/2)*d*e*x)*(f/4 - d/
4 - g/4 + i/4 + (3^(1/2)*d*1i)/12 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 +
(3^(1/2)*g*1i)/12 - (3^(1/2)*h*1i)/6 + (3^(1/2)*i*1i)/12) + log(d*f*9i - d*
e*6i + d*g*3i - d*h*3i + d*i*3i + e*h*6i + f*h*3i - g*h*3i - h*i*3i + 3*3^(
1/2)*d^2 - d^2*x*6i - f^2*x*3i - d^2*3i - f^2*6i - 2*3^(1/2)*d*e - 3*3^(1/2
)*d*f + 3^(1/2)*d*g + 4*3^(1/2)*e*f - 3*3^(1/2)*d*h + 3^(1/2)*d*i - 2*3^(1/
2)*e*h - 2*3^(1/2)*f*g + 3*3^(1/2)*f*h - 2*3^(1/2)*f*i + 3^(1/2)*g*h + 3^(1
/2)*h*i + d*f*x*9i - e*f*x*6i + d*h*x*3i + e*h*x*6i + f*g*x*3i - f*h*x*3i +
 f*i*x*3i - g*h*x*3i - h*i*x*3i - 3*3^(1/2)*f^2*x + 3*3^(1/2)*d*f*x + 2*3^(
1/2)*d*g*x + 2*3^(1/2)*e*f*x - 3*3^(1/2)*d*h*x + 2*3^(1/2)*d*i*x + 2*3^(1/2
)*e*h*x - 3^(1/2)*f*g*x + 3*3^(1/2)*f*h*x - 3^(1/2)*f*i*x - 3^(1/2)*g*h*x -
 3^(1/2)*h*i*x - 4*3^(1/2)*d*e*x)*(d/4 - f/4 + g/4 - i/4 + (3^(1/2)*d*1i)/1
2 - (3^(1/2)*e*1i)/6 + (3^(1/2)*f*1i)/12 + (3^(1/2)*g*1i)/12 - (3^(1/2)*h*1
i)/6 + (3^(1/2)*i*1i)/12) + (i*x^2)/2
```

3.20 $\int \frac{d+ex}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=189

$$\frac{\sqrt{2}\,\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+d*arctan(x*2^
(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-d*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1687, 12, 1107, 211, 1121, 632, 212}

$$\frac{\sqrt{2}\,\sqrt{c}\,d\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\,\sqrt{c}\,d\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*d*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqr
t[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{a+bx^2+cx^4}\,dx &= \int \frac{d}{a+bx^2+cx^4}\,dx + \int \frac{ex}{a+bx^2+cx^4}\,dx \\
&= d\int \frac{1}{a+bx^2+cx^4}\,dx + e\int \frac{x}{a+bx^2+cx^4}\,dx \\
&= \frac{(cd)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(cd)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{a+bx\ldots}\right. \\
&= \frac{\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - \ldots \\
&= \frac{\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - \ldots
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 194, normalized size = 1.03

$$
\frac{\frac{2\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}} + e\left(\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right) - \log\left(b+\sqrt{b^2-4ac}+2cx^2\right)\right)}{2\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] - (2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]] + e*(Log[-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]))/(
2*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 200, normalized size = 1.06

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$

default	$4c$	$\left(-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)}{4c}-\frac{d\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8ac-2b^2}+\frac{\sqrt{-4}\ldots}{\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*(1/4*e/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^
(1/2))-1/2*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+(-4*a*c+b^2)^(1/2)/(8*a*c-2*b^2)*(1/4*e
/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 3.34, size = 398481, normalized size = 2108.37

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/288*((-I*sqrt(3) + 1)*(6*(32*sqrt(2)*a^3*c^2*e*sqrt(-(b*d^2 - sqrt(b^2 -
4*a*c)*d^2)/(a*b^2 - 4*a^2*c)) + b^4*d^2 + (b^2 - 4*a*c)^(3/2)*b*d^2 - 8*(2
*sqrt(2)*b^2*c*e*sqrt(-(b*d^2 - sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c)) -
 2*c^2*d^2 + sqrt(b^2 - 4*a*c)*c*e^2)*a^2 + 2*(sqrt(2)*b^4*e*sqrt(-(b*d^2 -
 sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c)) - 2*(b^2 - 4*a*c)^(3/2)*e^2 - (4
```

```
*c*d^2 - sqrt(b^2 - 4*a*c)*e^2)*b^2)*a)/((b^2 - 4*a*c)^(3/2)*a*b^2 - 4*(b^2
 - 4*a*c)^(3/2)*a^2*c) - (2*b^2*e - 8*a*c*e + sqrt(2)*(b^2 - 4*a*c)^(3/2)*s
qrt(-(b*d^2 - sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c)))^2/(b^2 - 4*a*c)^3)
/(-1/64*(2*b^3*d^2*e - 6*sqrt(b^2 - 4*a*c)*b^2*d^2*e - 2*sqrt(2)*(b^2 - 4*a
*c)^(3/2)*b*d^2*sqrt(-(b*d^2 - sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c)) +
8*(b^2 - 4*a*c)^(3/2)*d^2*e + 4*(6*sqrt(2)*sqrt(b^2 - 4*a*c)*c*e^2*sqrt(-(b
*d^2 - sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c)) + sqrt(2)*(b^2 - 4*a*c)^(3
/2)*c*(-(b*d^2 - sqrt(b^2 - 4*a*c)*d^2)/(a*b^2 - 4*a^2*c))^(3/2) - 4*c*e^3)
*a^2 - (8*b*c*d^2*e - 24* ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. 2(150) = 300.
time = 5.38, size = 1437, normalized size = 7.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*d*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*
a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*a
bs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
- 4*a*c)*b*c^2)*d*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*abs(c)) + 1/4*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a
*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b^3*c^2 + 8*a^2*c
^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*e*log(x^2 + 1/2*(b +
 sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a
^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c
^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 - (b^4*c - 6*a*b^2*c^
2 - 2*b^3*c^2 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c
))*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b
^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2)
```

Mupad [B]
time = 1.32, size = 1308, normalized size = 6.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(c^2*(d*e^2 + e^3*x + 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4
- 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*
b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z
, k)^2*b^2*d - 8*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 +
16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*
a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^3*b^3*x - 16
*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2
 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*
b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^2*a*c*d + 2*root(128*a^2*b^2
*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z
^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d
^2*e^2 - c*d^4 - a*e^4, z, k)*b*e^2*x - 4*root(128*a^2*b^2*c*z^4 - 256*a^3*
c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*
z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 -
a*e^4, z, k)*c*d^2*x - 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^
4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z
^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^2*b^
2*e*x + 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*
c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2
*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)*b*d*e + 32*root(128
```

```
*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2
*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e
*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^3*a*b*c*x + 16*root(128*a^2*b^2*c*z^4
 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8
*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2
 - c*d^4 - a*e^4, z, k)^2*a*c*e*x))*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^
4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 -
4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4,
 z, k), k, 1, 4)
```

3.21 $\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=211

$$\frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{e\tan\text{l}}{}$$

```
[Out] -e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(f+(-b*f+2*c*d)/(-4*a*c+b^2)^
(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(f+(b*f-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1687, 1180, 211, 12, 1121, 632, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx = \int \frac{ex}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2}{a+bx^2+cx^4}\,dx$$

$$= e\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\Big(f + \cdots$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\cdots}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-\cdots}}}$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\cdots}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-\cdots}}}$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\cdots}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-\cdots}}}$$

Mathematica [A]
time = 0.14, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)-e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [A]
time = 0.05, size = 240, normalized size = 1.14

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2f+\_Re+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{2}+\frac{\left(-f\sqrt{-4ac+b^2}+bf-2cd\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{}}{\sqrt{\left(-b+\sqrt{-}\right.}}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{4c(4ac-b^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/2*e*ln(-b-2*c*x^2+(-4*a*c+b^2
)^(1/2))+1/2*(-f*(-4*a*c+b^2)^(1/2)+b*f-2*c*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4*(
-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-1/2*e*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/
2*(f*(-4*a*c+b^2)^(1/2)+b*f-2*c*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 20.40, size = 723401, normalized size = 3428.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/144*((-I*sqrt(3) + 1)*(3*(b^4*c*d^2 + (b^2 - 4*a*c)^(3/2)*b*c*d^2 + 16*(4
*sqrt(1/2)*c^3*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 - a*f^2)*sqr
t(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - c^2*f^2)*a^3 + 8*(2*c^3*d^2 - sqrt
(b^2 - 4*a*c)*c^2*e^2 - (4*sqrt(1/2)*c^2*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b
*f^2 - (c*d^2 - a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - c*f^2)*b
^2)*a^2 + ((4*sqrt(1/2)*c*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 -
 a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - f^2)*b^4 + (b^2 - 4*a*c
)^(3/2)*b*f^2 - 4*(b^2 - 4*a*c)^(3/2)*(e^2 + d*f)*c - 2*(4*c^2*d^2 - sqrt(b
^2 - 4*a*c)*c*e^2)*b^2)*a)/((b^2 - 4*a*c)^(3/2)*a*b^2*c - 4*(b^2 - 4*a*c)^(
3/2)*a^2*c^2) - 2*(b^2*e - 4*a*c*e + sqrt(1/2)*(b^2 - 4*a*c)^(3/2)*sqrt(-(b
*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 - a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c
- 4*a^2*c^2)))^2/(b^2 - 4*a*c)^3)/(-1/32*(b^3*c*d^2*e - 3*sqrt(b^2 - 4*a*c)
*b^2*c*d^2*e - 2*sqrt(1/2)*(b^2 - 4*a*c)^(3/2)*b*c*d^2*sqrt(-(b*c*d^2 - 4*a
*c*d*f + a*b*f^2 - (c*d^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(174) = 348$.
time = 4.70, size = 1714, normalized size = 8.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(
b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*
c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3
 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2
```

```
- 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*f)*arctan(2
*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b
^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^
4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^
2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*
d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 -
 2*(b^2 - 4*a*c)*a*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)) + 1/4*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*
c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b^3*c^2
+ 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*e*log(x^2 +
 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*
c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2)
```

Mupad [B]
time = 2.14, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(c^2*d*e^2 - c^2*d^2*f + c^2*e^3*x - a*c*f^3 - 8*root(16*a*b^4*c*
z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 + 64*a^2*c
^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2 +
32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*e*f^2*z +
 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^2*f + 2*a
*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2 + a*c*e^
4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*b^3*c^2*x + b*c*d*f^2 - 16*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z
```

^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c
^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^
2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*
d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2
*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^3*d - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*c^3*d^2*x + 4*root(16
*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 +
 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d
^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*
e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^
2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2
 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*b^2*c^2*d + 32*root(1
6*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*
d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c
*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e
^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^
2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*a*b*c^3*x + 16*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^3*e*x + 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*a*c^2*f^2*x + 2*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*b*c^2*e^2*x - 2*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*

```
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*b^2*c*f^2*x - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*b^2*c^2*e*x + 4*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z
^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c
^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^
2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*
d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2
*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^...
```

3.22 $\int \frac{d+ex+fx^2+gx^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=245

$$\frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{(2ce\ \cdots}{\cdots}$$

```
[Out] 1/4*g*ln(c*x^4+b*x^2+a)/c-1/2*(-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*(f+(-b*f+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(f+(b*f-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1687, 1180, 211, 1261, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*
g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*L
og[a + b*x^2 + c*x^4])/(4*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{a+bx^2+cx^4}\,dx = \int \frac{d+fx^2}{a+bx^2+cx^4}\,dx + \int \frac{x(e+gx^2)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{a+bx+cx^2}\,dx, x, x^2\right) + \frac{1}{2}\left(f - \frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\sqrt{c}\sqrt{b+}}$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\sqrt{c}\sqrt{b+}}$$

$$= \frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\sqrt{c}\sqrt{b+}}$$

Mathematica [A]
time = 0.19, size = 280, normalized size = 1.14

$$\frac{\frac{2\sqrt{2}\sqrt{c}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}+\left(2ce+\left(-b+\sqrt{b^2-4ac}\right)g\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)+\left(-2ce+\left(b+\sqrt{b^2-4ac}\right)g\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[2]*Sqrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + (2*Sqr
t[2]*Sqrt[c]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]] + (2*c*e + (-b
+ Sqrt[b^2 - 4*a*c])*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] + (-2*c*e + (
b + Sqrt[b^2 - 4*a*c])*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[b
^2 - 4*a*c])
```

Maple [A]
time = 0.05, size = 285, normalized size = 1.16

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^3g+\_R^2f+\_Re+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(-g\sqrt{-4ac+b^2}+bg-2ce\right)\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{\left(-f\sqrt{-4ac+b^2}+bf-2cd\right)\sqrt{}}{2\sqrt{(-}}\right.}{4c(4ac-b^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-1/4*(-g*(-4*a*c+b^2)^(1/2)+b*g
-2*c*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-f*(-4*a*c+b^2)^(1/2)+b*f-
2*c*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/4*(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(f*(-4
*a*c+b^2)^(1/2)+b*f-2*c*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3274 vs. 2(203) = 406.
time = 6.75, size = 3274, normalized size = 13.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*g*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a
^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*
a*c)*a*c^3)*c^2*f + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 - 4*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^3 +
8*(b^2 - 4*a*c)*a*c^4)*d*abs(c) + 2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d - (2*b^4*c^4 -
8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 -
```

```
 4*a*c)*b^2*c^4)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3
))/c^2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c
^4 + a*b^2*c^4 - 4*a^2*c^5)*c^2) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c
)*a*c^3)*c^2*f - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(
b^2 - 4*a*c)*a*c^4)*d*abs(c) + 2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d - (2*b^4*c^4 - 8*a
*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*
a*c)*b^2*c^4)*f)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/
c^2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4
+ a*b^2*c^4 - 4*a^2*c^5)*c^2) - 1/16*((b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b
^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c +
 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*g*abs
(c) - 2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3
*c^3 - 4*a*b*c^4 + (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^
3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*abs(c)*e + (b^6*c - 8*a*b^4*c^2 -
 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c
- 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*g - 2*(b^5*c^2 - 8*
a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 -
(b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*e)*log(x^2
 + 1/2*(b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3
*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) - 1/16*(
(b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b
^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
- 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*g*abs(c) - 2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c
^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 8*a*b^2*c^
2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^...
```

Mupad [B]
time = 2.54, size = 2500, normalized size = 10.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(c^2*d*e^2 + b^2*d*g^2 - c^2*d^2*f + c^2*e^3*x - a*c*f^3 - 8*root
(128*a^2*b^2*c^3*z^4 - 16*a*b^4*c^2*z^4 - 256*a^3*c^4*z^4 - 128*a^2*b^2*c^2
*g*z^3 + 16*a*b^4*c*g*z^3 + 256*a^3*c^3*g*z^3 + 32*a^2*b*c^2*e*g*z^2 + 16*a
*b^2*c^2*d*f*z^2 - 8*a*b^3*c*e*g*z^2 + 40*a^2*b^2*c*g^2*z^2 + 16*a^2*b*c^2*
f^2*z^2 + 8*a*b^2*c^2*e^2*z^2 - 64*a^2*c^3*d*f*z^2 - 4*a*b^3*c*f^2*z^2 + 16
*a*b*c^3*d^2*z^2 - 96*a^3*c^2*g^2*z^2 - 32*a^2*c^3*e^2*z^2 - 4*b^3*c^2*d^2*
z^2 - 4*a*b^4*g^2*z^2 - 8*a*b^2*c*d*f*g*z + 32*a^2*c^2*d*f*g*z - 16*a^2*b*c
*e*g^2*z - 4*a*b^2*c*e^2*g*z - 16*a*b*c^2*d^2*g*z + 4*a*b^2*c*e*f^2*z + 16*
a^2*c^2*e^2*g*z - 16*a^2*c^2*e*f^2*z - 4*b^2*c^2*d^2*e*z + 4*b^3*c*d^2*g*z
 + 4*a*b^3*e*g^2*z + 16*a*c^3*d^2*e*z + 16*a^3*c*g^3*z - 4*a^2*b^2*g^3*z - 4
*a*b*c*d*e*f*g + 2*a*b*c*e^3*g + 2*a*b*c*d*f^3 + 4*a^2*c*e*f^2*g - 4*a^2*c*
d*f*g^2 + 2*b^2*c*d^2*e*g - 4*a*c^2*d^2*e*g + 2*a*b^2*d*f*g^2 + 4*a*c^2*d*e
^2*f + 3*a*b*c*d^2*g^2 + 2*a^2*b*e*g^3 + 2*b*c^2*d^3*f - a*b*c*e^2*f^2 - 2*
a^2*c*e^2*g^2 - 2*a*c^2*d^2*f^2 - a^2*b*f^2*g^2 - b^2*c*d^2*f^2 - a*b^2*e^2
*g^2 - b*c^2*d^2*e^2 - b^3*d^2*g^2 - a^2*c*f^4 - a*c^2*e^4 - a^3*g^4 - c^3*
d^4, z, k)^3*b^3*c^2*x - a*c*d*g^2 + b*c*d*f^2 - a*b*f*g^2 - a*b*g^3*x - 16
*root(128*a^2*b^2*c^3*z^4 - 16*a*b^4*c^2*z^4 - 256*a^3*c^4*z^4 - 128*a^2*b^
2*c^2*g*z^3 + 16*a*b^4*c*g*z^3 + 256*a^3*c^3*g*z^3 + 32*a^2*b*c^2*e*g*z^2 +
 16*a*b^2*c^2*d*f*z^2 - 8*a*b^3*c*e*g*z^2 + 40*a^2*b^2*c*g^2*z^2 + 16*a^2*b
*c^2*f^2*z^2 + 8*a*b^2*c^2*e^2*z^2 - 64*a^2*c^3*d*f*z^2 - 4*a*b^3*c*f^2*z^2
 + 16*a*b*c^3*d^2*z^2 - 96*a^3*c^2*g^2*z^2 - 32*a^2*c^3*e^2*z^2 - 4*b^3*c^2
*d^2*z^2 - 4*a*b^4*g^2*z^2 - 8*a*b^2*c*d*f*g*z + 32*a^2*c^2*d*f*g*z - 16*a^
2*b*c*e*g^2*z - 4*a*b^2*c*e^2*g*z - 16*a*b*c^2*d^2*g*z + 4*a*b^2*c*e*f^2*z
 + 16*a^2*c^2*e^2*g*z - 16*a^2*c^2*e*f^2*z - 4*b^2*c^2*d^2*e*z + 4*b^3*c*d^2
*g*z + 4*a*b^3*e*g^2*z + 16*a*c^3*d^2*e*z + 16*a^3*c*g^3*z - 4*a^2*b^2*g^3*
z - 4*a*b*c*d*e*f*g + 2*a*b*c*e^3*g + 2*a*b*c*d*f^3 + 4*a^2*c*e*f^2*g - 4*a
^2*c*d*f*g^2 + 2*b^2*c*d^2*e*g - 4*a*c^2*d^2*e*g + 2*a*b^2*d*f*g^2 + 4*a*c^
2*d*e^2*f + 3*a*b*c*d^2*g^2 + 2*a^2*b*e*g^3 + 2*b*c^2*d^3*f - a*b*c*e^2*f^2
 - 2*a^2*c*e^2*g^2 - 2*a*c^2*d^2*f^2 - a^2*b*f^2*g^2 - b^2*c*d^2*f^2 - a*b^
2*e^2*g^2 - b*c^2*d^2*e^2 - b^3*d^2*g^2 - a^2*c*f^4 - a*c^2*e^4 - a^3*g^4 -
 c^3*d^4, z, k)^2*a*c^3*d - 4*root(128*a^2*b^2*c^3*z^4 - 16*a*b^4*c^2*z^4 -
 256*a^3*c^4*z^4 - 128*a^2*b^2*c^2*g*z^3 + 16*a*b^4*c*g*z^3 + 256*a^3*c^3*g
*z^3 + 32*a^2*b*c^2*e*g*z^2 + 16*a*b^2*c^2*d*f*z^2 - 8*a*b^3*c*e*g*z^2 + 40
*a^2*b^2*c*g^2*z^2 + 16*a^2*b*c^2*f^2*z^2 + 8*a*b^2*c^2*e^2*z^2 - 64*a^2*c^
3*d*f*z^2 - 4*a*b^3*c*f^2*z^2 + 16*a*b*c^3*d^2*z^2 - 96*a^3*c^2*g^2*z^2 - 3
2*a^2*c^3*e^2*z^2 - 4*b^3*c^2*d^2*z^2 - 4*a*b^4*g^2*z^2 - 8*a*b^2*c*d*f*g*z
 + 32*a^2*c^2*d*f*g*z - 16*a^2*b*c*e*g^2*z - 4*a*b^2*c*e^2*g*z - 16*a*b*c^2
```

```
*d^2*g*z + 4*a*b^2*c*e*f^2*z + 16*a^2*c^2*e^2*g*z - 16*a^2*c^2*e*f^2*z - 4*
b^2*c^2*d^2*e*z + 4*b^3*c*d^2*g*z + 4*a*b^3*e*g^2*z + 16*a*c^3*d^2*e*z + 16
*a^3*c*g^3*z - 4*a^2*b^2*g^3*z - 4*a*b*c*d*e*f*g + 2*a*b*c*e^3*g + 2*a*b*c*
d*f^3 + 4*a^2*c*e*f^2*g - 4*a^2*c*d*f*g^2 + 2*b^2*c*d^2*e*g - 4*a*c^2*d^2*e
*g + 2*a*b^2*d*f*g^2 + 4*a*c^2*d*e^2*f + 3*a*b*c*d^2*g^2 + 2*a^2*b*e*g^3 +
2*b*c^2*d^3*f - a*b*c*e^2*f^2 - 2*a^2*c*e^2*g^2 - 2*a*c^2*d^2*f^2 - a^2*b*f
^2*g^2 - b^2*c*d^2*f^2 - a*b^2*e^2*g^2 - b*c^2*d^2*e^2 - b^3*d^2*g^2 - a^2*
c*f^4 - a*c^2*e^4 - a^3*g^4 - c^3*d^4, z, k)*c^3*d^2*x - 2*root(128*a^2*b^2
*c^3*z^4 - 16*a*b^4*c^2*z^4 - 256*a^3*c^4*z^4 - 128*a^2*b^2*c^2*g*z^3 + 16*
a*b^4*c*g*z^3 + 256*a^3*c^3*g*z^3 + 32*a^2*b*c^2*e*g*z^2 + 16*a*b^2*c^2*d*f
*z^2 - 8*a*b^3*c*e*g*z^2 + 40*a^2*b^2*c*g^2*z^2 + 16*a^2*b*c^2*f^2*z^2 + 8*
a*b^2*c^2*e^2*z^2 - 64*a^2*c^3*d*f*z^2 - 4*a*b^3*c*f^2*z^2 + 16*a*b*c^3*d^2
*z^2 - 96*a^3*c^2*g^2*z^2 - 32*a^2*c^3*e^2*z^2 - 4*b^3*c^2*d^2*z^2 - 4*a*b^
4*g^2*z^2 - 8*a*b^2*c*d*f*g*z + 32*a^2*c^2*d*f*g*z - 16*a^2*b*c*e*g^2*z - 4
*a*b^2*c*e^2*g*z - 16*a*b*c^2*d^2*g*z + 4*a*b^2*c*e*f^2*z + 16*a^2*c^2*e^2*
g*z - 16*a^2*c^2*e*f^2*z - 4*b^2*c^2*d^2*e*z + 4*b^3*c*d^2*g*z + 4*a*b^3*e*
g^2*z + 16*a*c^3*d^2*e*z + 16*a^3*c*g^3*z - 4*a^2*b^2*g^3*z - 4*a*b*c*d*e*f
*g + 2*a*b*c*e^3*g + 2*a*b*c*d*f^3 + 4*a^2*c*e*f^2*g - 4*a^2*c*d*f*g^2 + 2*
b^2*c*d^2*e*g - 4*a*c^2*d^2*e*g + 2*a*b^2*d*f*g^2 + 4*a*c^2*d*e^2*f + 3*a*b
*c*d^2*g^2 + 2*a^2*b*e*g^3 + 2*b*c^2*d^3*f - a*b*c*e^2*f^2 - 2*a^2*c*e^2*g^
2 - 2*a*c^2*d^2*f^2 - a^2*b*f^2*g^2 - b^2*c*d^2*f^2 - a*b^2*e^2*g^2 - b*c^2
*d^2*e^2 - b^3*d^2*g^2 - a^2*c*f^4 - a*c^2*e^4 - a^3*g^4 - c^3*d^4, z, k)*b
^3*g^2*x + b^2*e*g^2*x + c^2*d^2*g*x + 4*root(128*a^2*b^2*c^3*z^4 - 16*a*b^
4*c^2*z^4 - 256*a^3*c^4*z^4 - 128*a^2*b^2*c^2*g*z^3 + 16*a*b^4*c*g*z^3 + 25
6*a^3*c^3*g*z^3 + 32*a^2*b*c^2*e*g*z^2 + 16*a*b^2*c^2*d*f*z^2 - 8*a*b^3*c*e
*g*z^2 + 40*a^2*b^2*c*g^2*z^2 + 16*a^2*b*c^2*f^2*z^2 + 8*a*b^2*c^2*e^2*z^2
- 64*a^2*c^3*d*f*z^2 - 4*a*b^3*c*f^2*z^2 + 16*a*b*c^3*d^2*z^2 - 96*a^3*c^2*
g^2*z^2 - 32*a^2*c^3*e^2*z^2 - 4*b^3*c^2*d^2*z^2 - 4*a*b^4*g^2*z^2 - 8*a*b^
2*c*d*f*g*z + 32*a^2*c^2*d*f*g*z - 16*a^2*b*c*e*g^2*z - 4*a*b^2*c*e^2*g*z -
 16*a*b*c^2*d^2*g*z + 4*a*b^2*c*e*f^2*z + 16*a^...
```

3.23 $\int \frac{d+ex+fx^2+gx^3+hx^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=290

$$\frac{hx}{c}+\frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(cf-bh-\frac{2c^2d-bcf+b^2h-2ach}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] h*x/c+1/4*g*ln(c*x^4+b*x^2+a)/c-1/2*(-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*(c*f-b*h+(2*c^2*d+b^2*h-c*(2*a*h+b*f))/(-4*a*c+b^2)^(1/2
))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c*f-b*h+(2*a*c*h-b^2*h+b*c*f-2*c^2*d)/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1687, 1690, 1180, 211, 1261, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{g\log\left(a+bx^2+cx^4\right)}{4c}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d - b*c*f + b^2*h -
2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*c*e - b*g)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (g*Log[a
+ b*x^2 + c*x^4])/(4*c)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{a+bx^2+cx^4}\,dx = \int \frac{x(e+gx^2)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+hx^4}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{a+bx+cx^2}\,dx, x, x^2\right) + \int \left(\frac{h}{c} + \frac{cd-ah+(cf-bh)x^2}{c\,(a+bx^2+cx^4)}\right)$$

$$= \frac{hx}{c} + \frac{\int \frac{cd-ah+(cf-bh)x^2}{a+bx^2+cx^4}\,dx}{c} + \frac{g\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{(2ce-bg)\text{Su}}{}$$

$$= \frac{hx}{c} + \frac{g\log\left(a+bx^2+cx^4\right)}{4c} - \frac{(2ce-bg)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right.}{2c}$$

$$= \frac{hx}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(c\right.}{}$$

Mathematica [A]
time = 0.31, size = 383, normalized size = 1.32

$$\frac{4\sqrt{c}\,hx + \frac{2\sqrt{2}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)h+c\left(-bf+\sqrt{b^2-4ac}\,f-2ah\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)h-c\left(bf+\sqrt{b^2-4ac}\,f+2ah\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{\sqrt{c}\left(2ce+\left(-b+\sqrt{b^2-4ac}\right)g\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\sqrt{c}\left(-2ce+\left(b+\sqrt{b^2-4ac}\right)g\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*h*x + (2*Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*h + c*(-(b
*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt
[2]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b*f + Sqrt[b^2 - 4*a*c]*f +
 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(2*c*e + (-b + Sqrt[b^2 -
 4*a*c])*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (Sqr
t[c]*(-2*c*e + (b + Sqrt[b^2 - 4*a*c])*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

Maple [A]
time = 0.06, size = 353, normalized size = 1.22

method	result

risch	$\frac{hx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(cg\_R^3+(-bh+cf)\_R^2+ce\_R-ah+cd\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{hx}{c}+\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(\sqrt{-4ac+b^2}\,cg-bcg+2c^2e\right)\ln\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)}{4c}+\frac{\left(-\sqrt{-4ac+b^2}\,bh+\sqrt{-4 \ldots\right.}{\ldots}\right.}{c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] h*x/c+(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-1/4*((-4*a*c+b^2)^(1/2)*c*g-b*c*g+
2*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-(-4*a*c+b^2)^(1/2)*b*h+(
-4*a*c+b^2)^(1/2)*f*c-2*a*c*h+b^2*h-f*b*c+2*c^2*d)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+(
-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/4*(-(-4*a*c+b^2)^(1/2)*c*g-b*c*g+2*c^2*e
)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*((-4*a*c+b^2)^(1/2)*b*h-(-4*a*c+b^
2)^(1/2)*f*c-2*a*c*h+b^2*h-f*b*c+2*c^2*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] h*x/c + integrate((c*g*x^3 + (c*f - b*h)*x^2 + c*x*e + c*d - a*h)/(c*x^4 +
b*x^2 + a), x)/c
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5203 vs. 2(248) = 496.
time = 5.07, size = 5203, normalized size = 17.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/4*g*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^
4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3
 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*f - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a
*c)*a*b*c^3)*c^2*h + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 +
 8*(b^2 - 4*a*c)*a*c^5)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
```

```
*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16
*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c
^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*h*abs(c) + 2*(2*b
^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^
2 - 4*a*c)*b*c^6)*d - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*f + (2*b^5*c^4 - 12*a*b
^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 -
4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*h)*arctan(2*sqrt(1/2)*x/sqrt((b*c
^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c
^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c
^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2
- 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*f - (2*b^5*c^2 - 16*a*b^3*c^3
 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^
2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*h - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*s...
```

Mupad [B]

time = 1.75, size = 2500, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log((x*(c^3*e^3 + c^3*d^2*g + b^3*e*h^2 - a*b*c*g^3 - 2*c^3*d*e*f +
a*c^2*e*g^2 + b*c^2*e*f^2 - a*c^2*f^2*g - 2*b*c^2*e^2*g + b^2*c*e*g^2 - a*b
^2*g*h^2 + a^2*c*g*h^2 - 2*a*b*c*e*h^2 + 2*b*c^2*d*e*h - 2*a*c^2*d*g*h + 2*
a*c^2*e*f*h - 2*b^2*c*e*f*h + 2*a*b*c*f*g*h))/c - root(128*a^2*b^2*c^4*z^4
- 16*a*b^4*c^3*z^4 - 256*a^3*c^5*z^4 - 128*a^2*b^2*c^3*g*z^3 + 16*a*b^4*c^2
*g*z^3 + 256*a^3*c^4*g*z^3 + 32*a^2*b*c^3*e*g*z^2 + 32*a^2*b*c^3*d*h*z^2 -
8*a*b^3*c^2*e*g*z^2 - 8*a*b^3*c^2*d*h*z^2 + 16*a*b^2*c^3*d*f*z^2 + 8*a*b^4*
c*f*h*z^2 - 48*a^2*b^2*c^2*f*h*z^2 - 48*a^3*b*c^2*h^2*z^2 + 28*a^2*b^3*c*h^
2*z^2 + 16*a^2*b*c^3*f^2*z^2 - 4*a*b^3*c^2*f^2*z^2 + 8*a*b^2*c^3*e^2*z^2 +
64*a^3*c^3*f*h*z^2 - 64*a^2*c^4*d*f*z^2 - 4*a*b^4*c*g^2*z^2 + 16*a*b*c^4*d^
2*z^2 + 40*a^2*b^2*c^2*g^2*z^2 - 96*a^3*c^3*g^2*z^2 - 32*a^2*c^4*e^2*z^2 -
4*b^3*c^3*d^2*z^2 - 4*a*b^5*h^2*z^2 + 8*a^2*b^2*c*f*g*h*z + 32*a^2*b*c^2*e*
f*h*z - 8*a*b^2*c^2*d*f*g*z + 8*a*b^2*c^2*d*e*h*z - 8*a*b^3*c*e*f*h*z - 20*
a^2*b^2*c*e*h^2*z - 16*a^2*b*c^2*e*g^2*z - 4*a*b^2*c^2*e^2*g*z + 4*a*b^2*c^
2*e*f^2*z - 32*a^3*c^2*f*g*h*z + 32*a^2*c^3*d*f*g*z - 32*a^2*c^3*d*e*h*z +
16*a^3*b*c*g*h^2*z + 4*a*b^3*c*e*g^2*z - 16*a*b*c^3*d^2*g*z - 4*a^2*b^3*g*h
^2*z + 16*a^3*c^2*e*h^2*z + 16*a^2*c^3*e^2*g*z + 4*b^3*c^2*d^2*g*z - 16*a^2
*c^3*e*f^2*z - 4*b^2*c^3*d^2*e*z - 4*a^2*b^2*c*g^3*z + 4*a*b^4*e*h^2*z + 16
*a*c^4*d^2*e*z + 16*a^3*c^2*g^3*z - 4*a^2*b*c*e*f*g*h - 4*a*b*c^2*d*e*f*g +
 8*a^2*c^2*d*e*g*h - 2*a^2*b*c*d*g^2*h + 2*a*b^2*c*e^2*f*h - 4*a*b^2*c*d*f^
2*h - 2*a^2*b*c*d*f*h^2 - 2*a*b*c^2*d^2*f*h + 2*a*b^2*c*d*f*g^2 - 2*a*b*c^2
*d*e^2*h - 4*a^2*c^2*e^2*f*h + 2*a^2*b^2*e*g*h^2 + 4*a^2*c^2*e*f^2*g + 4*a^
2*c^2*d*f^2*h - 4*a^2*c^2*d*f*g^2 + 2*b^2*c^2*d^2*e*g + 3*a^2*b*c*e^2*h^2 +
 4*a*b^2*c*d^2*h^2 + 3*a*b*c^2*d^2*g^2 + 4*a^3*c*f*g^2*h - 4*a^3*c*e*g*h^2
+ 2*b^3*c*d^2*f*h + 2*a*b^3*d*f*h^2 - 4*a*c^3*d^2*e*g + 2*a^2*b*c*f^3*h + 4
*a*c^3*d*e^2*f + 2*a^2*b*c*e*g^3 + 2*a*b*c^2*e^3*g + 2*a*b*c^2*d*f^3 + 2*a^
3*b*f*h^3 + 4*a^3*c*d*h^3 + 4*a*c^3*d^3*h + 2*b*c^3*d^3*f - a^2*b*c*f^2*g^2
 - a*b^2*c*e^2*g^2 - a*b*c^2*e^2*f^2 - 6*a^2*c^2*d^2*h^2 - 2*a^2*c^2*e^2*g^
2 - 2*a^3*c*f^2*h^2 - 2*b^2*c^2*d^3*h - 2*a^2*b^2*d*h^3 - 2*a*c^3*d^2*f^2 -
 a^2*b^2*f^2*h^2 - b^2*c^2*d^2*f^2 - a^3*b*g^2*h^2 - b^3*c*d^2*g^2 - a*b^3*
e^2*h^2 - b*c^3*d^2*e^2 - b^4*d^2*h^2 - a^2*c^2*f^4 - a^3*c*g^4 - a*c^3*e^4
 - a^4*h^4 - c^4*d^4, z, k)*(root(128*a^2*b^2*c^4*z^4 - 16*a*b^4*c^3*z^4 -
256*a^3*c^5*z^4 - 128*a^2*b^2*c^3*g*z^3 + 16*a*b^4*c^2*g*z^3 + 256*a^3*c^4*
g*z^3 + 32*a^2*b*c^3*e*g*z^2 + 32*a^2*b*c^3*d*h*z^2 - 8*a*b^3*c^2*e*g*z^2 -
 8*a*b^3*c^2*d*h*z^2 + 16*a*b^2*c^3*d*f*z^2 + 8*a*b^4*c*f*h*z^2 - 48*a^2*b^
2*c^2*f*h*z^2 - 48*a^3*b*c^2*h^2*z^2 + 28*a^2*b^3*c*h^2*z^2 + 16*a^2*b*c^3*
f^2*z^2 - 4*a*b^3*c^2*f^2*z^2 + 8*a*b^2*c^3*e^2*z^2 + 64*a^3*c^3*f*h*z^2 -
64*a^2*c^4*d*f*z^2 - 4*a*b^4*c*g^2*z^2 + 16*a*b*c^4*d^2*z^2 + 40*a^2*b^2*c^
2*g^2*z^2 - 96*a^3*c^3*g^2*z^2 - 32*a^2*c^4*e^2*z^2 - 4*b^3*c^3*d^2*z^2 - 4
*a*b^5*h^2*z^2 + 8*a^2*b^2*c*f*g*h*z + 32*a^2*b*c^2*e*f*h*z - 8*a*b^2*c^2*d
*f*g*z + 8*a*b^2*c^2*d*e*h*z - 8*a*b^3*c*e*f*h*z - 20*a^2*b^2*c*e*h^2*z - 1
```

```
6*a^2*b*c^2*e*g^2*z - 4*a*b^2*c^2*e^2*g*z + 4*a*b^2*c^2*e*f^2*z - 32*a^3*c^
2*f*g*h*z + 32*a^2*c^3*d*f*g*z - 32*a^2*c^3*d*e*h*z + 16*a^3*b*c*g*h^2*z +
4*a*b^3*c*e*g^2*z - 16*a*b*c^3*d^2*g*z - 4*a^2*b^3*g*h^2*z + 16*a^3*c^2*e*h
^2*z + 16*a^2*c^3*e^2*g*z + 4*b^3*c^2*d^2*g*z - 16*a^2*c^3*e*f^2*z - 4*b^2*
c^3*d^2*e*z - 4*a^2*b^2*c*g^3*z + 4*a*b^4*e*h^2*z + 16*a*c^4*d^2*e*z + 16*a
^3*c^2*g^3*z - 4*a^2*b*c*e*f*g*h - 4*a*b*c^2*d*e*f*g + 8*a^2*c^2*d*e*g*h -
2*a^2*b*c*d*g^2*h + 2*a*b^2*c*e^2*f*h - 4*a*b^2*c*d*f^2*h - 2*a^2*b*c*d*f*h
^2 - 2*a*b*c^2*d^2*f*h + 2*a*b^2*c*d*f*g^2 - 2*a*b*c^2*d*e^2*h - 4*a^2*c^2*
e^2*f*h + 2*a^2*b^2*e*g*h^2 + 4*a^2*c^2*e*f^2*g + 4*a^2*c^2*d*f^2*h - 4*a^2
*c^2*d*f*g^2 + 2*b^2*c^2*d^2*e*g + 3*a^2*b*c*e^2*h^2 + 4*a*b^2*c*d^2*h^2 +
3*a*b*c^2*d^2*g^2 + 4*a^3*c*f*g^2*h - 4*a^3*c*e*g*h^2 + 2*b^3*c*d^2*f*h + 2
*a*b^3*d*f*h^2 - 4*a*c^3*d^2*e*g + 2*a^2*b*c*f^3*h + 4*a*c^3*d*e^2*f + 2*a^
2*b*c*e*g^3 + 2*a*b*c^2*e^3*g + 2*a*b*c^2*d*f^3 + 2*a^3*b*f*h^3 + 4*a^3*c*d
*h^3 + 4*a*c^3*d^3*h + 2*b*c^3*d^3*f - a^2*b*c*f^2*g^2 - a*b^2*c*e^2*g^2 -
a*b*c^2*e^2*f^2 - 6*a^2*c^2*d^2*h^2 - 2*a^2*c^2*e^2*g^2 - 2*a^3*c*f^2*h^2 -
 2*b^2*c^2*d^3*h - 2*a^2*b^2*d*h^3 - 2*a*c^3*d^2*f^2 - a^2*b^2*f^2*h^2 - b^
2*c^2*d^2*f^2 - a^3*b*g^2*h^2 - b^3*c*d^2*g^2 - a*b^3*e^2*h^2 - b*c^3*d^2*e
^2 - b^4*d^2*h^2 - a^2*c^2*f^4 - a^3*c*g^4 - a*c^3*e^4 - a^4*h^4 - c^4*d^4,
 z, k)*((x*(4*b^2*c^3*e - 8*b^3*c^2*g - 16*a*c^4*e + 32*a*b*c^3*g))/c - (4*
b^2*c^3*d + 16*a^2*c^3*h - 16*a*c^4*d - 4*a*b^2*c^2*h)/c + (root(128*a^2*b^
2*c^4*z^4 - 16*a*b^4*c^3*z^4 - 256*a^3*c^5*z^4 - 128*a^2*b^2*c^3*g*z^3 + 16
*a*b^4*c^2*g*z^3 + 256*a^3*c^4*g*z^3 + 32*a^2*b*c^3*e*g*z^2 + 32*a^2*b*c^3*
d*h*z^2 - 8*a*b^3*c^2*e*g*z^2 - 8*a*b^3*c^2*d*h*z^2 + 16*a*b^2*c^3*d*f*z^2
+ 8*a*b^4*c*f*h*z^2 - 48*a^2*b^2*c^2*f*h*z^2 - 48*a^3*b*c^2*h^2*z^2 + 28*a^
2*b^3*c*h^2*z^2 + 16*a^2*b*c^3*f^2*z^2 - 4*a*b^3*c^2*f^2*z^2 + 8*a*b^2*c^3*
e^2*z^2 + 64*a^3*c^3*f*h*z^2 - 64*a^2*c^4*d*f*z...
```

3.24 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=321

$$\frac{hx}{c}+\frac{ix^2}{2c}+\frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(cf-bh-\frac{2c^2d-bcf+b^2h-2ach}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2}}}$$

```
[Out] h*x/c+1/2*i*x^2/c+1/4*(-b*i+c*g)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c*i+b^2*i-
b*c*g+2*c^2*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/
2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*f-b*h+(2*c
^2*d+b^2*h-c*(2*a*h+b*f))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*
(c*f-b*h+(2*a*c*h-b^2*h+b*c*f-2*c^2*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1687, 1690, 1180, 211, 1677, 1671, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2ah+bf)+b^2h+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2ach+b^2h-bcf+2c^2d}{\sqrt{b^2-4ac}}-bh+cf\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(-2aci+b^2i-bcg+2c^2e)}{2c^2\sqrt{b^2-4ac}}+\frac{(cg-bi)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{hx}{c}+\frac{ix^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (h*x)/c + (i*x^2)/(2*c) + ((c*f - b*h + (2*c^2*d + b^2*h - c*(b*f + 2*a*h))
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*f - b*h - (2*c^2*d -
b*c*f + b^2*h - 2*a*c*h)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
(2*c^2*e - b*c*g + b^2*i - 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]
])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*i)*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+24x^5}{a+bx^2+cx^4}\,dx = \int \frac{x(e+gx^2+24x^4)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+hx^4}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+24x^2}{a+bx+cx^2}\,dx, x, x^2\right) + \int \left(\frac{h}{c} + \frac{cd-ah+(cf-}{c\,(a+bx^2+c}\right.$$

$$= \frac{hx}{c} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{24}{c} - \frac{24a-ce+(24b-cg)x}{c\,(a+bx+cx^2)}\right) dx, x, x^2\right) +$$

$$= \frac{hx}{c} + \frac{12x^2}{c} - \frac{\text{Subst}\left(\int \frac{24a-ce+(24b-cg)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} + \frac{\left(cf-bh-\right.}{}$$

$$= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{hx}{c} + \frac{12x^2}{c} + \frac{\left(cf-bh+\frac{2c^2d+b^2h-c(bf+2ah)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2-}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]

time = 0.41, size = 441, normalized size = 1.37

$$\frac{4chx+2cix^2+\frac{2\sqrt{2}\sqrt{c}\left(2c^2d+b\left(b-\sqrt{b^2-4ac}\right)h+c\left(-bf+\sqrt{b^2-4ac}\,f-2ah\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}\left(2c^2d+b\left(b+\sqrt{b^2-4ac}\right)h-c\left(bf+\sqrt{b^2-4ac}\,f+2ah\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{\left(2c^2e+b\left(b-\sqrt{b^2-4ac}\right)i+c\left(-bg+\sqrt{b^2-4ac}\,g-2ai\right)\right)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(2c^2e+b\left(b+\sqrt{b^2-4ac}\right)i-c\left(bg+\sqrt{b^2-4ac}\,g+2ai\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*c*h*x + 2*c*i*x^2 + (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*
c])*h + c*(-(b*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (2*Sqrt[2]*Sqrt[c]*(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*h - c*(b*f +
 Sqrt[b^2 - 4*a*c]*f + 2*a*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^2*e +
b*(b - Sqrt[b^2 - 4*a*c])*i + c*(-(b*g) + Sqrt[b^2 - 4*a*c]*g - 2*a*i))*Log
[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((2*c^2*e + b*(b +
Sqrt[b^2 - 4*a*c])*i - c*(b*g + Sqrt[b^2 - 4*a*c]*g + 2*a*i))*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^2)
```

Maple [A]

time = 0.07, size = 408, normalized size = 1.27

method	result
risch	$\frac{hx}{c}+\frac{i x^2}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left((-bi+cg)\_R^3+(-bh+cf)\_R^2+(-ai+ce)\_R-ah+cd\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{hx+\frac{1}{2}i x^2}{c}+\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(-\sqrt{-4ac+b^2}\,bi+\sqrt{-4ac+b^2}\,cg-2aci+b^2i-bcg+2c^2e\right)\ln\left(-b-2c x^2+\sqrt{-4ac+\cdots}\right.}{4c}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/c*(h*x+1/2*i*x^2)+(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-1/4*(-(-4*a*c+b^2)^(
1/2)*b*i+(-4*a*c+b^2)^(1/2)*c*g-2*a*c*i+b^2*i-b*c*g+2*c^2*e)/c*ln(-b-2*c*x^
2+(-4*a*c+b^2)^(1/2))+1/2*(-(-4*a*c+b^2)^(1/2)*b*h+(-4*a*c+b^2)^(1/2)*f*c-2
*a*c*h+b^2*h-f*b*c+2*c^2*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+(-4*a*c+b^2)^(1/2)/c/(4*
a*c-b^2)*(1/4*((-4*a*c+b^2)^(1/2)*b*i-(-4*a*c+b^2)^(1/2)*c*g-2*a*c*i+b^2*i-
b*c*g+2*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*((-4*a*c+b^2)^(1/2)*b
*h-(-4*a*c+b^2)^(1/2)*f*c-2*a*c*h+b^2*h-f*b*c+2*c^2*d)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="max
ima")
```

```
[Out] 1/2*(2*h*x + I*x^2)/c + integrate(((c*g - I*b)*x^3 + (c*f - b*h)*x^2 + c*d
- a*h + (c*e - I*a)*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6100 vs. 2(274) = 548.
time = 7.71, size = 6100, normalized size = 19.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x, algorithm="gia
c")
```

```
[Out] 1/4*(c*g - I*b)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(2*c*h*x + I*c*x^2)/c
^2 + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*f - (2*b^5*
c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b
^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*h + 2*(sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^4 +
 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*
c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a
^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*d*abs(c) - 2*(sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 +
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 -
 4*a*c)*a^2*c^4)*h*abs(c) + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2*b^4*c^5 - 8*a*b^
2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c
)*b^2*c^5)*f + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)
*h)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a
*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c
^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2

```
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^
4)*c^2*f - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*h - 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b...
```

Mupad [B]
time = 2.03, size = 2500, normalized size = 7.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log((x*(c^4*e^3 - a^3*c*i^3 + c^4*d^2*g + b^4*e*i^2 + a^2*b^2*i^3 +
b^2*c^2*e*g^2 + 3*a^2*c^2*e*i^2 + a^2*c^2*g*h^2 + 2*b^2*c^2*e^2*i - a^2*c^2
*g^2*i - 2*c^4*d*e*f - a*b*c^2*g^3 + a*c^3*e*g^2 + b*c^3*e*f^2 - a*c^3*f^2*
g - 2*b*c^3*e^2*g - 3*a*c^3*e^2*i - b*c^3*d^2*i + b^3*c*e*h^2 - a*b^3*g*i^2
 - 2*a*b*c^2*e*h^2 - 3*a*b^2*c*e*i^2 - a*b^2*c*g*h^2 + 2*a*b^2*c*g^2*i + a^
2*b*c*h^2*i - 2*b^2*c^2*e*f*h - 2*a^2*c^2*f*h*i + 2*b*c^3*d*e*h + 2*a*c^3*d
*f*i - 2*a*c^3*d*g*h + 2*a*c^3*e*f*h - 2*b^3*c*e*g*i + 2*a*b*c^2*e*g*i + 2*
a*b*c^2*f*g*h))/c^2 - (a*c^3*f^3 - c^4*d*e^2 + c^4*d^2*f - b^4*d*i^2 - b^2*
c^2*d*g^2 - a^2*c^2*d*i^2 + a^2*c^2*f*h^2 - a^2*c^2*g^2*h - a^2*b^2*h*i^2 -
 a^2*b*c*h^3 + a*c^3*d*g^2 - b*c^3*d*f^2 + a*c^3*e^2*h - b*c^3*d^2*h - b^3*
c*d*h^2 + a*b^3*f*i^2 + a^3*c*h*i^2 + 2*a*b*c^2*d*h^2 + a*b*c^2*f*g^2 + 3*a
*b^2*c*d*i^2 - 2*a*b*c^2*f^2*h + a*b^2*c*f*h^2 - 2*a^2*b*c*f*i^2 - 2*b^2*c^
2*d*e*i + 2*b^2*c^2*d*f*h - 2*a^2*c^2*e*h*i + 2*a^2*c^2*f*g*i + 2*b*c^3*d*e
*g + 2*a*c^3*d*e*i - 2*a*c^3*d*f*h - 2*a*c^3*e*f*g + 2*b^3*c*d*g*i - 4*a*b*
c^2*d*g*i + 2*a*b*c^2*e*f*i - 2*a*b^2*c*f*g*i + 2*a^2*b*c*g*h*i)/c^2 - root
(128*a^2*b^2*c^5*z^4 - 16*a*b^4*c^4*z^4 - 256*a^3*c^6*z^4 + 128*a^2*b^3*c^3
*i*z^3 - 128*a^2*b^2*c^4*g*z^3 - 256*a^3*b*c^4*i*z^3 - 16*a*b^5*c^2*i*z^3 +
 16*a*b^4*c^3*g*z^3 + 256*a^3*c^5*g*z^3 + 160*a^3*b*c^3*g*i*z^2 + 8*a*b^4*c
```

^2*f*h*z^2 + 8*a*b^4*c^2*e*i*z^2 + 32*a^2*b*c^4*e*g*z^2 + 32*a^2*b*c^4*d*h*
z^2 - 8*a*b^3*c^3*e*g*z^2 - 8*a*b^3*c^3*d*h*z^2 + 16*a*b^2*c^4*d*f*z^2 + 8*
a*b^5*c*g*i*z^2 - 72*a^2*b^3*c^2*g*i*z^2 - 48*a^2*b^2*c^3*f*h*z^2 - 48*a^2*
b^2*c^3*e*i*z^2 + 32*a^2*b^4*c*i^2*z^2 - 48*a^3*b*c^3*h^2*z^2 - 4*a*b^4*c^2
*g^2*z^2 + 16*a^2*b*c^4*f^2*z^2 - 4*a*b^3*c^3*f^2*z^2 + 8*a*b^2*c^4*e^2*z^2
 + 64*a^3*c^4*f*h*z^2 + 64*a^3*c^4*e*i*z^2 - 64*a^2*c^5*d*f*z^2 - 4*a*b^5*c
*h^2*z^2 + 16*a*b*c^5*d^2*z^2 - 56*a^3*b^2*c^2*i^2*z^2 + 28*a^2*b^3*c^2*h^2
*z^2 + 40*a^2*b^2*c^3*g^2*z^2 - 32*a^4*c^3*i^2*z^2 - 96*a^3*c^4*g^2*z^2 - 3
2*a^2*c^5*e^2*z^2 - 4*b^3*c^4*d^2*z^2 - 4*a*b^6*i^2*z^2 + 32*a^2*b*c^3*e*f*
h*z - 32*a^2*b*c^3*d*f*i*z - 8*a*b^3*c^2*e*f*h*z + 8*a*b^3*c^2*d*f*i*z - 8*
a*b^2*c^3*d*f*g*z + 8*a*b^2*c^3*d*e*h*z - 8*a*b^4*c*e*g*i*z + 40*a^2*b^2*c^
2*e*g*i*z + 8*a^2*b^2*c^2*f*g*h*z - 8*a^2*b^2*c^2*d*h*i*z + 4*a^3*b^2*c*h^2
*i*z - 32*a^3*b*c^2*g^2*i*z + 12*a^3*b^2*c*g*i^2*z + 8*a^2*b^3*c*g^2*i*z +
16*a^3*b*c^2*g*h^2*z - 4*a^2*b^3*c*g*h^2*z + 32*a^3*b*c^2*e*i^2*z - 24*a^2*
b^3*c*e*i^2*z - 16*a^2*b*c^3*e^2*i*z + 4*a*b^3*c^2*e^2*i*z + 20*a*b^2*c^3*d
^2*i*z - 16*a^2*b*c^3*e*g^2*z + 4*a*b^3*c^2*e*g^2*z - 4*a*b^2*c^3*e^2*g*z +
 4*a*b^2*c^3*e*f^2*z - 32*a^3*c^3*f*g*h*z - 32*a^3*c^3*e*g*i*z + 32*a^3*c^3
*d*h*i*z + 32*a^2*c^4*d*f*g*z - 32*a^2*c^4*d*e*h*z + 4*a*b^4*c*e*h^2*z - 16
*a*b*c^4*d^2*g*z - 4*a^2*b^2*c^2*f^2*i*z - 20*a^2*b^2*c^2*e*h^2*z - 4*a^2*b
^2*c^2*g^3*z - 16*a^4*c^2*h^2*i*z + 16*a^4*c^2*g*i^2*z + 16*a^3*c^3*f^2*i*z
 - 4*a^2*b^4*g*i^2*z - 4*b^4*c^2*d^2*i*z + 16*a^3*c^3*e*h^2*z - 16*a^2*c^4*
d^2*i*z + 16*a^2*c^4*e^2*g*z + 4*b^3*c^3*d^2*g*z - 16*a^2*c^4*e*f^2*z - 4*b
^2*c^4*d^2*e*z + 4*a*b^5*e*i^2*z - 16*a^4*b*c*i^3*z + 16*a*c^5*d^2*e*z + 4*
a^3*b^3*i^3*z + 16*a^3*c^3*g^3*z + 4*a^2*b^2*c*d*g*h*i + 12*a^2*b*c^2*d*f*g
*i - 4*a^2*b*c^2*e*f*g*h - 4*a^2*b*c^2*d*e*h*i + 4*a*b^2*c^2*d*e*f*i - 4*a^
3*b*c*f*g*h*i - 4*a*b^3*c*d*f*g*i - 4*a*b*c^3*d*e*f*g + 2*a^2*b^2*c*f^2*g*i
 - 4*a^2*b^2*c*e*g^2*i - 2*a^2*b*c^2*e^2*g*i - 8*a*b^2*c^2*d^2*g*i + 2*a^2*
b^2*c*e*g*h^2 - 2*a^2*b*c^2*e*f^2*i - 8*a^2*b^2*c*d*f*i^2 - 2*a^2*b*c^2*d*g
^2*h + 2*a*b^2*c^2*e^2*f*h - 4*a*b^2*c^2*d*f^2*h - 2*a^2*b*c^2*d*f*h^2 + 2*
a*b^2*c^2*d*f*g^2 + 8*a^3*c^2*e*f*h*i - 8*a^3*c^2*d*g*h*i + 8*a^2*c^3*d*e*g
*h - 8*a^2*c^3*d*e*f*i - 2*a^3*b*c*e*h^2*i + 6*a^3*b*c*d*h*i^2 - 2*a^3*b*c*
e*g*i^2 + 2*a*b^3*c*e^2*g*i + 6*a*b*c^3*d^2*e*i + 2*a*b^3*c*d*f*h^2 - 2*a*b
*c^3*d^2*f*h - 2*a*b*c^3*d*e^2*h + 4*a^2*b^2*c*e^2*i^2 - 5*a^2*b*c^2*d^2*i^
2 + 3*a^2*b*c^2*e^2*h^2 + 4*a*b^2*c^2*d^2*h^2 - 4*a^3*c^2*f^2*g*i + 2*a^3*b
^2*f*h*i^2 + 4*a^3*c^2*f*g^2*h + 4*a^3*c^2*e*g^2*i - 4*a^3*c^2*e*g*h^2 + 4*
a^2*c^3*d^2*g*i + 2*a^2*b^3*e*g*i^2 - 2*a^2*b^3*d*h*i^2 + 4*a^3*c^2*d*f*i^2
 - 4*a^2*c^3*e^2*f*h + 2*b^3*c^2*d^2*f*h - 2*b^3*c^2*d^2*e*i + 4*a^2*c^3*e*
f^2*g + 4*a^2*c^3*d*f^2*h - 4*a^2*c^3*d*f*g^2 + 3*a^3*b*c*f^2*i^2 + 2*b^2*c
^3*d^2*e*g + 2*a^2*b*c^2*f^3*h - 2*a*b^2*c^2*e^3*i + 5*a*b^3*c*d^2*i^2 - 2*
a^2*b^2*c*d*h^3 + 2*a^2*b*c^2*e*g^3 + 3*a*b*c^3*d^2*g^2 + 4*a^4*c*g*h^2*i -
 4*a^4*c*f*h*i^2 + 2*b^4*c*d^2*g*i + 2*a^3*b*c*g^3*i + 2*a*b^4*d*f*i^2 - 4*
a*c^4*d^2*e*g + 2*a^3*b*c*f*h^3 + 4*a*c^4*d*e^2*f + 2*a*b*c^3*e^3*g + 2*a*b
*c^3*d*f^3 - a^2*b^2*c*f^2*h^2 - a^2*b*c^2*f^2*g^2 - a*b^2*c^2*e^2*g^2 + 2*
a^4*b*g*i^3 + 4*a^4*c*e*i^3 + 4*a*c^4*d^3*h + 2*b*c^4*d^3*f - a^3*b*c*g^2*h
^2 - a*b^3*c*e^2*h^2 - 6*a^3*c^2*e^2*i^2 - 2*a^3*c^2*f^2*h^2 - a*b*c^3*e^2*

```
f^2 - 6*a^2*c^3*d^2*h^2 - 2*a^2*c^3*e^2*g^2 - 2*a^4*c*g^2*i^2 + 4*a^2*c^3*e
^3*i - 2*b^2*c^3*d^3*h - 2*a^3*b^2*e*i^3 + 4*a^3*c^2*d*h^3 - 2*a*c^4*d^2*f^
2 - a^3*b^2*g^2*i^2 - a^2*b^3*f^2*i^2 - b^3*c^2*d^2*g^2 - b^2*c^3*d^2*f^2 -
 a^4*b*h^2*i^2 - b^4*c*d^2*h^2 - a*b^4*e^2*i^2 ...
```

3.25 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=545

$$\frac{(c^2h+b^2m-c(bk+am))\,x}{c^3}+\frac{(cj-bl)x^2}{2c^2}+\frac{(ck-bm)x^3}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c}+\frac{\left(c^3f-c^2(bh+ak)-b^3m+bc(bk+\right.}{}$$

```
[Out] (c^2*h+b^2*m-c*(a*m+b*k))*x/c^3+1/2*(-b*l+c*j)*x^2/c^2+1/3*(-b*m+c*k)*x^3/c
^2+1/4*l*x^4/c+1/5*m*x^5/c+1/4*(c^2*g+b^2*l-c*(a*l+b*j))*ln(c*x^4+b*x^2+a)/
c^3-1/2*(2*c^3*e-c^2*(2*a*j+b*g)-b^3*l+b*c*(3*a*l+b*j))*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2))*(c^3*f-c^2*(a*k+b*h)-b^3*m+b*c*(2*a*m+b*k)+(2*c
^4*d-c^3*(2*a*h+b*f)+b^4*m-b^2*c*(4*a*m+b*k)+c^2*(2*a^2*m+3*a*b*k+b^2*h))/(
-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c^3*f-c^2*(a*k+b*h)-b^3*m+
b*c*(2*a*m+b*k)+(-2*c^4*d+c^3*(2*a*h+b*f)-b^4*m+b^2*c*(4*a*m+b*k)-c^2*(2*a^
2*m+3*a*b*k+b^2*h))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)
```

Rubi [A]
time = 2.99, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1687, 1690, 1180, 211, 1677, 1671, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{c^2(2a^2m+3abk+b^2h)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{c^2(2a^2m+3abk+b^2h)-b^2c(4am+bk)-c^3(2ah+bf)+b^4m+2c^4d}{\sqrt{b^2-4ac}}-c^2(ak+bh)+bc(2am+bk)+b^3(-m)+c^3f\right)}{\sqrt{2}\,c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(al+bj)+b^2l+c^2g\right)}{4c^3}+\frac{x\left(-c(am+bk)+b^2m+c^2h\right)}{c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(cj-bl)}{2c^2}+\frac{x^3(ck-bm)}{3c^2}+\frac{lx^4}{4c}+\frac{mx^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*k
 - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((c^3*f - c^2*(b*h +
 a*k) - b^3*m + b*c*(b*k + 2*a*m) + (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m -
b^2*c*(b*k + 4*a*m) + c^2*(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + ((c^3*f - c^2*(b*h + a*k) - b^3*m + b*c*(b*k +
 2*a*m) - (2*c^4*d - c^3*(b*f + 2*a*h) + b^4*m - b^2*c*(b*k + 4*a*m) + c^2*
(b^2*h + 3*a*b*k + 2*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - ((2*c^3*e - c^2*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*g + b^2*l -
c*(b*j + a*l))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx^2+cx^4}\,dx = \int \frac{x(e+gx^2+jx^4+lx^6)}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2+h\ldots}{a+b\ldots}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{e+gx+jx^2+lx^3}{a+bx+cx^2}\,dx, x, x^2\right) + \int \ldots$$

$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3} + \frac{(ck-bm)x^3}{3c^2} + \ldots$$

$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \ldots$$

$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \ldots$$

$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \ldots$$

$$= \frac{(c^2h+b^2m-c(bk+am))\,x}{c^3} + \frac{(cj-bl)x^2}{2c^2} + \ldots$$

Mathematica [A]
time = 0.82, size = 816, normalized size = 1.50

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*h + b^2*m - c*(b*k + a*m))*x)/c^3 + ((c*j - b*l)*x^2)/(2*c^2) + ((c*k
 - b*m)*x^3)/(3*c^2) + (l*x^4)/(4*c) + (m*x^5)/(5*c) + ((2*c^4*d + c^3*(-(b
*f) + Sqrt[b^2 - 4*a*c]*f - 2*a*h) + b^3*(b - Sqrt[b^2 - 4*a*c])*m + c^2*(b
^2*h - b*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k - a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m) +
 b*c*(-(b^2*k) + b*Sqrt[b^2 - 4*a*c]*k - 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m)
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)
```

```
*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d - c^3*(b*f + Sq
rt[b^2 - 4*a*c]*f + 2*a*h) + b^3*(b + Sqrt[b^2 - 4*a*c])*m + c^2*(b^2*h + b
*Sqrt[b^2 - 4*a*c]*h + 3*a*b*k + a*Sqrt[b^2 - 4*a*c]*k + 2*a^2*m) - b*c*(b^
2*k + b*Sqrt[b^2 - 4*a*c]*k + 4*a*b*m + 2*a*Sqrt[b^2 - 4*a*c]*m))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 -
 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c^3*e + c^2*(-(b*g) + Sqrt[b^2 -
 4*a*c]*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + c*(b^2*j - b*Sqrt[b^2
 - 4*a*c]*j + 3*a*b*l - a*Sqrt[b^2 - 4*a*c]*l))*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c]) + ((-2*c^3*e + c^2*(b*g + Sqrt[b^2 -
4*a*c]*g + 2*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c])*l - c*(b^2*j + b*Sqrt[b^2 -
 4*a*c]*j + 3*a*b*l + a*Sqrt[b^2 - 4*a*c]*l))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(4*c^3*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.62, size = 829, normalized size = 1.52

method	result
risch	$\frac{m x^5}{5c}+\frac{l x^4}{4c}-\frac{bm x^3}{3c^2}+\frac{k x^3}{3c}-\frac{bl x^2}{2c^2}+\frac{j x^2}{2c}-\frac{amx}{c^2}+\frac{b^2mx}{c^3}-\frac{bkx}{c^2}+\frac{hx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)} \frac{(c(-acl+b^2 \cdots}{\cdots}}{\cdots}$
default	$-\frac{-\frac{1}{5}m x^5c^2-\frac{1}{4}l x^4c^2+\frac{1}{3}bcm x^3-\frac{1}{3}c^2k x^3+\frac{1}{2}bcl x^2-\frac{1}{2}c^2j x^2+acmx-b^2mx+bckx-c^2hx}{c^3}+\frac{-\frac{\sqrt{-4ac+b^2}\left(-\frac{(\sqrt{-4ac \cdots}{\cdots}\cdots\right)}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a),x,met
hod=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/5*m*x^5*c^2-1/4*l*x^4*c^2+1/3*b*c*m*x^3-1/3*c^2*k*x^3+1/2*b*c*l*
x^2-1/2*c^2*j*x^2+a*c*m*x-b^2*m*x+b*c*k*x-c^2*h*x)+4/c^2*(-1/4*(-4*a*c+b^2)
^(1/2)/c/(4*a*c-b^2)*(-1/4*((-4*a*c+b^2)^(1/2)*a*c^2*l-(-4*a*c+b^2)^(1/2)*b
^2*c*l+(-4*a*c+b^2)^(1/2)*b*c^2*j-(-4*a*c+b^2)^(1/2)*c^3*g-3*a*b*c^2*l+2*a*
c^3*j+b^3*c*l-b^2*c^2*j+b*c^3*g-2*c^4*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2)
)+1/2*(-2*(-4*a*c+b^2)^(1/2)*a*b*m*c+c^2*(-4*a*c+b^2)^(1/2)*a*k+(-4*a*c+b^2
)^(1/2)*b^3*m-(-4*a*c+b^2)^(1/2)*b^2*k*c+c^2*(-4*a*c+b^2)^(1/2)*b*h-c^3*(-4
*a*c+b^2)^(1/2)*f-2*a^2*c^2*m+4*a*b^2*m*c-3*a*b*c^2*k+2*a*c^3*h-b^4*m+b^3*k
*c-b^2*c^2*h+b*c^3*f-2*c^4*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)
/c/(4*a*c-b^2)*(1/4*(-(-4*a*c+b^2)^(1/2)*a*c^2*l+(-4*a*c+b^2)^(1/2)*b^2*c*l
-(-4*a*c+b^2)^(1/2)*b*c^2*j+(-4*a*c+b^2)^(1/2)*c^3*g-3*a*b*c^2*l+2*a*c^3*j+
```

```
b^3*c*l-b^2*c^2*j+b*c^3*g-2*c^4*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(
2*(-4*a*c+b^2)^(1/2)*a*b*m*c-c^2*(-4*a*c+b^2)^(1/2)*a*k-(-4*a*c+b^2)^(1/2)*
b^3*m+(-4*a*c+b^2)^(1/2)*b^2*k*c-c^2*(-4*a*c+b^2)^(1/2)*b*h+c^3*(-4*a*c+b^2
)^(1/2)*f-2*a^2*c^2*m+4*a*b^2*m*c-3*a*b*c^2*k+2*a*c^3*h-b^4*m+b^3*k*c-b^2*c
^2*h+b*c^3*f-2*c^4*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="maxima")
```

```
[Out] 1/60*(12*c^2*m*x^5 + 15*c^2*l*x^4 + 20*(c^2*k - b*c*m)*x^3 + 30*(c^2*j - b*
c*l)*x^2 + 60*(c^2*h - b*c*k + (b^2 - a*c)*m)*x)/c^3 - integrate(-(c^3*d -
a*c^2*h + a*b*c*k + (c^3*g - b*c^2*j + (b^2*c - a*c^2)*l)*x^3 + (c^3*f - b*
c^2*h + (b^2*c - a*c^2)*k - (b^3 - 2*a*b*c)*m)*x^2 - (a*b^2 - a^2*c)*m - (a
*c^2*j - a*b*c*l - c^3*e)*x)/(c*x^4 + b*x^2 + a), x)/c^3
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11833 vs. $2(495) = 990$.
time = 7.86, size = 11833, normalized size = 21.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
,x, algorithm="giac")
```

```
[Out] -1/8*((2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*f - (2*b^5*c^
4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3
*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5
- 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*h + (2*b^6*c^3 - 1
8*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5
 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a
^2*c^5)*c^2*k - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*
a*c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*m - 2*(sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
```

```
b^2*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*b^4*c^6 + 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^
7 - 16*a*b^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^8 + 32*a^2
*c^8 - 2*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*d*abs(c) + 2*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 - 4
*a*c)*a^2*c^6)*h*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*
c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c^4 + 16*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 16*a^2*
b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + 32*a^3*b*c^
6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^2*b*c^5)*k*abs(c) + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^5*c^3 + 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 48*a^
3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6
- 2*(b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a*c)*...
```

Mupad [B]
time = 4.31, size = 2500, normalized size = 4.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4),x)
```

```
[Out] x^2*(j/(2*c) - (b*l)/(2*c^2)) - x*((b*(k/c - (b*m)/c^2))/c - h/c + (a*m)/c^
2) + x^3*(k/(3*c) - (b*m)/(3*c^2)) + symsum(log((c^7*d*e^2 - a*c^6*f^3 - c^
7*d^2*f + b^7*d*m^2 + a^4*c^3*k^3 + a^4*b^3*m^3 + a^2*b*c^4*h^3 + b^2*c^5*d
*g^2 + b^3*c^4*d*h^2 + a^2*c^5*d*j^2 - a^2*c^5*f*h^2 + a^2*c^5*g^2*h + b^4*
c^3*d*j^2 - a^3*c^4*d*l^2 - b^2*c^5*d^2*k + b^5*c^2*d*k^2 + 3*a^2*c^5*f^2*k
 - 3*a^3*c^4*f*k^2 + a^2*c^5*e^2*m - a^3*c^4*h*j^2 + b^3*c^4*d^2*m + a^3*c^
4*h^2*k - a^4*c^3*f*m^2 + a^2*b^5*h*m^2 - a^3*c^4*g^2*m + a^4*c^3*h*l^2 - a
^3*b^4*k*m^2 + a^4*c^3*j^2*m + a^5*c^2*k*m^2 - a^5*c^2*l^2*m - a^3*b^2*c^2*
```

k^3 - a*c^6*d*g^2 + b*c^6*d*f^2 - a*c^6*e^2*h + b*c^6*d^2*h + a*c^6*d^2*k - 2*a^5*b*c*m^3 + b^6*c*d*l^2 - a*b^6*f*m^2 - 2*a*b*c^5*d*h^2 - a*b*c^5*f*g^2 + 2*a*b*c^5*f^2*h + a*b*c^5*e^2*k - 2*a*b*c^5*d^2*m - 6*a*b^5*c*d*m^2 - 2*b^2*c^5*d*f*h - a*b^5*c*f*l^2 + 2*b^2*c^5*d*e*j - 2*b^3*c^4*d*e*l + 2*b^3*c^4*d*f*k - 2*b^3*c^4*d*g*j - 2*a^2*c^5*d*f*m + 2*a^2*c^5*d*g*l - 2*a^2*c^5*d*h*k - 2*a^2*c^5*e*f*l - 2*a^2*c^5*e*g*k + 2*a^2*c^5*e*h*j - 2*a^2*c^5*f*g*j - 2*b^4*c^3*d*f*m + 2*b^4*c^3*d*g*l - 2*b^4*c^3*d*h*k + 2*b^5*c^2*d*h*m + 2*a^3*c^4*f*h*m - 2*a^3*c^4*g*h*l - 2*b^5*c^2*d*j*l + 2*a^3*c^4*d*k*m - 2*a^3*c^4*e*j*m + 2*a^3*c^4*e*k*l + 2*a^3*c^4*f*j*l + 2*a^3*c^4*g*j*k + 2*a^4*c^3*g*l*m - 2*a^4*c^3*h*k*m - 2*a^4*c^3*j*k*l - 3*a*b^2*c^4*d*j^2 - a*b^2*c^4*f*h^2 - 4*a*b^3*c^3*d*k^2 + 3*a^2*b*c^4*d*k^2 - a*b^3*c^3*f*j^2 - 5*a*b^4*c^2*d*l^2 + 2*a^2*b*c^4*f*j^2 - 2*a*b^2*c^4*f^2*k - a*b^4*c^2*f*k^2 - 4*a^3*b*c^3*d*m^2 - a*b^2*c^4*e^2*m - 3*a^3*b*c^3*f*l^2 + 2*a*b^3*c^3*f^2*m - 5*a^2*b*c^4*f^2*m + 5*a^2*b^4*c*f*m^2 + a^2*b^4*c*h*l^2 - 4*a^3*b*c^3*h^2*m - a^3*b*c^3*j^2*k - 4*a^3*b^3*c*h*m^2 + 5*a^4*b*c^2*h*m^2 - a^3*b^3*c*k*l^2 + 2*a^4*b*c^2*k*l^2 + 2*a^3*b^3*c*k^2*m - 3*a^4*b*c^2*k^2*m + a^4*b^2*c*k*m^2 + a^4*b^2*c*l^2*m - 2*b*c^6*d*e*g + 2*a*c^6*d*f*h + 2*a*c^6*e*f*g - 2*a*c^6*d*e*j - 2*b^6*c*d*k*m + 6*a^2*b^2*c^3*d*l^2 + 3*a^2*b^2*c^3*f*k^2 + 10*a^2*b^3*c^2*d*m^2 + a^2*b^2*c^3*h*j^2 + 4*a^2*b^3*c^2*f*l^2 - 2*a^2*b^2*c^3*h^2*k + a^2*b^3*c^2*h*k^2 - 6*a^3*b^2*c^2*f*m^2 - 3*a^3*b^2*c^2*h*l^2 + 2*a^2*b^3*c^2*h^2*m + 4*a*b*c^5*d*e*l - 4*a*b*c^5*d*f*k + 4*a*b*c^5*d*g*j - 2*a*b*c^5*e*f*j + 2*a*b^5*c*f*k*m + 6*a*b^2*c^4*d*f*m - 6*a*b^2*c^4*d*g*l + 6*a*b^2*c^4*d*h*k + 2*a*b^2*c^4*e*f*l + 2*a*b^2*c^4*f*g*j - 8*a*b^3*c^3*d*h*m - 2*a*b^3*c^3*f*g*l + 2*a*b^3*c^3*f*h*k + 6*a^2*b*c^4*d*h*m + 2*a^2*b*c^4*e*g*m - 2*a^2*b*c^4*e*h*l + 4*a^2*b*c^4*f*g*l - 2*a^2*b*c^4*f*h*k - 2*a^2*b*c^4*g*h*j + 8*a*b^3*c^3*d*j*l - 6*a^2*b*c^4*d*j*l - 2*a*b^4*c^2*f*h*m + 10*a*b^4*c^2*d*k*m + 2*a*b^4*c^2*f*j*l + 8*a^3*b*c^3*f*k*m - 2*a^3*b*c^3*g*k*l + 4*a^3*b*c^3*h*j*l - 2*a^2*b^4*c*h*k*m - 2*a^4*b*c^2*j*l*m + 4*a^2*b^2*c^3*f*h*m + 2*a^2*b^2*c^3*g*h*l - 12*a^2*b^2*c^3*d*k*m - 6*a^2*b^2*c^3*f*j*l - 8*a^2*b^3*c^2*f*k*m - 2*a^2*b^3*c^2*h*j*l + 4*a^3*b^2*c^2*h*k*m + 2*a^3*b^2*c^2*j*k*l)/c^5 - root(128*a^2*b^2*c^8*z^4 - 16*a*b^4*c^7*z^4 - 256*a^3*c^9*z^4 + 384*a^3*b^2*c^6*l*z^3 - 144*a^2*b^4*c^5*l*z^3 + 128*a^2*b^3*c^6*j*z^3 - 128*a^2*b^2*c^7*g*z^3 + 16*a*b^6*c^4*l*z^3 - 256*a^3*b*c^7*j*z^3 - 16*a*b^5*c^5*j*z^3 + 16*a*b^4*c^6*g*z^3 - 256*a^4*c^7*l*z^3 + 256*a^3*c^8*g*z^3 - 96*a^4*b*c^5*j*l*z^2 + 8*a*b^7*c^2*j*l*z^2 + 160*a^4*b*c^5*h*m*z^2 - 8*a*b^7*c^2*h*m*z^2 + 8*a*b^6*c^3*h*k*z^2 - 8*a*b^6*c^3*g*l*z^2 + 8*a*b^6*c^3*f*m*z^2 + 160*a^3*b*c^6*g*j*z^2 - 96*a^3*b*c^6*f*k*z^2 - 96*a^3*b*c^6*e*l*z^2 - 96*a^3*b*c^6*d*m*z^2 + 8*a*b^5*c^4*g*j*z^2 - 8*a*b^5*c^4*f*k*z^2 - 8*a*b^5*c^4*e*l*z^2 - 8*a*b^5*c^4*d*m*z^2 + 8*a*b^4*c^5*e*j*z^2 + 8*a*b^4*c^5*d*k*z^2 + 8*a*b^4*c^5*f*h*z^2 + 32*a^2*b*c^7*e*g*z^2 + 32*a^2*b*c^7*d*h*z^2 - 8*a*b^3*c^6*e*g*z^2 - 8*a*b^3*c^6*d*h*z^2 + 16*a*b^2*c^7*d*f*z^2 + 8*a*b^8*c*k*m*z^2 - 304*a^4*b^2*c^4*k*m*z^2 + 264*a^3*b^4*c^3*k*m*z^2 - 80*a^2*b^6*c^2*k*m*z^2 + 184*a^3*b^3*c^4*j*l*z^2 - 72*a^2*b^5*c^3*j*l*z^2 - 200*a^3*b^3*c^4*h*m*z^2 + 72*a^2*b^5*c^3*h*m*z^2 - 240*a^3*b^2*c^5*g*l*z^2 + 144*a^3*b^2*c^5*h*k*z^2 + 144*a^3*b^2*c^5*f*m*z^2 + 80*a^2*b^4*c^4*g*l

```
*z^2 - 64*a^2*b^4*c^4*h*k*z^2 - 64*a^2*b^4*c^4*f*m*z^2 - 72*a^2*b^3*c^5*g*j
*z^2 + 56*a^2*b^3*c^5*f*k*z^2 + 56*a^2*b^3*c^5*e*l*z^2 + 56*a^2*b^3*c^5*d*m
*z^2 - 48*a^2*b^2*c^6*e*j*z^2 - 48*a^2*b^2*c^6*d*k*z^2 - 48*a^2*b^2*c^6*f*h
*z^2 - 112*a^5*b*c^4*m^2*z^2 + 44*a^2*b^7*c*m^2*z^2 + 80*a^4*b*c^5*k^2*z^2
- 4*a*b^7*c^2*k^2*z^2 - 4*a*b^6*c^3*j^2*z^2 - 48*a^3*b*c^6*h^2*z^2 - 4*a*b^
5*c^4*h^2*z^2 - 4*a*b^4*c^5*g^2*z^2 + 16*a^2*b*c^7*f^2*z^2 - 4*a*b^3*c^6*f^
2*z^2 + 8*a*b^2*c^7*e^2*z^2 + 64*a^5*c^5*k*m*z^2 + 192*a^4*c^6*g*l*z^2 - 64
*a^4*c^6*h*k*z^2 - 64*a^4*c^6*f*m*z^2 + 64*a^3*c^7*e*j*z^2 + 64*a^3*c^7*d*k
*z^2 + 64*a^3*c^7*f*h*z^2 - 4*a*b^8*c*l^2*z^2 - 64*a^2*c^8*d*f*z^2 + 16*a*b
*c^8*d^2*z^2 + 252*a^4*b^3*c^3*m^2*z^2 - 168*a^3*b^5*c^2*m^2*z^2 + 168*a^4*
b^2*c^4*l^2*z^2 - 132*a^3*b^4*c^3*l^2*z^2 + 40*a^2*b^6*c^2*l^2*z^2 - 100*a^
3*b^3*c^4*k^2*z^2 + 36*a^2*b^5*c^3*k^2*z^2 - 56*a^3*b^2*c^5*j^2*z^2 + 32*a^
2*b^4*c^4*j^2*z^2 + 28*a^2*b^3*c^5*h^2*z^2 + 40*a^2*b^2*c^6*g^2*z^2 - 96*a^
5*c^5*l^2*z^2 - 32*a^4*c^6*j^2*z^2 - 96*a^3*c^7...
```

3.26 $\int \frac{d+ex}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=94

$$\frac{dx(17-5x^2)}{72(4-5x^2+x^4)}+\frac{e(5-2x^2)}{18(4-5x^2+x^4)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)$$

```
[Out] 1/72*d*x*(-5*x^2+17)/(x^4-5*x^2+4)+1/18*e*(-2*x^2+5)/(x^4-5*x^2+4)+19/432*d
*arctanh(1/2*x)-1/54*d*arctanh(x)+1/27*e*ln(-x^2+1)-1/27*e*ln(-x^2+4)
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1687, 12, 1106, 1180, 213, 1121, 628, 630, 31}

$$\frac{dx(17-5x^2)}{72(x^4-5x^2+4)}+\frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}d\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e(5-2x^2)}{18(x^4-5x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(72*(4 - 5*x^2 + x^4)) + (e*(5 - 2*x^2))/(18*(4 - 5*x^2
+ x^4)) + (19*d*ArcTanh[x/2])/432 - (d*ArcTanh[x])/54 + (e*Log[1 - x^2])/27
 - (e*Log[4 - x^2])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex}{(4-5x^2+x^4)^2}\,dx = \int \frac{d}{(4-5x^2+x^4)^2}\,dx + \int \frac{ex}{(4-5x^2+x^4)^2}\,dx$$
$$= d\int \frac{1}{(4-5x^2+x^4)^2}\,dx + e\int \frac{x}{(4-5x^2+x^4)^2}\,dx$$
$$= \frac{dx(17-5x^2)}{72\,(4-5x^2+x^4)} - \frac{1}{72}d\int \frac{-1+5x^2}{4-5x^2+x^4}\,dx + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{(4-5x+x^2)^2}\,dx, x,$$
$$= \frac{dx(17-5x^2)}{72\,(4-5x^2+x^4)} + \frac{e(5-2x^2)}{18\,(4-5x^2+x^4)} + \frac{1}{54}d\int \frac{1}{-1+x^2}\,dx - \frac{1}{216}(19d)\int \frac{1}{-4+}$$
$$= \frac{dx(17-5x^2)}{72\,(4-5x^2+x^4)} + \frac{e(5-2x^2)}{18\,(4-5x^2+x^4)} + \frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}d\tanh^{-1}(x) - \frac{1}{27}$$
$$= \frac{dx(17-5x^2)}{72\,(4-5x^2+x^4)} + \frac{e(5-2x^2)}{18\,(4-5x^2+x^4)} + \frac{19}{432}d\tanh^{-1}\left(\frac{x}{2}\right) - \frac{1}{54}d\tanh^{-1}(x) + \frac{1}{27}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.96

$$\frac{1}{864}\left(\frac{12(e(20-8x^2)+dx(17-5x^2))}{4-5x^2+x^4} + 8(d+4e)\log(1-x) - (19d+32e)\log(2-x) - 8(d-4e)\log(1+x) + (19d-32e)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4) + 8*(d + 4*e)*L
og[1 - x] - (19*d + 32*e)*Log[2 - x] - 8*(d - 4*e)*Log[1 + x] + (19*d - 32*
e)*Log[2 + x])/864
```

Maple [A]
time = 0.04, size = 106, normalized size = 1.13

method	result
norman	$\frac{-\frac{1}{9}e\,x^2+\frac{17}{72}dx-\frac{5}{72}d\,x^3+\frac{5}{18}e}{x^4-5x^2+4} + \left(-\frac{19d}{864}-\frac{e}{27}\right)\ln(x-2) + \left(-\frac{d}{108}+\frac{e}{27}\right)\ln(1+x) + \left(\frac{d}{108}+\frac{e}{27}\right)\ln(-1+x$
risch	$\frac{-\frac{1}{9}e\,x^2+\frac{17}{72}dx-\frac{5}{72}d\,x^3+\frac{5}{18}e}{x^4-5x^2+4} + \frac{\ln(1-x)d}{108} + \frac{\ln(1-x)e}{27} + \frac{19\ln(x+2)d}{864} - \frac{\ln(x+2)e}{27} - \frac{\ln(1+x)d}{108} + \frac{\ln(1+x)e}{27} - \frac{19\ln(2-x)d}{864}$
default	$-\frac{\frac{d}{144}-\frac{e}{72}}{x+2} + \left(\frac{19d}{864}-\frac{e}{27}\right)\ln(x+2) + \left(-\frac{19d}{864}-\frac{e}{27}\right)\ln(x-2) - \frac{\frac{d}{144}+\frac{e}{72}}{x-2} - \frac{\frac{d}{36}+\frac{e}{36}}{-1+x} + \left(\frac{d}{108}+\frac{e}{27}\right)\ln(-1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/144*d-1/72*e)/(x+2)+(19/864*d-1/27*e)*ln(x+2)+(-19/864*d-1/27*e)*ln(x-2
)-(1/144*d+1/72*e)/(x-2)-(1/36*d+1/36*e)/(-1+x)+(1/108*d+1/27*e)*ln(-1+x)+(
-1/108*d+1/27*e)*ln(1+x)-(1/36*d-1/36*e)/(1+x)
```

Maxima [A]
time = 0.27, size = 89, normalized size = 0.95

$$\frac{1}{864}\left(19\,d-32\,e\right)\log\left(x+2\right)-\frac{1}{108}\left(d-4\,e\right)\log\left(x+1\right)+\frac{1}{108}\left(d+4\,e\right)\log\left(x-1\right)-\frac{1}{864}\left(19\,d+32\,e\right)\log\left(x-2\right)-\frac{5\,dx^{3}+8\,x^{2}e-17\,dx-20\,e}{72\,\left(x^{4}-5\,x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d - 32*e)*log(x + 2) - 1/108*(d - 4*e)*log(x + 1) + 1/108*(d + 4*
e)*log(x - 1) - 1/864*(19*d + 32*e)*log(x - 2) - 1/72*(5*d*x^3 + 8*x^2*e -
17*d*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(80) = 160.
time = 0.41, size = 169, normalized size = 1.80

$$-\frac{60\,dx^{3}+96\,ex^{2}-204\,dx-\left(\left(19\,d-32\,e\right)x^{4}-5\,\left(19\,d-32\,e\right)x^{2}+76\,d-128\,e\right)\log\left(x+2\right)+8\,\left(\left(d-4\,e\right)x^{4}-5\,\left(d-4\,e\right)x^{2}+4\,d-16\,e\right)\log\left(x+1\right)-8\,\left(\left(d+4\,e\right)x^{4}-5\,\left(d+4\,e\right)x^{2}+4\,d+16\,e\right)\log\left(x-1\right)+\left(\left(19\,d+32\,e\right)x^{4}-5\,\left(19\,d+32\,e\right)x^{2}+76\,d+128\,e\right)\log\left(x-2\right)-240\,e}{864\,\left(x^{4}-5\,x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(60*d*x^3 + 96*e*x^2 - 204*d*x - ((19*d - 32*e)*x^4 - 5*(19*d - 32*e
)*x^2 + 76*d - 128*e)*log(x + 2) + 8*((d - 4*e)*x^4 - 5*(d - 4*e)*x^2 + 4*d
 - 16*e)*log(x + 1) - 8*((d + 4*e)*x^4 - 5*(d + 4*e)*x^2 + 4*d + 16*e)*log(
x - 1) + ((19*d + 32*e)*x^4 - 5*(19*d + 32*e)*x^2 + 76*d + 128*e)*log(x - 2
) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(78) = 156.
time = 2.16, size = 604, normalized size = 6.43

$$-\frac{(d-4e)\log\left(x+[illegible]\right)}{108}+\frac{(d+4e)\log\left(x+[illegible]\right)}{108}+\frac{(19d-32e)\log\left(x+[illegible]\right)}{864}-\frac{(19d+32e)\log\left(x+[illegible]\right)}{864}+\frac{-5dx^3+17dx-8ex^2+20e}{72x^4-360x^2+288}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d - 4*e)*log(x + (-6006260*d**4*e + 2341251*d**4*(d - 4*e) - 18247680*d**
2*e**3 + 24099840*d**2*e**2*(d - 4*e) + 7387904*d**2*e*(d - 4*e)**2 - 66528
0*d**2*(d - 4*e)**3 + 587202560*e**5 - 12582912*e**4*(d - 4*e) - 36700160*e
**3*(d - 4*e)**2 + 786432*e**2*(d - 4*e)**3)/(1675971*d**5 - 66150400*d**3*
e**2 + 318767104*d*e**4))/108 + (d + 4*e)*log(x + (-6006260*d**4*e - 234125
1*d**4*(d + 4*e) - 18247680*d**2*e**3 - 24099840*d**2*e**2*(d + 4*e) + 7387
904*d**2*e*(d + 4*e)**2 + 665280*d**2*(d + 4*e)**3 + 587202560*e**5 + 12582
912*e**4*(d + 4*e) - 36700160*e**3*(d + 4*e)**2 - 786432*e**2*(d + 4*e)**3)
/(1675971*d**5 - 66150400*d**3*e**2 + 318767104*d*e**4))/108 + (19*d - 32*e
```

```
)*log(x + (-6006260*d**4*e - 2341251*d**4*(19*d - 32*e)/8 - 18247680*d**2*e
**3 - 3012480*d**2*e**2*(19*d - 32*e) + 115436*d**2*e*(19*d - 32*e)**2 + 10
395*d**2*(19*d - 32*e)**3/8 + 587202560*e**5 + 1572864*e**4*(19*d - 32*e) -
 573440*e**3*(19*d - 32*e)**2 - 1536*e**2*(19*d - 32*e)**3)/(1675971*d**5 -
 66150400*d**3*e**2 + 318767104*d*e**4))/864 - (19*d + 32*e)*log(x + (-6006
260*d**4*e + 2341251*d**4*(19*d + 32*e)/8 - 18247680*d**2*e**3 + 3012480*d*
*2*e**2*(19*d + 32*e) + 115436*d**2*e*(19*d + 32*e)**2 - 10395*d**2*(19*d +
 32*e)**3/8 + 587202560*e**5 - 1572864*e**4*(19*d + 32*e) - 573440*e**3*(19
*d + 32*e)**2 + 1536*e**2*(19*d + 32*e)**3)/(1675971*d**5 - 66150400*d**3*e
**2 + 318767104*d*e**4))/864 + (-5*d*x**3 + 17*d*x - 8*e*x**2 + 20*e)/(72*x
**4 - 360*x**2 + 288)
```

Giac [A]
time = 4.50, size = 93, normalized size = 0.99

$$\frac{1}{864}\left(19\,d-32\,e\right)\log\left(|x+2|\right)-\frac{1}{108}\left(d-4\,e\right)\log\left(|x+1|\right)+\frac{1}{108}\left(d+4\,e\right)\log\left(|x-1|\right)-\frac{1}{864}\left(19\,d+32\,e\right)\log\left(|x-2|\right)-\frac{5\,dx^3+8\,x^2e-17\,dx-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d - 32*e)*log(abs(x + 2)) - 1/108*(d - 4*e)*log(abs(x + 1)) + 1/1
08*(d + 4*e)*log(abs(x - 1)) - 1/864*(19*d + 32*e)*log(abs(x - 2)) - 1/72*(
5*d*x^3 + 8*x^2*e - 17*d*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Mupad [B]
time = 0.09, size = 84, normalized size = 0.89

$$\ln\left(x-1\right)\,\left(\frac{d}{108}+\frac{e}{27}\right)-\ln\left(x+1\right)\,\left(\frac{d}{108}-\frac{e}{27}\right)-\ln\left(x-2\right)\,\left(\frac{19\,d}{864}+\frac{e}{27}\right)+\ln\left(x+2\right)\,\left(\frac{19\,d}{864}-\frac{e}{27}\right)+\frac{-\frac{5\,d\,x^3}{72}-\frac{e\,x^2}{9}+\frac{17\,d\,x}{72}+\frac{5\,e}{18}}{x^4-5\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/108 + e/27) - log(x + 1)*(d/108 - e/27) - log(x - 2)*((19*d)/
864 + e/27) + log(x + 2)*((19*d)/864 - e/27) + ((5*e)/18 + (17*d*x)/72 - (5
*d*x^3)/72 - (e*x^2)/9)/(x^4 - 5*x^2 + 4)
```

3.27 $\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=115

$$\frac{e(5-2x^2)}{18\left(4-5x^2+x^4\right)}+\frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{27}e\,\mathrm{lo}$$

```
[Out] 1/18*e*(-2*x^2+5)/(x^4-5*x^2+4)+1/72*x*(17*d+20*f-(5*d+8*f)*x^2)/(x^4-5*x^2
+4)+1/432*(19*d+52*f)*arctanh(1/2*x)-1/54*(d+7*f)*arctanh(x)+1/27*e*ln(-x^2
+1)-1/27*e*ln(-x^2+4)
```

Rubi [A]

time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1687, 1192, 1180, 213, 12, 1121, 628, 630, 31}

$$\frac{x(-(x^2(5d+8f))+17d+20f)}{72\left(x^4-5x^2+4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{27}e\log\left(1-x^2\right)-\frac{1}{27}e\log\left(4-x^2\right)+\frac{e(5-2x^2)}{18\left(x^4-5x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (e*(5 - 2*x^2))/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f - (5*d + 8*f)*x^2)
)/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 - ((d + 7*f)*Ar
cTanh[x])/54 + (e*Log[1 - x^2])/27 - (e*Log[4 - x^2])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx &= \int \frac{ex}{\left(4-5x^2+x^4\right)^2}\,dx+\int \frac{d+fx^2}{\left(4-5x^2+x^4\right)^2}\,dx \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}-\frac{1}{72}\int \frac{-d+20f+(5d+8f)x^2}{4-5x^2+x^4}\,dx+e\int \frac{x}{\left(4-5x^2+\right.} \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{\left(4-5x+x^2\right)^2}\,dx,x,x^2\right)-\frac{1}{54}(-d-7 \\
&= \frac{e(5-2x^2)}{18\left(4-5x^2+x^4\right)}+\frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)- \\
&= \frac{e(5-2x^2)}{18\left(4-5x^2+x^4\right)}+\frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)- \\
&= \frac{e(5-2x^2)}{18\left(4-5x^2+x^4\right)}+\frac{x(17d+20f-(5d+8f)x^2)}{72\left(4-5x^2+x^4\right)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 112, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12(17dx+20fx-5dx^3-8fx^3+e(20-8x^2))}{4-5x^2+x^4}+8(d+4e+7f)\log(1-x)-(19d+32e+52f)\log(2-x)-8(d-4e+7f)\log(1+x)+(19d-32e+52f)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 + x
^4) + 8*(d + 4*e + 7*f)*Log[1 - x] - (19*d + 32*e + 52*f)*Log[2 - x] - 8*(d
 - 4*e + 7*f)*Log[1 + x] + (19*d - 32*e + 52*f)*Log[2 + x])/864
```

Maple [A]
time = 0.04, size = 130, normalized size = 1.13

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x-\frac{e\,x^2}{9}+\frac{5e}{18}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}\right)\ln\left(x-2\right)+\left(-\frac{d}{108}+\frac{e}{27}-\frac{7f}{108}\right)\ln\left(1+x\right)+\left(\frac{d}{108}\right.$
default	$-\frac{\frac{d}{144}-\frac{e}{72}+\frac{f}{36}}{x+2}+\left(\frac{19d}{864}-\frac{e}{27}+\frac{13f}{216}\right)\ln\left(x+2\right)+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}\right)\ln\left(x-2\right)-\frac{\frac{d}{144}+\frac{e}{72}+\frac{f}{36}}{x-2}-\frac{\frac{d}{36}+\frac{e}{36}+\frac{f}{36}}{-1+x}$
risch	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x-\frac{e\,x^2}{9}+\frac{5e}{18}}{x^4-5x^2+4}+\frac{19\ln(x+2)d}{864}-\frac{\ln(x+2)e}{27}+\frac{13\ln(x+2)f}{216}+\frac{\ln(1-x)d}{108}+\frac{\ln(1-x)e}{27}+\frac{7\ln(1-x)f}{108}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/144*d-1/72*e+1/36*f)/(x+2)+(19/864*d-1/27*e+13/216*f)*ln(x+2)+(-19/864*
d-1/27*e-13/216*f)*ln(x-2)-(1/144*d+1/72*e+1/36*f)/(x-2)-(1/36*d+1/36*e+1/3
```

```
6*f)/(-1+x)+(1/108*d+1/27*e+7/108*f)*ln(-1+x)+(-1/108*d+1/27*e-7/108*f)*ln(
1+x)-(1/36*d-1/36*e+1/36*f)/(1+x)
```

Maxima [A]
time = 0.28, size = 112, normalized size = 0.97

$$\frac{1}{864}\,(19\,d+52\,f-32\,e)\log\left(x+2\right)-\frac{1}{108}\,(d+7\,f-4\,e)\log\left(x+1\right)+\frac{1}{108}\,(d+7\,f+4\,e)\log\left(x-1\right)-\frac{1}{864}\,(19\,d+52\,f+32\,e)\log\left(x-2\right)-\frac{(5\,d+8\,f)x^3+8\,x^2e-(17\,d+20\,f)x-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d + 52*f - 32*e)*log(x + 2) - 1/108*(d + 7*f - 4*e)*log(x + 1) +
1/108*(d + 7*f + 4*e)*log(x - 1) - 1/864*(19*d + 52*f + 32*e)*log(x - 2) -
1/72*((5*d + 8*f)*x^3 + 8*x^2*e - (17*d + 20*f)*x - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(100) = 200.
time = 0.52, size = 217, normalized size = 1.89

$$-\frac{12\,(5\,d+8\,f)x^3+96\,ex^2-12\,(17\,d+20\,f)x-((19\,d-32\,e+52\,f)x^4-5\,(19\,d-32\,e+52\,f)x^2+76\,d-128\,e+208\,f)\log\left(x+2\right)+8\,((d-4\,e+7\,f)x^4-5\,(d-4\,e+7\,f)x^2+4\,d-16\,e+28\,f)\log\left(x+1\right)-8\,((d+4\,e+7\,f)x^4-5\,(d+4\,e+7\,f)x^2+4\,d+16\,e+28\,f)\log\left(x-1\right)+((19\,d+32\,e+52\,f)x^4-5\,(19\,d+32\,e+52\,f)x^2+76\,d+128\,e+208\,f)\log\left(x-2\right)-240\,e}{864\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 96*e*x^2 - 12*(17*d + 20*f)*x - ((19*d - 32*e
+ 52*f)*x^4 - 5*(19*d - 32*e + 52*f)*x^2 + 76*d - 128*e + 208*f)*log(x + 2)
 + 8*((d - 4*e + 7*f)*x^4 - 5*(d - 4*e + 7*f)*x^2 + 4*d - 16*e + 28*f)*log(
x + 1) - 8*((d + 4*e + 7*f)*x^4 - 5*(d + 4*e + 7*f)*x^2 + 4*d + 16*e + 28*f
)*log(x - 1) + ((19*d + 32*e + 52*f)*x^4 - 5*(19*d + 32*e + 52*f)*x^2 + 76*
d + 128*e + 208*f)*log(x - 2) - 240*e)/(x^4 - 5*x^2 + 4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.73, size = 115, normalized size = 1.00

$$\frac{1}{864}\,(19\,d+52\,f-32\,e)\log\left(|x+2|\right)-\frac{1}{108}\,(d+7\,f-4\,e)\log\left(|x+1|\right)+\frac{1}{108}\,(d+7\,f+4\,e)\log\left(|x-1|\right)-\frac{1}{864}\,(19\,d+52\,f+32\,e)\log\left(|x-2|\right)-\frac{5\,dx^3+8\,fx^3+8\,x^2e-17\,dx-20\,fx-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f - 4*e)*log(abs(
x + 1)) + 1/108*(d + 7*f + 4*e)*log(abs(x - 1)) - 1/864*(19*d + 52*f + 32*e
)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 8*x^2*e - 17*d*x - 20*f*x - 2
0*e)/(x^4 - 5*x^2 + 4)
```

Mupad [B]
time = 0.10, size = 107, normalized size = 0.93

$$\ln\left(x-1\right)\,\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}\right)-\ln\left(x+1\right)\,\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}\right)-\ln\left(x-2\right)\,\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}\right)+\ln\left(x+2\right)\,\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}\right)+\frac{\left(-\frac{5\,d}{72}-\frac{f}{9}\right)\,x^3-\frac{e\,x^2}{9}+\left(\frac{17\,d}{72}+\frac{5\,f}{18}\right)\,x+\frac{5\,e}{18}}{x^4-5\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/108 + e/27 + (7*f)/108) - log(x + 1)*(d/108 - e/27 + (7*f)/10
8) - log(x - 2)*((19*d)/864 + e/27 + (13*f)/216) + log(x + 2)*((19*d)/864 -
 e/27 + (13*f)/216) + ((5*e)/18 - x^3*((5*d)/72 + f/9) - (e*x^2)/9 + x*((17
*d)/72 + (5*f)/18))/(x^4 - 5*x^2 + 4)
```

3.28 $\int \frac{d+ex+fx^2+gx^3}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=138

$$\frac{x(17d+20f-(5d+8f)x^2)}{72\,(4-5x^2+x^4)}+\frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+$$

```
[Out] 1/72*x*(17*d+20*f-(5*d+8*f)*x^2)/(x^4-5*x^2+4)+1/18*(5*e+8*g-(2*e+5*g)*x^2)
/(x^4-5*x^2+4)+1/432*(19*d+52*f)*arctanh(1/2*x)-1/54*(d+7*f)*arctanh(x)+1/5
4*(2*e+5*g)*ln(-x^2+1)-1/54*(2*e+5*g)*ln(-x^2+4)
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1687, 1192, 1180, 213, 1261, 652, 630, 31}

$$\frac{x(-(x^2(5d+8f))+17d+20f)}{72\,(x^4-5x^2+4)}+\frac{1}{432}(19d+52f)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d+7f)\tanh^{-1}(x)+\frac{1}{54}(2e+5g)\log\left(1-x^2\right)-\frac{1}{54}(2e+5g)\log\left(4-x^2\right)+\frac{-(x^2(2e+5g))+5e+8g}{18\,(x^4-5x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(72*(4 - 5*x^2 + x^4)) + (5*e + 8*g - (
2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d + 52*f)*ArcTanh[x/2])/432 -
 ((d + 7*f)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[1 - x^2])/54 - ((2*e + 5*g)*L
og[4 - x^2])/54
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{(4-5x^2+x^4)^2}\,dx &= \int \frac{d+fx^2}{(4-5x^2+x^4)^2}\,dx + \int \frac{x(e+gx^2)}{(4-5x^2+x^4)^2}\,dx \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\,(4-5x^2+x^4)} - \frac{1}{72}\int \frac{-d+20f+(5d+8f)x^2}{4-5x^2+x^4}\,dx + \frac{1}{2}\text{Subst}\Big(\\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\,(4-5x^2+x^4)} + \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} - \frac{1}{54}(-d-7f)\int \frac{1}{-1} \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\,(4-5x^2+x^4)} + \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} + \frac{1}{432}(19d+52f)\tanh \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{72\,(4-5x^2+x^4)} + \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} + \frac{1}{432}(19d+52f)\tanh
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 134, normalized size = 0.97

$$\frac{1}{864}\left(\frac{12(17dx+20fx-5dx^3-8fx^3+e(20-8x^2)-4g(-8+5x^2))}{4-5x^2+x^4}+8(d+4e+7f+10g)\log(1-x)-(19d+32e+52f+80g)\log(2-x)-8(d-4e+7f-10g)\log(1+x)+(19d-32e+52f-80g)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g*(-8 + 5*x^
2)))/(4 - 5*x^2 + x^4) + 8*(d + 4*e + 7*f + 10*g)*Log[1 - x] - (19*d + 32*e
 + 52*f + 80*g)*Log[2 - x] - 8*(d - 4*e + 7*f - 10*g)*Log[1 + x] + (19*d -
32*e + 52*f - 80*g)*Log[2 + x])/864
```

Maple [A]
time = 0.05, size = 154, normalized size = 1.12

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x+\left(-\frac{e}{9}-\frac{5g}{18}\right)x^2+\frac{5e}{18}+\frac{4g}{9}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}\right)\ln(x-2)+\left(-\frac{d}{108}+\frac{e}{27}-\frac{7f}{108}\right.$
default	$-\frac{\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}}{x+2}+\left(\frac{19d}{864}-\frac{e}{27}+\frac{13f}{216}-\frac{5g}{54}\right)\ln(x+2)+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}\right)\ln(x-2)-$
risch	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}\right)x+\left(-\frac{e}{9}-\frac{5g}{18}\right)x^2+\frac{5e}{18}+\frac{4g}{9}}{x^4-5x^2+4}+\frac{\ln(1-x)d}{108}+\frac{\ln(1-x)e}{27}+\frac{7\ln(1-x)f}{108}+\frac{5\ln(1-x)g}{54}+\frac{19\ln(x+2)d}{864}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/144*d-1/72*e+1/36*f-1/18*g)/(x+2)+(19/864*d-1/27*e+13/216*f-5/54*g)*ln(
x+2)+(-19/864*d-1/27*e-13/216*f-5/54*g)*ln(x-2)-(1/144*d+1/72*e+1/36*f+1/18
*g)/(x-2)-(1/36*d+1/36*e+1/36*f+1/36*g)/(-1+x)+(1/108*d+1/27*e+7/108*f+5/54
```

```
*g)*ln(-1+x)+(-1/108*d+1/27*e-7/108*f+5/54*g)*ln(1+x)-(1/36*d-1/36*e+1/36*f
-1/36*g)/(1+x)
```

Maxima [A]
time = 0.30, size = 133, normalized size = 0.96

$$\frac{1}{864}\,(19\,d+52\,f-80\,g-32\,e)\log\left(x+2\right)-\frac{1}{108}\,(d+7\,f-10\,g-4\,e)\log\left(x+1\right)+\frac{1}{108}\,(d+7\,f+10\,g+4\,e)\log\left(x-1\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+32\,e)\log\left(x-2\right)-\frac{(5\,d+8\,f)x^3+4\,(5\,g+2\,e)x^2-(17\,d+20\,f)x-32\,g-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d + 52*f - 80*g - 32*e)*log(x + 2) - 1/108*(d + 7*f - 10*g - 4*e)
*log(x + 1) + 1/108*(d + 7*f + 10*g + 4*e)*log(x - 1) - 1/864*(19*d + 52*f
+ 80*g + 32*e)*log(x - 2) - 1/72*((5*d + 8*f)*x^3 + 4*(5*g + 2*e)*x^2 - (17
*d + 20*f)*x - 32*g - 20*e)/(x^4 - 5*x^2 + 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(122) = 244$.
time = 0.97, size = 262, normalized size = 1.90

$$-\frac{12\,(5\,d+8\,f)x^3+48\,(2\,e+5\,g)x^2-12\,(17\,d+20\,f)x-((19\,d-32\,e+52\,f-80\,g)x^4-5\,(19\,d-32\,e+52\,f-80\,g)x^2+76\,d-128\,e+208\,f-320\,g)\log\left(x+2\right)+8\,((d-4\,e+7\,f-10\,g)x^4-5\,(d-4\,e+7\,f-10\,g)x^2+4\,d-16\,e+28\,f-40\,g)\log\left(x+1\right)-8\,((d+4\,e+7\,f+10\,g)x^4-5\,(d+4\,e+7\,f+10\,g)x^2+4\,d+16\,e+28\,f+40\,g)\log\left(x-1\right)+((19\,d+32\,e+52\,f+80\,g)x^4-5\,(19\,d+32\,e+52\,f+80\,g)x^2+76\,d+128\,e+208\,f+320\,g)\log\left(x-2\right)-240\,e-384\,g}{864\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d + 20*f)*x - ((19
*d - 32*e + 52*f - 80*g)*x^4 - 5*(19*d - 32*e + 52*f - 80*g)*x^2 + 76*d - 1
28*e + 208*f - 320*g)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g)*x^4 - 5*(d - 4
*e + 7*f - 10*g)*x^2 + 4*d - 16*e + 28*f - 40*g)*log(x + 1) - 8*((d + 4*e +
 7*f + 10*g)*x^4 - 5*(d + 4*e + 7*f + 10*g)*x^2 + 4*d + 16*e + 28*f + 40*g)
*log(x - 1) + ((19*d + 32*e + 52*f + 80*g)*x^4 - 5*(19*d + 32*e + 52*f + 80
*g)*x^2 + 76*d + 128*e + 208*f + 320*g)*log(x - 2) - 240*e - 384*g)/(x^4 -
5*x^2 + 4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.63, size = 136, normalized size = 0.99

$$\frac{1}{864}\,(19\,d+52\,f-80\,g-32\,e)\log\left(|x+2|\right)-\frac{1}{108}\,(d+7\,f-10\,g-4\,e)\log\left(|x+1|\right)+\frac{1}{108}\,(d+7\,f+10\,g+4\,e)\log\left(|x-1|\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+32\,e)\log\left(|x-2|\right)-\frac{5\,dx^3+8\,fx^3+20\,gx^2+8\,x^2e-17\,dx-20\,fx-32\,g-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 80*g - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f - 10*g -
 4*e)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g + 4*e)*log(abs(x - 1)) - 1/86
4*(19*d + 52*f + 80*g + 32*e)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 2
0*g*x^2 + 8*x^2*e - 17*d*x - 20*f*x - 32*g - 20*e)/(x^4 - 5*x^2 + 4)
```

Mupad [B]
time = 0.14, size = 128, normalized size = 0.93

$$\ln(x-1)\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}+\frac{5\,g}{54}\right)-\ln(x+1)\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}-\frac{5\,g}{54}\right)-\ln(x-2)\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}+\frac{5\,g}{54}\right)+\ln(x+2)\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}-\frac{5\,g}{54}\right)+\frac{\left(-\frac{5\,d}{72}-\frac{f}{9}\right)x^3+\left(-\frac{e}{9}-\frac{5\,g}{18}\right)x^2+\left(\frac{17\,d}{72}+\frac{5\,f}{18}\right)x+\frac{5\,e}{18}+\frac{4\,g}{9}}{x^4-5\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/108 + e/27 + (7*f)/108 + (5*g)/54) - log(x + 1)*(d/108 - e/27
 + (7*f)/108 - (5*g)/54) - log(x - 2)*((19*d)/864 + e/27 + (13*f)/216 + (5*
g)/54) + log(x + 2)*((19*d)/864 - e/27 + (13*f)/216 - (5*g)/54) + ((5*e)/18
 + (4*g)/9 - x^3*((5*d)/72 + f/9) - x^2*(e/9 + (5*g)/18) + x*((17*d)/72 + (
5*f)/18))/(x^4 - 5*x^2 + 4)
```

3.29 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=150

$$\frac{5e+8g-(2e+5g)x^2}{18(4-5x^2+x^4)}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72(4-5x^2+x^4)}+\frac{1}{432}(19d+52f+112h)\tanh^{-1}\left(\frac{x}{2}\right)-\frac{1}{54}(d$$

```
[Out] 1/18*(5*e+8*g-(2*e+5*g)*x^2)/(x^4-5*x^2+4)+1/72*x*(17*d+20*f+32*h-(5*d+8*f+
20*h)*x^2)/(x^4-5*x^2+4)+1/432*(19*d+52*f+112*h)*arctanh(1/2*x)-1/54*(d+7*f
+13*h)*arctanh(x)+1/54*(2*e+5*g)*ln(-x^2+1)-1/54*(2*e+5*g)*ln(-x^2+4)
```

Rubi [A]
time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1687, 1692, 1180, 213, 1261, 652, 630, 31}

$$\frac{x(-(x^2(5d+8f+20h))+17d+20f+32h)}{72(x^4-5x^2+4)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}(2e+5g)\log(1-x^2)-\frac{1}{54}(2e+5g)\log(4-x^2)+\frac{-(x^2(2e+5g))+5e+8g}{18(x^4-5x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(18*(4 - 5*x^2 + x^4)) + (x*(17*d + 20*f + 32
*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) + ((19*d + 52*f + 112*
h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2*e + 5*g)*Log[
1 - x^2])/54 - ((2*e + 5*g)*Log[4 - x^2])/54
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^2}\,dx = \int \frac{x(e+gx^2)}{(4-5x^2+x^4)^2}\,dx + \int \frac{d+fx^2+hx^4}{(4-5x^2+x^4)^2}\,dx$$

$$= \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)} - \frac{1}{72}\int \frac{-d+20f+32h+(5c}{4-5x^2+}$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} + \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)} + \frac{1}{18}$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} + \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)} + \frac{1}{43}$$

$$= \frac{5e+8g-(2e+5g)x^2}{18\,(4-5x^2+x^4)} + \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)} + \frac{1}{43}$$

Mathematica [A]
time = 0.05, size = 159, normalized size = 1.06

$$\frac{1}{864}\left(-\frac{12(4e(-5+2x^2)+4g(-8+5x^2)+x(4f(-5+2x^2)+d(-17+5x^2)+4h(-8+5x^2)))}{4-5x^2+x^4}+8(d+4e+7f+10g+13h)\log(1-x)-(19d+32e+52f+80g+112h)\log(2-x)-8(d-4e+7f-10g+13h)\log(1+x)+(19d-32e+52f-80g+112h)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(4*e*(-5 + 2*x^2) + 4*g*(-8 + 5*x^2) + x*(4*f*(-5 + 2*x^2) + d*(-17 +
 5*x^2) + 4*h*(-8 + 5*x^2))))/(4 - 5*x^2 + x^4) + 8*(d + 4*e + 7*f + 10*g +
 13*h)*Log[1 - x] - (19*d + 32*e + 52*f + 80*g + 112*h)*Log[2 - x] - 8*(d -
 4*e + 7*f - 10*g + 13*h)*Log[1 + x] + (19*d - 32*e + 52*f - 80*g + 112*h)*
Log[2 + x])/864
```

Maple [A]
time = 0.05, size = 178, normalized size = 1.19

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}-\frac{5h}{18}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}+\frac{4h}{9}\right)x+\left(-\frac{e}{9}-\frac{5g}{18}\right)x^2+\frac{5e}{18}+\frac{4g}{9}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}\right)\ln(x-2)+\left(-\frac{d}{108}\right.$
default	$-\frac{\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}}{x+2}+\left(\frac{19d}{864}-\frac{e}{27}+\frac{13f}{216}-\frac{5g}{54}+\frac{7h}{54}\right)\ln(x+2)+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}\right)\ln(x-$
risch	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}-\frac{5h}{18}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}+\frac{4h}{9}\right)x+\left(-\frac{e}{9}-\frac{5g}{18}\right)x^2+\frac{5e}{18}+\frac{4g}{9}}{x^4-5x^2+4}+\frac{\ln(1-x)d}{108}+\frac{\ln(1-x)e}{27}+\frac{7\ln(1-x)f}{108}+\frac{5\ln(1-x)g}{54}+\frac{13\ln(1-}{108}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/144*d-1/72*e+1/36*f-1/18*g+1/9*h)/(x+2)+(19/864*d-1/27*e+13/216*f-5/54*
g+7/54*h)*ln(x+2)+(-19/864*d-1/27*e-13/216*f-5/54*g-7/54*h)*ln(x-2)-(1/144*
```

```
d+1/72*e+1/36*f+1/18*g+1/9*h)/(x-2)-(1/36*d+1/36*e+1/36*f+1/36*g+1/36*h)/(-
1+x)+(1/108*d+1/27*e+7/108*f+5/54*g+13/108*h)*ln(-1+x)+(-1/108*d+1/27*e-7/1
08*f+5/54*g-13/108*h)*ln(1+x)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h)/(1+x)
```

Maxima [**A**]
time = 0.29, size = 151, normalized size = 1.01

$$\frac{1}{864}\,(19\,d+52\,f-80\,g+112\,h-32\,e)\log\left(x+2\right)-\frac{1}{108}\,(d+7\,f-10\,g+13\,h-4\,e)\log\left(x+1\right)+\frac{1}{108}\,(d+7\,f+10\,g+13\,h+4\,e)\log\left(x-1\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+112\,h+32\,e)\log\left(x-2\right)-\frac{(5\,d+8\,f+20\,h)x^3+4\,(5\,g+2\,e)x^2-(17\,d+20\,f+32\,h)x-32\,g-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e)*log(x + 2) - 1/108*(d + 7*f - 10*
g + 13*h - 4*e)*log(x + 1) + 1/108*(d + 7*f + 10*g + 13*h + 4*e)*log(x - 1)
 - 1/864*(19*d + 52*f + 80*g + 112*h + 32*e)*log(x - 2) - 1/72*((5*d + 8*f
+ 20*h)*x^3 + 4*(5*g + 2*e)*x^2 - (17*d + 20*f + 32*h)*x - 32*g - 20*e)/(x^
4 - 5*x^2 + 4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(134) = 268$.
time = 1.88, size = 304, normalized size = 2.03

$$-\frac{12\,(5\,d+8\,f+20\,h)x^3+48\,(2\,e+5\,g)x^2-12\,(17\,d+20\,f+32\,h)x-((19\,d-32\,e+52\,f-80\,g+112\,h)x^4-5\,(19\,d-32\,e+52\,f-80\,g+112\,h)x^2+76\,d-128\,e+208\,f-320\,g+448\,h)\log\left(x+2\right)+8\,((d-4\,e+7\,f-10\,g+13\,h)x^4-5\,(d-4\,e+7\,f-10\,g+13\,h)x^2+4\,d-16\,e+28\,f-40\,g+52\,h)\log\left(x+1\right)-8\,((d+4\,e+7\,f+10\,g+13\,h)x^4-5\,(d+4\,e+7\,f+10\,g+13\,h)x^2+4\,d+16\,e+28\,f+40\,g+52\,h)\log\left(x-1\right)+((19\,d+32\,e+52\,f+80\,g+112\,h)x^4-5\,(19\,d+32\,e+52\,f+80\,g+112\,h)x^2+76\,d+128\,e+208\,f+320\,g+448\,h)\log\left(x-2\right)-240\,e-384\,g}{864\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g)*x^2 - 12*(17*d + 20*f +
32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h)*x^4 - 5*(19*d - 32*e + 52*f
- 80*g + 112*h)*x^2 + 76*d - 128*e + 208*f - 320*g + 448*h)*log(x + 2) + 8*
((d - 4*e + 7*f - 10*g + 13*h)*x^4 - 5*(d - 4*e + 7*f - 10*g + 13*h)*x^2 +
4*d - 16*e + 28*f - 40*g + 52*h)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13
*h)*x^4 - 5*(d + 4*e + 7*f + 10*g + 13*h)*x^2 + 4*d + 16*e + 28*f + 40*g +
52*h)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h)*x^4 - 5*(19*d + 32*
e + 52*f + 80*g + 112*h)*x^2 + 76*d + 128*e + 208*f + 320*g + 448*h)*log(x
- 2) - 240*e - 384*g)/(x^4 - 5*x^2 + 4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.24, size = 158, normalized size = 1.05

$$\frac{1}{864}\,(19\,d+52\,f-80\,g+112\,h-32\,e)\log\left(|x+2|\right)-\frac{1}{108}\,(d+7\,f-10\,g+13\,h-4\,e)\log\left(|x+1|\right)+\frac{1}{108}\,(d+7\,f+10\,g+13\,h+4\,e)\log\left(|x-1|\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+112\,h+32\,e)\log\left(|x-2|\right)-\frac{5\,dx^3+8\,fx^3+20\,hx^3+20\,gx^2+8\,x^2e-17\,dx-20\,fx-32\,hx-32\,g-20\,e}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e)*log(abs(x + 2)) - 1/108*(d + 7*f
- 10*g + 13*h - 4*e)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g + 13*h + 4*e)*
log(abs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 112*h + 32*e)*log(abs(x - 2))
 - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 20*g*x^2 + 8*x^2*e - 17*d*x - 20*f*
x - 32*h*x - 32*g - 20*e)/(x^4 - 5*x^2 + 4)
```

Mupad [B]
time = 0.87, size = 146, normalized size = 0.97

$$\frac{\left(-\frac{5\,d}{72}-\frac{f}{9}-\frac{5\,h}{18}\right)\,x^3+\left(-\frac{e}{9}-\frac{5\,g}{18}\right)\,x^2+\left(\frac{17\,d}{72}+\frac{5\,f}{18}+\frac{4\,h}{9}\right)\,x+\frac{5\,e}{18}+\frac{4\,g}{9}}{x^4-5\,x^2+4}+\ln\left(x-1\right)\,\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}+\frac{5\,g}{54}+\frac{13\,h}{108}\right)-\ln\left(x+1\right)\,\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}-\frac{5\,g}{54}+\frac{13\,h}{108}\right)-\ln\left(x-2\right)\,\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}+\frac{5\,g}{54}+\frac{7\,h}{54}\right)+\ln\left(x+2\right)\,\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}-\frac{5\,g}{54}+\frac{7\,h}{54}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] ((5*e)/18 + (4*g)/9 - x^2*(e/9 + (5*g)/18) + x*((17*d)/72 + (5*f)/18 + (4*h
)/9) - x^3*((5*d)/72 + f/9 + (5*h)/18))/(x^4 - 5*x^2 + 4) + log(x - 1)*(d/1
08 + e/27 + (7*f)/108 + (5*g)/54 + (13*h)/108) - log(x + 1)*(d/108 - e/27 +
 (7*f)/108 - (5*g)/54 + (13*h)/108) - log(x - 2)*((19*d)/864 + e/27 + (13*f
)/216 + (5*g)/54 + (7*h)/54) + log(x + 2)*((19*d)/864 - e/27 + (13*f)/216 -
 (5*g)/54 + (7*h)/54)
```

3.30 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=162

$$\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\,(4-5x^2+x^4)}+\frac{5e+8g+20i-(2e+5g+17i)x^2}{18\,(4-5x^2+x^4)}+\frac{1}{432}(19d+52f+112h)\tanh$$

```
[Out] 1/72*x*(17*d+20*f+32*h-(5*d+8*f+20*h)*x^2)/(x^4-5*x^2+4)+1/18*(5*e+8*g+20*i
-(2*e+5*g+17*i)*x^2)/(x^4-5*x^2+4)+1/432*(19*d+52*f+112*h)*arctanh(1/2*x)-1
/54*(d+7*f+13*h)*arctanh(x)+1/54*(2*e+5*g+8*i)*ln(-x^2+1)-1/54*(2*e+5*g+8*i
)*ln(-x^2+4)
```

Rubi [A]
time = 0.15, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1687, 1692, 1180, 213, 1677, 1674, 12, 630, 31}

$$\frac{x(-(x^2(5d+8f+20h))+17d+20f+32h)}{72\,(x^4-5x^2+4)}+\frac{1}{432}\tanh^{-1}\left(\frac{x}{2}\right)(19d+52f+112h)-\frac{1}{54}\tanh^{-1}(x)(d+7f+13h)+\frac{1}{54}\log\left(1-x^2\right)(2e+5g+8i)-\frac{1}{54}\log\left(4-x^2\right)(2e+5g+8i)+\frac{-(x^2(2e+5g+17i))+5e+8g+20i}{18\,(x^4-5x^2+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(72*(4 - 5*x^2 + x^4)) +
(5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(18*(4 - 5*x^2 + x^4)) + ((19*d
 + 52*f + 112*h)*ArcTanh[x/2])/432 - ((d + 7*f + 13*h)*ArcTanh[x])/54 + ((2
*e + 5*g + 8*i)*Log[1 - x^2])/54 - ((2*e + 5*g + 8*i)*Log[4 - x^2])/54
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
```

```
Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
```

```
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+30x^5}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{x(e+gx^2+30x^4)}{\left(4-5x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(4-5x^2+x^4\right)^2}\,dx$$

$$= \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{72\left(4-5x^2+x^4\right)} - \frac{1}{72}\int \frac{-d+20f+\ldots}{\ldots}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x(17d+20f+32h-(5d\ldots}{72\left(4-5x^2+\ldots\right.}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x(17d+20f+32h-(5d\ldots}{72\left(4-5x^2+\ldots\right.}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x(17d+20f+32h-(5d\ldots}{72\left(4-5x^2+\ldots\right.}$$

$$= \frac{600+5e+8g-(510+2e+5g)x^2}{18\left(4-5x^2+x^4\right)} + \frac{x(17d+20f+32h-(5d\ldots}{72\left(4-5x^2+\ldots\right.}$$

Mathematica [A]
time = 0.06, size = 185, normalized size = 1.14

$$\frac{20e+32g+80i+17dx+20fx+32hx-8ex^2-20gx^2-68ix^2-5dx^3-8fx^3-20hx^3}{72\left(4-5x^2+x^4\right)} + \frac{1}{108}(d+4e+7f+10g+13h+16i)\log(1-x) + \frac{1}{864}(-19d-32e-52f-80g-112h-128i)\log(2-x) + \frac{1}{108}(-d+4e-7f+10g-13h+16i)\log(1+x) + \frac{1}{864}(19d-32e+52f-80g+112h-128i)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i*
x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(72*(4 - 5*x^2 + x^4)) + ((d + 4*e + 7*
f + 10*g + 13*h + 16*i)*Log[1 - x])/108 + ((-19*d - 32*e - 52*f - 80*g - 11
2*h - 128*i)*Log[2 - x])/864 + ((-d + 4*e - 7*f + 10*g - 13*h + 16*i)*Log[1
 + x])/108 + ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*Log[2 + x])/864
```

Maple [A]
time = 0.06, size = 202, normalized size = 1.25

method	result
norman	$\frac{\left(-\frac{5d}{72}-\frac{f}{9}-\frac{5h}{18}\right)x^3+\left(\frac{17d}{72}+\frac{5f}{18}+\frac{4h}{9}\right)x+\left(-\frac{5g}{18}-\frac{e}{9}-\frac{17i}{18}\right)x^2+\frac{4g}{9}+\frac{5e}{18}+\frac{10i}{9}}{x^4-5x^2+4}+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}-\frac{4i}{27}\right)\ln\left(x\ldots\right.$
default	$-\frac{\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}-\frac{2i}{9}}{x+2}+\left(\frac{19d}{864}-\frac{e}{27}+\frac{13f}{216}-\frac{5g}{54}+\frac{7h}{54}-\frac{4i}{27}\right)\ln\left(x+2\right)+\left(-\frac{19d}{864}-\frac{e}{27}-\frac{13f}{216}-\frac{5g}{54}-\frac{7h}{54}\ldots\right.$

risch	$\frac{13\ln(1-x)h}{108}+\frac{7\ln(x+2)h}{54}-\frac{4\ln(x+2)i}{27}-\frac{4\ln(2-x)i}{27}-\frac{5\ln(x+2)g}{54}-\frac{5\ln(2-x)g}{54}+\frac{5\ln(1+x)g}{54}+\frac{5\ln(1-x)g}{54}+\frac{4\ln(1+x}{27}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -(1/144*d-1/72*e+1/36*f-1/18*g+1/9*h-2/9*i)/(x+2)+(19/864*d-1/27*e+13/216*f
-5/54*g+7/54*h-4/27*i)*ln(x+2)+(-19/864*d-1/27*e-13/216*f-5/54*g-7/54*h-4/2
7*i)*ln(x-2)-(1/144*d+1/72*e+1/36*f+1/18*g+1/9*h+2/9*i)/(x-2)-(1/36*d+1/36*
e+1/36*f+1/36*g+1/36*h+1/36*i)/(-1+x)+(1/108*d+1/27*e+7/108*f+5/54*g+13/108
*h+4/27*i)*ln(-1+x)+(-1/108*d+1/27*e-7/108*f+5/54*g-13/108*h+4/27*i)*ln(1+x
)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h-1/36*i)/(1+x)
```

Maxima [A]
time = 0.30, size = 157, normalized size = 0.97

$$\frac{1}{864}\,(19\,d+52\,f-80\,g+112\,h-32\,e-128i)\log(x+2)-\frac{1}{108}\,(d+7\,f-10\,g+13\,h-4\,e-16i)\log(x+1)+\frac{1}{108}\,(d+7\,f+10\,g+13\,h+4\,e+16i)\log(x-1)-\frac{1}{864}\,(19\,d+52\,f+80\,g+112\,h+32\,e+128i)\log(x-2)-\frac{(5\,d+8\,f+20\,h)x^3+4\,(5\,g+2\,e+17i)x^2-(17\,d+20\,f+32\,h)x-32\,g-20\,e-80i}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e - 128*I)*log(x + 2) - 1/108*(d + 7
*f - 10*g + 13*h - 4*e - 16*I)*log(x + 1) + 1/108*(d + 7*f + 10*g + 13*h +
4*e + 16*I)*log(x - 1) - 1/864*(19*d + 52*f + 80*g + 112*h + 32*e + 128*I)*
log(x - 2) - 1/72*((5*d + 8*f + 20*h)*x^3 + 4*(5*g + 2*e + 17*I)*x^2 - (17*
d + 20*f + 32*h)*x - 32*g - 20*e - 80*I)/(x^4 - 5*x^2 + 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(146) = 292$.
time = 7.43, size = 346, normalized size = 2.14

$$-\frac{12\,(5\,d+8\,f+20\,h)x^3+48\,(2\,e+5\,g+17\,i)x^2-12\,(17\,d+20\,f+32\,h)x-((19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)x^4-5\,(19\,d-32\,e+52\,f-80\,g+112\,h-128\,i)x^2+76\,d-128\,e+208\,f-320\,g+448\,h-512\,i)\log(x+2)+8\,((d-4\,e+7\,f-10\,g+13\,h-16\,i)x^4-5\,(d-4\,e+7\,f-10\,g+13\,h-16\,i)x^2+4\,d-16\,e+28\,f-40\,g+52\,h-64\,i)\log(x+1)-8\,((d+4\,e+7\,f+10\,g+13\,h+16\,i)x^4-5\,(d+4\,e+7\,f+10\,g+13\,h+16\,i)x^2+4\,d+16\,e+28\,f+40\,g+52\,h+64\,i)\log(x-1)+((19\,d+32\,e+52\,f+80\,g+112\,h+128\,i)x^4-5\,(19\,d+32\,e+52\,f+80\,g+112\,h+128\,i)x^2+76\,d+128\,e+208\,f+320\,g+448\,h+512\,i)\log(x-2)-240\,e-384\,g-960\,i}{864\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/864*(12*(5*d + 8*f + 20*h)*x^3 + 48*(2*e + 5*g + 17*i)*x^2 - 12*(17*d +
20*f + 32*h)*x - ((19*d - 32*e + 52*f - 80*g + 112*h - 128*i)*x^4 - 5*(19*d
 - 32*e + 52*f - 80*g + 112*h - 128*i)*x^2 + 76*d - 128*e + 208*f - 320*g +
 448*h - 512*i)*log(x + 2) + 8*((d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^4 -
5*(d - 4*e + 7*f - 10*g + 13*h - 16*i)*x^2 + 4*d - 16*e + 28*f - 40*g + 52*
h - 64*i)*log(x + 1) - 8*((d + 4*e + 7*f + 10*g + 13*h + 16*i)*x^4 - 5*(d +
```

```
 4*e + 7*f + 10*g + 13*h + 16*i)*x^2 + 4*d + 16*e + 28*f + 40*g + 52*h + 64
*i)*log(x - 1) + ((19*d + 32*e + 52*f + 80*g + 112*h + 128*i)*x^4 - 5*(19*d
 + 32*e + 52*f + 80*g + 112*h + 128*i)*x^2 + 76*d + 128*e + 208*f + 320*g +
 448*h + 512*i)*log(x - 2) - 240*e - 384*g - 960*i)/(x^4 - 5*x^2 + 4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.13, size = 168, normalized size = 1.04

$$\frac{1}{864}\,(19\,d+52\,f-80\,g+112\,h-32\,e-128i)\log\left(|x+2|\right)-\frac{1}{108}\,(d+7\,f-10\,g+13\,h-4\,e-16i)\log\left(|x+1|\right)+\frac{1}{108}\,(d+7\,f+10\,g+13\,h+4\,e+16i)\log\left(|x-1|\right)-\frac{1}{864}\,(19\,d+52\,f+80\,g+112\,h+32\,e+128i)\log\left(|x-2|\right)-\frac{5\,dx^3+8\,fx^3+20\,hx^3+20\,gx^2+8\,x^2e-17\,dx-20\,fx-32\,hx+68i\,x^2-32\,g-20\,e-80i}{72\,(x^4-5\,x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/864*(19*d + 52*f - 80*g + 112*h - 32*e - 128*I)*log(abs(x + 2)) - 1/108*(
d + 7*f - 10*g + 13*h - 4*e - 16*I)*log(abs(x + 1)) + 1/108*(d + 7*f + 10*g
 + 13*h + 4*e + 16*I)*log(abs(x - 1)) - 1/864*(19*d + 52*f + 80*g + 112*h +
 32*e + 128*I)*log(abs(x - 2)) - 1/72*(5*d*x^3 + 8*f*x^3 + 20*h*x^3 + 20*g*
x^2 + 8*x^2*e - 17*d*x - 20*f*x - 32*h*x + 68*I*x^2 - 32*g - 20*e - 80*I)/(
x^4 - 5*x^2 + 4)
```

Mupad [**B**]
time = 0.58, size = 164, normalized size = 1.01

$$\frac{\left(-\frac{5\,d}{72}-\frac{f}{9}-\frac{5\,h}{18}\right)\,x^3+\left(-\frac{e}{9}-\frac{5\,g}{18}-\frac{17\,i}{18}\right)\,x^2+\left(\frac{17\,d}{72}+\frac{5\,f}{18}+\frac{4\,h}{9}\right)\,x+\frac{5\,e}{18}+\frac{4\,g}{9}+\frac{10\,i}{9}}{x^4-5\,x^2+4}+\ln\left(x-1\right)\,\left(\frac{d}{108}+\frac{e}{27}+\frac{7\,f}{108}+\frac{5\,g}{54}+\frac{13\,h}{108}+\frac{4\,i}{27}\right)-\ln\left(x+1\right)\,\left(\frac{d}{108}-\frac{e}{27}+\frac{7\,f}{108}-\frac{5\,g}{54}+\frac{13\,h}{108}-\frac{4\,i}{27}\right)-\ln\left(x-2\right)\,\left(\frac{19\,d}{864}+\frac{e}{27}+\frac{13\,f}{216}+\frac{5\,g}{54}+\frac{7\,h}{54}+\frac{4\,i}{27}\right)+\ln\left(x+2\right)\,\left(\frac{19\,d}{864}-\frac{e}{27}+\frac{13\,f}{216}-\frac{5\,g}{54}+\frac{7\,h}{54}-\frac{4\,i}{27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] ((5*e)/18 + (4*g)/9 + (10*i)/9 + x*((17*d)/72 + (5*f)/18 + (4*h)/9) - x^3*(
(5*d)/72 + f/9 + (5*h)/18) - x^2*(e/9 + (5*g)/18 + (17*i)/18))/(x^4 - 5*x^2
 + 4) + log(x - 1)*(d/108 + e/27 + (7*f)/108 + (5*g)/54 + (13*h)/108 + (4*i
)/27) - log(x + 1)*(d/108 - e/27 + (7*f)/108 - (5*g)/54 + (13*h)/108 - (4*i
)/27) - log(x - 2)*((19*d)/864 + e/27 + (13*f)/216 + (5*g)/54 + (7*h)/54 +
(4*i)/27) + log(x + 2)*((19*d)/864 - e/27 + (13*f)/216 - (5*g)/54 + (7*h)/5
4 - (4*i)/27)
```

3.31 $\int \frac{d+ex}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=140

$$\frac{dx(1-x^2)}{6(1+x^2+x^4)}+\frac{e(1+2x^2)}{6(1+x^2+x^4)}-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{4}d\log\left(1-x+x^2\right.$$

```
[Out] 1/6*d*x*(-x^2+1)/(x^4+x^2+1)+1/6*e*(2*x^2+1)/(x^4+x^2+1)-1/4*d*ln(x^2-x+1)+
1/4*d*ln(x^2+x+1)-1/9*d*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/9*d*arctan(1/
3*(1+2*x)*3^(1/2))*3^(1/2)+2/9*e*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$,
Rules used = {1687, 12, 1106, 1183, 648, 632, 210, 642, 1121, 628}

$$-\frac{d\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{4}d\log\left(x^2-x+1\right)+\frac{1}{4}d\log\left(x^2+x+1\right)+\frac{dx(1-x^2)}{6(x^4+x^2+1)}+\frac{e(2x^2+1)}{6(x^4+x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (d*x*(1 - x^2))/(6*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) -
 (d*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (d*ArcTan[(1 + 2*x)/Sqrt[3]])/
(3*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - (d*Log[1 - x
+ x^2])/4 + (d*Log[1 + x + x^2])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{(1+x^2+x^4)^2}\,dx &= \int \frac{d}{(1+x^2+x^4)^2}\,dx+\int \frac{ex}{(1+x^2+x^4)^2}\,dx\\ &= d\int \frac{1}{(1+x^2+x^4)^2}\,dx+e\int \frac{x}{(1+x^2+x^4)^2}\,dx\\ &= \frac{dx(1-x^2)}{6\left(1+x^2+x^4\right)}+\frac{1}{6}d\int \frac{5-x^2}{1+x^2+x^4}\,dx+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{(1+x+x^2)^2}\,dx,x,x^2\right)\\ &= \frac{dx(1-x^2)}{6\left(1+x^2+x^4\right)}+\frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)}+\frac{1}{12}d\int \frac{5-6x}{1-x+x^2}\,dx+\frac{1}{12}d\int \frac{5+6x}{1+x+x^2}\,dx+\\ &= \frac{dx(1-x^2)}{6\left(1+x^2+x^4\right)}+\frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)}+\frac{1}{6}d\int \frac{1}{1-x+x^2}\,dx+\frac{1}{6}d\int \frac{1}{1+x+x^2}\,dx-\frac{1}{4}\\ &= \frac{dx(1-x^2)}{6\left(1+x^2+x^4\right)}+\frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)}+\frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{4}d\log\left(1-x+x^2\right)+\frac{1}{4}d\,\text{lo}\\ &= \frac{dx(1-x^2)}{6\left(1+x^2+x^4\right)}+\frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)}-\frac{d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{d\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2e\tan^{-1}\left(\right.}{3\sqrt{}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.31, size = 146, normalized size = 1.04

$$\frac{e+2ex^2+d(x-x^3)}{6\left(1+x^2+x^4\right)}-\frac{\left(-11i+\sqrt{3}\right)d\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(11i+\sqrt{3}\right)d\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}-\frac{2e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e + 2*e*x^2 + d*(x - x^3))/(6*(1 + x^2 + x^4)) - ((-11*I + Sqrt[3])*d*ArcT
an[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt[3]]) - ((11*I + Sqrt[3])*d
*ArcTan[((I + Sqrt[3])*x)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]]) - (2*e*ArcTan[Sqr
t[3]/(1 + 2*x^2)])/(3*Sqrt[3])
```

Maple [A]
time = 0.07, size = 124, normalized size = 0.89

method	result
default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}\right)x-\frac{2d}{3}+\frac{e}{3}}{4x^2+4x+4}+\frac{d\ln\left(x^2+x+1\right)}{4}+\frac{(2d-4e)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{\left(\frac{d}{3}-\frac{e}{3}\right)x-\frac{2d}{3}-\frac{e}{3}}{4(x^2-x+1)}-\frac{d\ln\left(x^2-x+1\right)}{4}-\frac{(-2d}{}$
risch	$\frac{d\ln\left(10044d^2x^2+5952e^2x^2+10044d^2x+5952e^2x+10044d^2+5952e^2\right)}{4}+\frac{-\frac{1}{6}d\,x^3+\frac{1}{3}e\,x^2+\frac{1}{6}dx+\frac{1}{6}e}{x^4+x^2+1}-\frac{d\ln\left(10044d^2x^2+5952e^2x^2-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*d-1/3*e)*x-2/3*d+1/3*e)/(x^2+x+1)+1/4*d*ln(x^2+x+1)+1/18*(2*d-4*
e)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*((1/3*d-1/3*e)*x-2/3*d-1/3*e)/(x
^2-x+1)-1/4*d*ln(x^2-x+1)-1/18*(-2*d-4*e)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

Maxima [A]
time = 0.50, size = 100, normalized size = 0.71

$$\frac{1}{9}\sqrt{3}\,(d-2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{9}\sqrt{3}\,(d+2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\,d\log\left(x^2+x+1\right)-\frac{1}{4}\,d\log\left(x^2-x+1\right)-\frac{dx^3-2\,x^2e-dx-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1) - 1/6*(d*x^3 - 2*x^2*e - d*x - e)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 0.44, size = 154, normalized size = 1.10

$$-\frac{6\,dx^3-12\,ex^2-4\,\sqrt{3}\,((d-2\,e)x^4+(d-2\,e)x^2+d-2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-4\,\sqrt{3}\,((d+2\,e)x^4+(d+2\,e)x^2+d+2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-6\,dx-9\,(dx^4+dx^2+d)\log\left(x^2+x+1\right)+9\,(dx^4+dx^2+d)\log\left(x^2-x+1\right)-6\,e}{36\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(6*d*x^3 - 12*e*x^2 - 4*sqrt(3)*((d - 2*e)*x^4 + (d - 2*e)*x^2 + d -
2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*((d + 2*e)*x^4 + (d + 2*e)*x
^2 + d + 2*e)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*d*x - 9*(d*x^4 + d*x^2 + d)
*log(x^2 + x + 1) + 9*(d*x^4 + d*x^2 + d)*log(x^2 - x + 1) - 6*e)/(x^4 + x^
2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.21, size = 952, normalized size = 6.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 - sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4*e + 1026*d**4*(-d/4 -
sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 - 31536*d**2*e**2*(-d/4 - sqrt(3)*
I*(d + 2*e)/18) + 108432*d**2*e*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**2 + 163296
*d**2*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**3 + 1792*e**5 + 11520*e**4*(-d/4 - s
qrt(3)*I*(d + 2*e)/18) + 48384*e**3*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**2 + 31
1040*e**2*(-d/4 - sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5 - 11408*d**3*e**2
- 7936*d*e**4)) + (-d/4 + sqrt(3)*I*(d + 2*e)/18)*log(x + (-10309*d**4*e +
1026*d**4*(-d/4 + sqrt(3)*I*(d + 2*e)/18) - 7200*d**2*e**3 - 31536*d**2*e**
2*(-d/4 + sqrt(3)*I*(d + 2*e)/18) + 108432*d**2*e*(-d/4 + sqrt(3)*I*(d + 2*
e)/18)**2 + 163296*d**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**3 + 1792*e**5 + 11
520*e**4*(-d/4 + sqrt(3)*I*(d + 2*e)/18) + 48384*e**3*(-d/4 + sqrt(3)*I*(d
+ 2*e)/18)**2 + 311040*e**2*(-d/4 + sqrt(3)*I*(d + 2*e)/18)**3)/(3348*d**5
- 11408*d**3*e**2 - 7936*d*e**4)) + (d/4 - sqrt(3)*I*(d - 2*e)/18)*log(x +
(-10309*d**4*e + 1026*d**4*(d/4 - sqrt(3)*I*(d - 2*e)/18) - 7200*d**2*e**3
- 31536*d**2*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18) + 108432*d**2*e*(d/4 - sqr
t(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d/4 - sqrt(3)*I*(d - 2*e)/18)**3 + 1
792*e**5 + 11520*e**4*(d/4 - sqrt(3)*I*(d - 2*e)/18) + 48384*e**3*(d/4 - sq
rt(3)*I*(d - 2*e)/18)**2 + 311040*e**2*(d/4 - sqrt(3)*I*(d - 2*e)/18)**3)/(
3348*d**5 - 11408*d**3*e**2 - 7936*d*e**4)) + (d/4 + sqrt(3)*I*(d - 2*e)/18
)*log(x + (-10309*d**4*e + 1026*d**4*(d/4 + sqrt(3)*I*(d - 2*e)/18) - 7200*
d**2*e**3 - 31536*d**2*e**2*(d/4 + sqrt(3)*I*(d - 2*e)/18) + 108432*d**2*e*
(d/4 + sqrt(3)*I*(d - 2*e)/18)**2 + 163296*d**2*(d/4 + sqrt(3)*I*(d - 2*e)/
18)**3 + 1792*e**5 + 11520*e**4*(d/4 + sqrt(3)*I*(d - 2*e)/18) + 48384*e**3
*(d/4 + sqrt(3)*I*(d - 2*e)/18)**2 + 311040*e**2*(d/4 + sqrt(3)*I*(d - 2*e)
/18)**3)/(3348*d**5 - 11408*d**3*e**2 - 7936*d*e**4)) + (-d*x**3 + d*x + 2*
e*x**2 + e)/(6*x**4 + 6*x**2 + 6)
```

Giac [A]
time = 3.20, size = 100, normalized size = 0.71

$$\frac{1}{9}\sqrt{3}\,(d-2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{9}\sqrt{3}\,(d+2\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\,d\log\left(x^2+x+1\right)-\frac{1}{4}\,d\log\left(x^2-x+1\right)-\frac{dx^3-2\,x^2e-dx-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(d - 2*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/9*sqrt(3)*(d + 2*e)
*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/4*d*log(x^2 + x + 1) - 1/4*d*log(x^2 - x
 + 1) - 1/6*(d*x^3 - 2*x^2*e - d*x - e)/(x^4 + x^2 + 1)
```

Mupad [B]
time = 0.25, size = 149, normalized size = 1.06

$$\frac{-\frac{d\,x^3}{6}+\frac{e\,x^2}{3}+\frac{d\,x}{6}+\frac{e}{6}}{x^4+x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{d}{4}-\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] (e/6 + (d*x)/6 - (d*x^3)/6 + (e*x^2)/3)/(x^2 + x^4 + 1) - log(x - (3^(1/2)*
1i)/2 - 1/2)*(d/4 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9) + log(x - (3^(1/2
)*1i)/2 + 1/2)*(d/4 - (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9) + log(x + (3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*d*1i)/18 - d/4 + (3^(1/2)*e*1i)/9) + log(x + (3^
(1/2)*1i)/2 + 1/2)*(d/4 + (3^(1/2)*d*1i)/18 - (3^(1/2)*e*1i)/9)
```

3.32 $\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=165

$$\frac{e(1+2x^2)}{6\,(1+x^2+x^4)}+\frac{x(d+f-(d-2f)x^2)}{6\,(1+x^2+x^4)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{2e\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right.}{3\sqrt{3}}$$

```
[Out] 1/6*e*(2*x^2+1)/(x^4+x^2+1)+1/6*x*(d+f-(d-2*f)*x^2)/(x^4+x^2+1)-1/8*(2*d-f)
*ln(x^2-x+1)+1/8*(2*d-f)*ln(x^2+x+1)-1/36*(4*d+f)*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)+1/36*(4*d+f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+2/9*e*arctan(1/
3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1687, 1192, 1183, 648, 632, 210, 642, 12, 1121, 628}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f)}{12\sqrt{3}}+\frac{2e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x(-(x^2(d-2f))+d+f)}{6\left(x^4+x^2+1\right)}+\frac{e(2x^2+1)}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x
^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*d + f
)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]
])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f)*Log[1 + x + x^
2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{\left(1+x^2+x^4\right)^2}\,dx &= \int \frac{ex}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^2}\,dx \\
&= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+(-d+2f)x^2}{1+x^2+x^4}\,dx + e\int \frac{x}{\left(1+x^2+x^4\right)^2}\,dx \\
&= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-f-(6d-3f)x}{1-x+x^2}\,dx + \frac{1}{12}\int \frac{5d-f+(6d-3f)}{1+x+x^2} \\
&= \frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)} + \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{1}{3}e\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) + \frac{1}{8}(\\
&= \frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)} + \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} - \frac{1}{8}(2d-f)\log\left(1-x+x^2\right) + \frac{1}{8}(2d-f) \\
&= \frac{e(1+2x^2)}{6\left(1+x^2+x^4\right)} + \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} - \frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4d+f)\tan^{-}}{12\sqrt{3}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 186, normalized size = 1.13

$$\frac{1}{36}\left(\frac{6(e+2ex^2+x(d+f-dx^2+2fx^2))}{1+x^2+x^4} - \frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 8\sqrt{3}\,e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4) - (((-11*I
 + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1
 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f)*ArcTan[((I +
 Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 8*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 +
2*x^2)])/36
```

Maple [A]
time = 0.11, size = 154, normalized size = 0.93

method	result
default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}+\frac{f}{3}}{4x^2+4x+4}+\frac{(6d-3f)\ln(x^2+x+1)}{24}+\frac{\left(2d-4e+\frac{f}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{\left(\frac{d}{3}-\frac{e}{3}-\frac{2f}{3}\right)x-\frac{2d}{3}-\frac{e}{3}+\frac{f}{3}}{4(x^2-x+1)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*d-1/3*e+2/3*f)*x-2/3*d+1/3*e+1/3*f)/(x^2+x+1)+1/24*(6*d-3*f)*ln(
x^2+x+1)+1/18*(2*d-4*e+1/2*f)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*((1/3
*d-1/3*e-2/3*f)*x-2/3*d-1/3*e+1/3*f)/(x^2-x+1)-1/24*(6*d-3*f)*ln(x^2-x+1)-1
/18*(-2*d-4*e-1/2*f)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 124, normalized size = 0.75

$$\frac{1}{36}\sqrt{3}\,(4\,d+f-8\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{36}\sqrt{3}\,(4\,d+f+8\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{8}(2\,d-f)\log\left(x^2+x+1\right)-\frac{1}{8}(2\,d-f)\log\left(x^2-x+1\right)-\frac{(d-2\,f)x^3-2\,x^2e-(d+f)x-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d + f - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(
4*d + f + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x +
1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d - 2*f)*x^3 - 2*x^2*e - (d + f
)*x - e)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 0.45, size = 212, normalized size = 1.28

$$-\frac{12\,(d-2\,f)x^3-24\,ex^2-2\,\sqrt{3}\,((4\,d-8\,e+f)x^4+(4\,d-8\,e+f)x^2+4\,d-8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-2\,\sqrt{3}\,((4\,d+8\,e+f)x^4+(4\,d+8\,e+f)x^2+4\,d+8\,e+f)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-12\,(d+f)x-9\,((2\,d-f)x^4+(2\,d-f)x^2+2\,d-f)\log\left(x^2+x+1\right)+9\,((2\,d-f)x^4+(2\,d-f)x^2+2\,d-f)\log\left(x^2-x+1\right)-12\,e}{72\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f)*x^3 - 24*e*x^2 - 2*sqrt(3)*((4*d - 8*e + f)*x^4 + (4*d
- 8*e + f)*x^2 + 4*d - 8*e + f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(
(4*d + 8*e + f)*x^4 + (4*d + 8*e + f)*x^2 + 4*d + 8*e + f)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 12*(d + f)*x - 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*
log(x^2 + x + 1) + 9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 - x
+ 1) - 12*e)/(x^4 + x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 89.50, size = 4106, normalized size = 24.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] (-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5*e + 16416
*d**5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 336520*d**4*e*f + 20066
4*d**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 115200*d**3*e**3 - 5
04576*d**3*e**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e
*f**2 + 1734912*d**3*e*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229
500*d**3*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(-
d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d
**2*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 119420*d**2*e*f**3
 - 2477952*d**2*e*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 50436*
d**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) - 2519424*d**2*f*(-d/
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(
-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e*
*3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**2*(-d/
4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/4 + f/8 - sqrt
(3)*I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(-d/4 + f/
8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*d*f**4*(-d/4 + f/8 - sqrt(3)*I
*(4*d + 8*e + f)/72) + 139968*d*f**2*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f
)/72)**3 - 20480*e**5*f - 36864*e**4*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e +
 f)/72) - 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e
 + f)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)
- 995328*e**2*f*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 3956*e*f**
5 - 209088*e*f**3*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 3996*f**
5*(-d/4 + f/8 - sqrt(3)*I*(4*d + 8*e + f)/72) + 233280*f**3*(-d/4 + f/8 - s
qrt(3)*I*(4*d + 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e
**2 + 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e
**4 - 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e*
*2*f**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**6)) + (-d/
4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)*log(x + (-164944*d**5*e + 16416*d**
5*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 336520*d**4*e*f + 200664*d*
*4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 115200*d**3*e**3 - 50457
6*d**3*e**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 272380*d**3*e*f**
2 + 1734912*d**3*e*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 229500*
d**3*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 2612736*d**3*(-d/4
+ f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d**2*
e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 119420*d**2*e*f**3 - 2
477952*d**2*e*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 50436*d**2
*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 2519424*d**2*f*(-d/4 +
f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(-d/4
 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(
-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 409536*d*e**2*f**2*(-d/4 +
f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 4976640*d*e**2*(-d/4 + f/8 + sqrt(3)*
```

```
I*(4*d + 8*e + f)/72)**3 - 31040*d*e*f**4 + 1270080*d*e*f**2*(-d/4 + f/8 +
sqrt(3)*I*(4*d + 8*e + f)/72)**2 + 14040*d*f**4*(-d/4 + f/8 + sqrt(3)*I*(4*
d + 8*e + f)/72) + 139968*d*f**2*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72
)**3 - 20480*e**5*f - 36864*e**4*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/
72) - 2880*e**3*f**3 - 552960*e**3*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f
)/72)**2 + 70848*e**2*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) - 99
5328*e**2*f*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**3 + 3956*e*f**5 -
209088*e*f**3*(-d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72)**2 - 3996*f**5*(-
d/4 + f/8 + sqrt(3)*I*(4*d + 8*e + f)/72) + 233280*f**3*(-d/4 + f/8 + sqrt(
3)*I*(4*d + 8*e + f)/72)**3)/(53568*d**6 - 69984*d**5*f - 182528*d**4*e**2
+ 23652*d**4*f**2 + 377344*d**3*e**2*f + 5400*d**3*f**3 - 126976*d**2*e**4
- 278400*d**2*e**2*f**2 - 4131*d**2*f**4 + 102400*d*e**4*f + 93568*d*e**2*f
**3 + 81*d*f**5 - 28672*e**4*f**2 - 11648*e**2*f**4 + 189*f**6)) + (d/4 - f
/8 - sqrt(3)*I*(4*d - 8*e + f)/72)*log(x + (-164944*d**5*e + 16416*d**5*(d/
4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 336520*d**4*e*f + 200664*d**4*f*(
d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 115200*d**3*e**3 - 504576*d**3*
e**2*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 272380*d**3*e*f**2 + 1734
912*d**3*e*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 - 229500*d**3*f**2
*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 2612736*d**3*(d/4 - f/8 - sqr
t(3)*I*(4*d - 8*e + f)/72)**3 + 51840*d**2*e**3*f + 881280*d**2*e**2*f*(d/4
 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) + 119420*d**2*e*f**3 - 2477952*d**2*
e*f*(d/4 - f/8 - sqrt(3)*I*(4*d - 8*e + f)/72)**2 + 50436*d**2*f**3*(d/4 -
f/8 - sqrt(3)*I*(4*d - 8*e + f)/72) - 2519424*d**2*f*(d/4 - f/8 - sqrt(3)*I
*(4*d - 8*e + f)/72)**3 + 28672*d*e**5 + 184320*d*e**4*(d/4 - f/8 - sqrt(3)
*I*(4*d - 8*e + f)/72) + 8640*d*e**3*f**2 + 774144*d*e**3*(d/4 - f/8 - sqrt
(3)*I*(4*d - 8*e + f)/72)**2 - 409536*d*e**2*f*...
```

Giac [A]
time = 2.66, size = 128, normalized size = 0.78

$$\frac{1}{36}\sqrt{3}\,(4\,d+f-8\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{36}\,\sqrt{3}\,(4\,d+f+8\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{8}\,(2\,d-f)\log\left(x^2+x+1\right)-\frac{1}{8}\,(2\,d-f)\log\left(x^2-x+1\right)-\frac{dx^3-2\,fx^3-2\,x^2e-dx-fx-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(
4*d + f + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*log(x^2 + x +
1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 - 2*x^2*e - d*x
- f*x - e)/(x^4 + x^2 + 1)
```

Mupad [B]
time = 0.32, size = 201, normalized size = 1.22

$$\frac{\left(\frac{f}{3}-\frac{d}{6}\right)\,x^3+\frac{e\,x^2}{3}+\left(\frac{d}{6}+\frac{f}{6}\right)\,x+\frac{e}{6}}{x^4+x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{72}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{72}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{72}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1{}\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] (e/6 - x^3*(d/6 - f/3) + (e*x^2)/3 + x*(d/6 + f/6))/(x^2 + x^4 + 1) - log(x
 - (3^(1/2)*1i)/2 - 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9
+ (3^(1/2)*f*1i)/72) - log(x - (3^(1/2)*1i)/2 + 1/2)*(f/8 - d/4 + (3^(1/2)*
d*1i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72) + log(x + (3^(1/2)*1i)/2 -
 1/2)*(f/8 - d/4 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72
) + log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 - (3^(1/2)
*e*1i)/9 + (3^(1/2)*f*1i)/72)
```

3.33 $\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=179

$$\frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)}+\frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)}-\frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(2e-g)\,\mathrm{t}}{}$$

```
[Out] 1/6*x*(d+f-(d-2*f)*x^2)/(x^4+x^2+1)+1/6*(e-2*g+(2*e-g)*x^2)/(x^4+x^2+1)-1/8
*(2*d-f)*ln(x^2-x+1)+1/8*(2*d-f)*ln(x^2+x+1)-1/36*(4*d+f)*arctan(1/3*(1-2*x
)*3^(1/2))*3^(1/2)+1/36*(4*d+f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/9*(2*
e-g)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1687, 1192, 1183, 648, 632, 210, 642, 1261, 652}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{3\sqrt{3}}-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)+\frac{x(-(x^2(d-2f))+d+f)}{6\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + (2*e - g)*x^2)
/(6*(1 + x^2 + x^4)) - ((4*d + f)*ArcTan[(1 - 2*x)/Sqrt[3]])/(12*Sqrt[3]) +
 ((4*d + f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2*e - g)*ArcTan[(1
+ 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f)*Log[1 - x + x^2])/8 + ((2*d - f
)*Log[1 + x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
```

```
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\left(1+x^2+x^4\right)^2}\,dx = \int \frac{d+fx^2}{\left(1+x^2+x^4\right)^2}\,dx + \int \frac{x(e+gx^2)}{\left(1+x^2+x^4\right)^2}\,dx$$

$$= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{1}{6}\int \frac{5d-f+(-d+2f)x^2}{1+x^2+x^4}\,dx + \frac{1}{2}\text{Subst}\Bigg(\int \frac{e+}{(1+x}$$

$$= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{1}{12}\int \frac{5d-f-(6d-3f)x}{1-x+x^2}\,d$$

$$= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{1}{8}(2d-f)\int \frac{1+2x}{1+x+x^2}\,dx +$$

$$= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} + \frac{(2e-g)\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{8}($$

$$= \frac{x(d+f-(d-2f)x^2)}{6\left(1+x^2+x^4\right)} + \frac{e-2g+(2e-g)x^2}{6\left(1+x^2+x^4\right)} - \frac{(4d+f)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{(4}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 200, normalized size = 1.12

$$\frac{1}{36}\left(\frac{6(e+2ex^2-g(2+x^2)+x(d+f-dx^2+2fx^2))}{1+x^2+x^4} - \frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 4\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + 2*e*x^2 - g*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x
^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*
x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f
)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g
)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Maple [A]
time = 0.13, size = 172, normalized size = 0.96

method	result

default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}+\frac{2f}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{f}{3}}{4x^2+4x+4}+\frac{(6d-3f)\ln(x^2+x+1)}{24}+\frac{\left(2d-4e+\frac{f}{2}+2g\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{\left(\frac{d}{3}-\frac{e}{3}-\frac{g}{3}-\frac{2f}{3}\right)}{4(x^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*d-1/3*e-1/3*g+2/3*f)*x-2/3*d+1/3*e-2/3*g+1/3*f)/(x^2+x+1)+1/24*(
6*d-3*f)*ln(x^2+x+1)+1/18*(2*d-4*e+1/2*f+2*g)*arctan(1/3*(2*x+1)*3^(1/2))*3
^(1/2)-1/4*((1/3*d-1/3*e-1/3*g-2/3*f)*x-2/3*d-1/3*e+2/3*g+1/3*f)/(x^2-x+1)-
1/24*(6*d-3*f)*ln(x^2-x+1)-1/18*(-2*d-4*e-1/2*f+2*g)*3^(1/2)*arctan(1/3*(2*
x-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 136, normalized size = 0.76

$$\frac{1}{36}\sqrt{3}(4d+f+4g-8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+f-4g+8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)-\frac{(d-2f)x^3+(g-2e)x^2-(d+f)x+2g-e}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqr
t(3)*(4*d + f - 4*g + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*lo
g(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*((d - 2*f)*x^3 + (g -
 2*e)*x^2 - (d + f)*x + 2*g - e)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 0.57, size = 239, normalized size = 1.34

$$-\frac{12(d-2f)x^3-12(2e-g)x^2-2\sqrt{3}\left((4d-8e+f+4g)x^4+(4d-8e+f+4g)x^2+4d-8e+f+4g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((4d+8e+f-4g)x^4+(4d+8e+f-4g)x^2+4d+8e+f-4g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(d+f)x-9\left((2d-f)x^4+(2d-f)x^2+2d-f\right)\log\left(x^2+x+1\right)+9\left((2d-f)x^4+(2d-f)x^2+2d-f\right)\log\left(x^2-x+1\right)-12e+24g}{72(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f)*x^3 - 12*(2*e - g)*x^2 - 2*sqrt(3)*((4*d - 8*e + f + 4*
g)*x^4 + (4*d - 8*e + f + 4*g)*x^2 + 4*d - 8*e + f + 4*g)*arctan(1/3*sqrt(3
)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e + f - 4*g)*x^4 + (4*d + 8*e + f - 4*g)
*x^2 + 4*d + 8*e + f - 4*g)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(d + f)*x -
9*((2*d - f)*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 + x + 1) + 9*((2*d - f)
*x^4 + (2*d - f)*x^2 + 2*d - f)*log(x^2 - x + 1) - 12*e + 24*g)/(x^4 + x^2
+ 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.72, size = 142, normalized size = 0.79

$$\frac{1}{36}\sqrt{3}(4d+f+4g-8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+f-4g+8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f)\log\left(x^2+x+1\right)-\frac{1}{8}(2d-f)\log\left(x^2-x+1\right)-\frac{dx^3-2fx^3+gx^2-2x^2e-dx-fx+2g-e}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqr
t(3)*(4*d + f - 4*g + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d - f)*lo
g(x^2 + x + 1) - 1/8*(2*d - f)*log(x^2 - x + 1) - 1/6*(d*x^3 - 2*f*x^3 + g*
x^2 - 2*x^2*e - d*x - f*x + 2*g - e)/(x^4 + x^2 + 1)
```

Mupad [B]
time = 1.15, size = 237, normalized size = 1.32

$$\frac{\left(\frac{f}{3}-\frac{d}{6}\right)x^3+\left(\frac{e}{3}-\frac{g}{6}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{e}{6}-\frac{g}{3}}{x^4+x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}-\frac{\sqrt{3}\,g\,1\mathrm{i}}{18}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}+\frac{\sqrt{3}\,g\,1\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{d}{4}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}-\frac{\sqrt{3}\,g\,1\mathrm{i}}{18}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{d}{4}-\frac{f}{8}+\frac{\sqrt{3}\,d\,1\mathrm{i}}{18}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{72}+\frac{\sqrt{3}\,g\,1\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] (e/6 - g/3 - x^3*(d/6 - f/3) + x^2*(e/3 - g/6) + x*(d/6 + f/6))/(x^2 + x^4
+ 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 + (3^(1
/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*1i)/18) - log(x - (3^(1/2)*1i)
/2 + 1/2)*(f/8 - d/4 + (3^(1/2)*d*1i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i
)/72 + (3^(1/2)*g*1i)/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*(f/8 - d/4 + (3^(
1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*1i)/18) +
 log(x + (3^(1/2)*1i)/2 + 1/2)*(d/4 - f/8 + (3^(1/2)*d*1i)/18 - (3^(1/2)*e*
1i)/9 + (3^(1/2)*f*1i)/72 + (3^(1/2)*g*1i)/18)
```

3.34 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=187

$$\frac{e-2g+(2e-g)x^2}{6(1+x^2+x^4)}+\frac{x(d+f-2h-(d-2f+h)x^2)}{6(1+x^2+x^4)}-\frac{(4d+f+h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f+h)\tan^{-1}\left(\frac{1}{\right.}}{12\sqrt{3}}$$

```
[Out] 1/6*(e-2*g+(2*e-g)*x^2)/(x^4+x^2+1)+1/6*x*(d+f-2*h-(d-2*f+h)*x^2)/(x^4+x^2+
1)-1/8*(2*d-f+h)*ln(x^2-x+1)+1/8*(2*d-f+h)*ln(x^2+x+1)-1/36*(4*d+f+h)*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*d+f+h)*arctan(1/3*(1+2*x)*3^(1/2))*3
^(1/2)+1/9*(2*e-g)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {1687, 1692, 1183, 648, 632, 210, 642, 1261, 652}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{3\sqrt{3}}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x(-(x^2(d-2f+h))+d+f-2h)}{6\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (e - 2*g + (2*e - g)*x^2)/(6*(1 + x^2 + x^4)) + (x*(d + f - 2*h - (d - 2*f
+ h)*x^2))/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/
(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3]) + ((2
*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f + h)*Log[1 - x
 + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
```

```
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{(1+x^2+x^4)^2}\,dx &= \int \frac{x(e+gx^2)}{(1+x^2+x^4)^2}\,dx + \int \frac{d+fx^2+hx^4}{(1+x^2+x^4)^2}\,dx \\
&= \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} + \frac{1}{6}\int \frac{5d-f+2h+(-d+2f-h)x^2}{1+x^2+x^4} \\
&= \frac{e-2g+(2e-g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} + \frac{1}{12}\int \frac{5d-f\,-}{} \\
&= \frac{e-2g+(2e-g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} + \frac{1}{3}(-2e+g)\mathrm{Su} \\
&= \frac{e-2g+(2e-g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} + \frac{(2e-g)\tan^{-1}}{3\sqrt{3}} \\
&= \frac{e-2g+(2e-g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} - \frac{(4d+f+h)\,\mathrm{t}}{12\surd}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 234, normalized size = 1.25

$$\frac{1}{36}\left(-\frac{6(g(2+x^2)-e(1+2x^2)+x(d(-1+x^2)+h(2+x^2)-f(1+2x^2)))}{1+x^2+x^4}-\frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f+\left(-5i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f+\left(5i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-4\sqrt{3}\,(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((-6*(g*(2 + x^2) - e*(1 + 2*x^2) + x*(d*(-1 + x^2) + h*(2 + x^2) - f*(1 +
2*x^2))))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3])*f +
(-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] -
 (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*ArcTan[((I
 + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g)*ArcTan[Sqr
t[3]/(1 + 2*x^2)])/36
```

Maple [A]
time = 0.18, size = 196, normalized size = 1.05

method	result
default	$\frac{\left(-\frac{d}{3}+\frac{2f}{3}-\frac{g}{3}-\frac{e}{3}-\frac{h}{3}\right)x-\frac{2d}{3}+\frac{f}{3}-\frac{2g}{3}+\frac{e}{3}+\frac{h}{3}}{4x^2+4x+4}+\frac{(6d-3f+3h)\ln(x^2+x+1)}{24}+\frac{\left(2d-4e+\frac{f}{2}+2g+\frac{h}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{18}-$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*d+2/3*f-1/3*g-1/3*e-1/3*h)*x-2/3*d+1/3*f-2/3*g+1/3*e+1/3*h)/(x^2
+x+1)+1/24*(6*d-3*f+3*h)*ln(x^2+x+1)+1/18*(2*d-4*e+1/2*f+2*g+1/2*h)*arctan(
1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*((1/3*d-2/3*f-1/3*g-1/3*e+1/3*h)*x-2/3*d+1
/3*f+2/3*g-1/3*e+1/3*h)/(x^2-x+1)-1/24*(6*d-3*f+3*h)*ln(x^2-x+1)-1/18*(-2*d
-4*e-1/2*f+2*g-1/2*h)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 144, normalized size = 0.77

$$\frac{1}{36}\sqrt{3}(4d+f+4g+h-8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+f-4g+h+8e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f+h)\log(x^2+x+1)-\frac{1}{8}(2d-f+h)\log(x^2-x+1)-\frac{(d-2f+h)x^3+(g-2e)x^2-(d+f-2h)x+2g-e}{6(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + f - 4*g + h + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1) - 1/6*((d -
 2*f + h)*x^3 + (g - 2*e)*x^2 - (d + f - 2*h)*x + 2*g - e)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 1.64, size = 255, normalized size = 1.36

$$-\frac{12(d-2f+h)x^3-12(2e-g)x^2-2\sqrt{3}\left((4d-8e+f+4g+h)x^4+(4d-8e+f+4g+h)x^2+4d-8e+f+4g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((4d+8e+f-4g+h)x^4+(4d+8e+f-4g+h)x^2+4d+8e+f-4g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(d+f-2h)x-9\left((2d-f+h)x^4+(2d-f+h)x^2+2d-f+h\right)\log(x^2+x+1)+9\left((2d-f+h)x^4+(2d-f+h)x^2+2d-f+h\right)\log(x^2-x+1)-12e+24g}{72(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(d - 2*f + h)*x^3 - 12*(2*e - g)*x^2 - 2*sqrt(3)*((4*d - 8*e + f
+ 4*g + h)*x^4 + (4*d - 8*e + f + 4*g + h)*x^2 + 4*d - 8*e + f + 4*g + h)*a
rctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e + f - 4*g + h)*x^4 + (
4*d + 8*e + f - 4*g + h)*x^2 + 4*d + 8*e + f - 4*g + h)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 12*(d + f - 2*h)*x - 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2
+ 2*d - f + h)*log(x^2 + x + 1) + 9*((2*d - f + h)*x^4 + (2*d - f + h)*x^2
+ 2*d - f + h)*log(x^2 - x + 1) - 12*e + 24*g)/(x^4 + x^2 + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.44, size = 155, normalized size = 0.83

$$\frac{1}{36}\sqrt{3}\,(4\,d+f+4\,g+h-8\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{36}\sqrt{3}\,(4\,d+f-4\,g+h+8\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{8}\,(2\,d-f+h)\log\left(x^2+x+1\right)-\frac{1}{8}\,(2\,d-f+h)\log\left(x^2-x+1\right)-\frac{dx^3-2\,fx^3+hx^3+gx^2-2\,x^2e-dx-fx+2\,hx+2\,g-e}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36
*sqrt(3)*(4*d + f - 4*g + h + 8*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*d
 - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1) - 1/6*(d*x^
3 - 2*f*x^3 + h*x^3 + g*x^2 - 2*x^2*e - d*x - f*x + 2*h*x + 2*g - e)/(x^4 +
 x^2 + 1)
```

Mupad [B]
time = 5.35, size = 1547, normalized size = 8.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] (e/6 - g/3 + x^2*(e/3 - g/6) + x*(d/6 + f/6 - h/3) - x^3*(d/6 - f/3 + h/6))
/(x^2 + x^4 + 1) - log(60*d*g - 153*d*f - 120*d*e + 24*e*f + 135*d*h - 48*e
*h - 12*f*g - 81*f*h + 24*g*h + 3^(1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*
h^2*18i - 198*d^2*x - 36*f^2*x - 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 + 3^(
1/2)*d*e*56i - 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i - 3^(1/2)*e*f*40i + 3^(1/2
)*d*h*81i + 3^(1/2)*e*h*32i + 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i - 3^(1/2)*g
*h*16i - 24*d*e*x + 171*d*f*x + 12*d*g*x + 48*e*f*x - 189*d*h*x - 24*e*h*x
- 24*f*g*x + 81*f*h*x + 12*g*h*x + 3^(1/2)*d^2*x*18i + 3^(1/2)*f^2*x*18i +
3^(1/2)*h^2*x*9i - 3^(1/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32
i + 3^(1/2)*d*h*x*27i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h
*x*27i + 3^(1/2)*g*h*x*20i - 3^(1/2)*d*e*x*88i)*(d/4 - f/8 + h/8 + (3^(1/2)
*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*1i)/18 + (3^(
1/2)*h*1i)/72) - log(120*d*e - 153*d*f - 60*d*g - 24*e*f + 135*d*h + 48*e*h
```

```
 + 12*f*g - 81*f*h - 24*g*h - 3^(1/2)*d^2*90i - 3^(1/2)*f^2*9i - 3^(1/2)*h^
2*18i + 198*d^2*x + 36*f^2*x + 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 + 3^(1/
2)*d*e*56i + 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i - 3^(1/2)*e*f*40i - 3^(1/2)*
d*h*81i + 3^(1/2)*e*h*32i + 3^(1/2)*f*g*20i + 3^(1/2)*f*h*27i - 3^(1/2)*g*h
*16i - 24*d*e*x - 171*d*f*x + 12*d*g*x + 48*e*f*x + 189*d*h*x - 24*e*h*x -
24*f*g*x - 81*f*h*x + 12*g*h*x + 3^(1/2)*d^2*x*18i + 3^(1/2)*f^2*x*18i + 3^
(1/2)*h^2*x*9i - 3^(1/2)*d*f*x*45i - 3^(1/2)*d*g*x*44i - 3^(1/2)*e*f*x*32i
+ 3^(1/2)*d*h*x*27i + 3^(1/2)*e*h*x*40i + 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h*x
*27i - 3^(1/2)*g*h*x*20i + 3^(1/2)*d*e*x*88i)*(f/8 - d/4 - h/8 + (3^(1/2)*d
*1i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 + (3^(1/2)*g*1i)/18 + (3^(1/
2)*h*1i)/72) + log(120*d*e - 153*d*f - 60*d*g - 24*e*f + 135*d*h + 48*e*h +
 12*f*g - 81*f*h - 24*g*h + 3^(1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*h^2*
18i + 198*d^2*x + 36*f^2*x + 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 - 3^(1/2)
*d*e*56i - 3^(1/2)*d*f*63i + 3^(1/2)*d*g*28i + 3^(1/2)*e*f*40i + 3^(1/2)*d*
h*81i - 3^(1/2)*e*h*32i - 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i + 3^(1/2)*g*h*1
6i - 24*d*e*x - 171*d*f*x + 12*d*g*x + 48*e*f*x + 189*d*h*x - 24*e*h*x - 24
*f*g*x - 81*f*h*x + 12*g*h*x - 3^(1/2)*d^2*x*18i - 3^(1/2)*f^2*x*18i - 3^(1
/2)*h^2*x*9i + 3^(1/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32i -
3^(1/2)*d*h*x*27i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i + 3^(1/2)*f*h*x*2
7i + 3^(1/2)*g*h*x*20i - 3^(1/2)*d*e*x*88i)*(d/4 - f/8 + h/8 + (3^(1/2)*d*1
i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 + (3^(1/2)*g*1i)/18 + (3^(1/2)
*h*1i)/72) + log(120*d*e + 153*d*f - 60*d*g - 24*e*f - 135*d*h + 48*e*h + 1
2*f*g + 81*f*h - 24*g*h + 3^(1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*h^2*18
i + 198*d^2*x + 36*f^2*x + 45*h^2*x - 126*d^2 - 45*f^2 - 36*h^2 + 3^(1/2)*d
*e*56i - 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i - 3^(1/2)*e*f*40i + 3^(1/2)*d*h*
81i + 3^(1/2)*e*h*32i + 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i - 3^(1/2)*g*h*16i
 + 24*d*e*x - 171*d*f*x - 12*d*g*x - 48*e*f*x + 189*d*h*x + 24*e*h*x + 24*f
*g*x - 81*f*h*x - 12*g*h*x + 3^(1/2)*d^2*x*18i + 3^(1/2)*f^2*x*18i + 3^(1/2
)*h^2*x*9i - 3^(1/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32i + 3^
(1/2)*d*h*x*27i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h*x*27i
 + 3^(1/2)*g*h*x*20i - 3^(1/2)*d*e*x*88i)*(f/8 - d/4 - h/8 + (3^(1/2)*d*1i)
/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h
*1i)/72)
```

3.35 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^2}\,dx$

Optimal. Leaf size=194

$$\frac{x(d+f-2h-(d-2f+h)x^2)}{6(1+x^2+x^4)}+\frac{e-2g+i+(2e-g-i)x^2}{6(1+x^2+x^4)}-\frac{(4d+f+h)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4d+f+h)\,\mathrm{ta}}{12\sqrt{}}$$

```
[Out] 1/6*x*(d+f-2*h-(d-2*f+h)*x^2)/(x^4+x^2+1)+1/6*(e-2*g+i+(2*e-g-i)*x^2)/(x^4+
x^2+1)-1/8*(2*d-f+h)*ln(x^2-x+1)+1/8*(2*d-f+h)*ln(x^2+x+1)-1/36*(4*d+f+h)*a
rctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*d+f+h)*arctan(1/3*(1+2*x)*3^(1/2
))*3^(1/2)+1/9*(2*e-g+2*i)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1687, 1692, 1183, 648, 632, 210, 642, 1677, 1674, 12}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(4d+f+h)}{12\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+2i)}{3\sqrt{3}}-\frac{1}{8}\log\left(x^2-x+1\right)(2d-f+h)+\frac{1}{8}\log\left(x^2+x+1\right)(2d-f+h)+\frac{x(-(x^2(d-2f+h))+d+f-2h)}{6(x^4+x^2+1)}+\frac{x^2(2e-g-i)+e-2g+i}{6(x^4+x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(6*(1 + x^2 + x^4)) + (e - 2*g + i +
(2*e - g - i)*x^2)/(6*(1 + x^2 + x^4)) - ((4*d + f + h)*ArcTan[(1 - 2*x)/Sq
rt[3]])/(12*Sqrt[3]) + ((4*d + f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[3
]) + ((2*e - g + 2*i)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((2*d - f
+ h)*Log[1 - x + x^2])/8 + ((2*d - f + h)*Log[1 + x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+35x^5}{(1+x^2+x^4)^2}\,dx = \int \frac{x(e+gx^2+35x^4)}{(1+x^2+x^4)^2}\,dx + \int \frac{d+fx^2+hx^4}{(1+x^2+x^4)^2}\,dx$$

$$= \frac{x(d+f-2h-(d-2f+h)x^2)}{6\,(1+x^2+x^4)} + \frac{1}{6}\int \frac{5d-f+2h+(-d+2f}{1+x^2+x^4}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2}{6\,(1+x^2+x^4)}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2}{6\,(1+x^2+x^4)}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2}{6\,(1+x^2+x^4)}$$

$$= \frac{35+e-2g-(35-2e+g)x^2}{6\,(1+x^2+x^4)} + \frac{x(d+f-2h-(d-2f+h)x^2}{6\,(1+x^2+x^4)}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 0.34, size = 243, normalized size = 1.25

$$\frac{1}{36}\left(\frac{6(e+i+dx+fx-2hx+2ex^2-ix^2-dx^3+2fx^3-hx^3-g(2+x^2))}{1+x^2+x^4} - \frac{\left(\left(-11i+\sqrt{3}\right)d-2\left(-2i+\sqrt{3}\right)f+\left(-5i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(\left(11i+\sqrt{3}\right)d-2\left(2i+\sqrt{3}\right)f+\left(5i+\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 4\sqrt{3}\,(2e-g+2i)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^2,x]
```

```
[Out] ((6*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^3
- g*(2 + x^2)))/(1 + x^2 + x^4) - (((-11*I + Sqrt[3])*d - 2*(-2*I + Sqrt[3]
)*f + (-5*I + Sqrt[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3]
)/6] - (((11*I + Sqrt[3])*d - 2*(2*I + Sqrt[3])*f + (5*I + Sqrt[3])*h)*ArcT
```

```
an[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 4*Sqrt[3]*(2*e - g + 2*i
)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/36
```

Maple [A]
time = 0.21, size = 214, normalized size = 1.10

method	result
default	$\frac{\left(-\frac{d}{3}-\frac{e}{3}-\frac{g}{3}-\frac{h}{3}+\frac{2f}{3}+\frac{2i}{3}\right)x-\frac{2d}{3}+\frac{e}{3}-\frac{2g}{3}+\frac{h}{3}+\frac{f}{3}+\frac{i}{3}}{4x^{2}+4x+4}+\frac{(6d-3f+3h)\ln\left(x^{2}+x+1\right)}{24}+\frac{\left(2d-4e+\frac{f}{2}+2g+\frac{h}{2}-4i\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right.}{18}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*d-1/3*e-1/3*g-1/3*h+2/3*f+2/3*i)*x-2/3*d+1/3*e-2/3*g+1/3*h+1/3*f
+1/3*i)/(x^2+x+1)+1/24*(6*d-3*f+3*h)*ln(x^2+x+1)+1/18*(2*d-4*e+1/2*f+2*g+1/
2*h-4*i)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*((1/3*d-1/3*e-1/3*g+1/3*h-
2/3*f+2/3*i)*x-2/3*d-1/3*e+2/3*g+1/3*h+1/3*f-1/3*i)/(x^2-x+1)-1/24*(6*d-3*f
+3*h)*ln(x^2-x+1)-1/18*(-2*d-4*e-1/2*f+2*g-1/2*h-4*i)*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 148, normalized size = 0.76

$$\frac{1}{36}\sqrt{3}(4d+f+4g+h-8e-8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{36}\sqrt{3}(4d+f-4g+h+8e+8i)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}(2d-f+h)\log\left(x^{2}+x+1\right)-\frac{1}{8}(2d-f+h)\log\left(x^{2}-x+1\right)-\frac{(d-2f+h)x^{3}+(g-2e+i)x^{2}-(d+f-2h)x+2g-e-i}{6(x^{4}+x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="maxim
a")
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e - 8*I)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + f - 4*g + h + 8*e + 8*I)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1)
 - 1/6*((d - 2*f + h)*x^3 + (g - 2*e + I)*x^2 - (d + f - 2*h)*x + 2*g - e -
 I)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 5.46, size = 279, normalized size = 1.44

$$-\frac{12(d-2f+h)x^{3}-12(2e-g-i)x^{2}-2\sqrt{3}\left((4d-8e+f+4g+h-8i)x^{4}+(4d-8e+f+4g+h-8i)x^{2}+4d-8e+f+4g+h-8i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((4d+8e+f-4g+h+8i)x^{4}+(4d+8e+f-4g+h+8i)x^{2}+4d+8e+f-4g+h+8i\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(d+f-2h)x-9\left((2d-f+h)x^{4}+(2d-f+h)x^{2}+2d-f+h\right)\log\left(x^{2}+x+1\right)+9\left((2d-f+h)x^{4}+(2d-f+h)x^{2}+2d-f+h\right)\log\left(x^{2}-x+1\right)-12e+24g-12i}{72(x^{4}+x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="frica
s")
```

```
[Out] -1/72*(12*(d - 2*f + h)*x^3 - 12*(2*e - g - i)*x^2 - 2*sqrt(3)*((4*d - 8*e
+ f + 4*g + h - 8*i)*x^4 + (4*d - 8*e + f + 4*g + h - 8*i)*x^2 + 4*d - 8*e
+ f + 4*g + h - 8*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*d + 8*e
+ f - 4*g + h + 8*i)*x^4 + (4*d + 8*e + f - 4*g + h + 8*i)*x^2 + 4*d + 8*e
+ f - 4*g + h + 8*i)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(d + f - 2*h)*x - 9
*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 + x + 1) + 9
*((2*d - f + h)*x^4 + (2*d - f + h)*x^2 + 2*d - f + h)*log(x^2 - x + 1) - 1
2*e + 24*g - 12*i)/(x^4 + x^2 + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.62, size = 163, normalized size = 0.84

$$\frac{1}{36}\sqrt{3}\,(4\,d+f+4\,g+h-8\,e-8i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{36}\sqrt{3}\,(4\,d+f-4\,g+h+8\,e+8i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{8}(2\,d-f+h)\log\left(x^2+x+1\right)-\frac{1}{8}(2\,d-f+h)\log\left(x^2-x+1\right)-\frac{dx^3-2\,fx^3+hx^3+gx^2-2\,x^2e-dx-fx+2\,hx+ix^2+2\,g-e-i}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^2,x, algorithm="giac"
)
```

```
[Out] 1/36*sqrt(3)*(4*d + f + 4*g + h - 8*e - 8*I)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/36*sqrt(3)*(4*d + f - 4*g + h + 8*e + 8*I)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/8*(2*d - f + h)*log(x^2 + x + 1) - 1/8*(2*d - f + h)*log(x^2 - x + 1)
 - 1/6*(d*x^3 - 2*f*x^3 + h*x^3 + g*x^2 - 2*x^2*e - d*x - f*x + 2*h*x + I*x
^2 + 2*g - e - I)/(x^4 + x^2 + 1)
```

Mupad [B]
time = 8.18, size = 1894, normalized size = 9.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] (e/6 - g/3 + i/6 + x*(d/6 + f/6 - h/3) - x^3*(d/6 - f/3 + h/6) - x^2*(g/6 -
 e/3 + i/6))/(x^2 + x^4 + 1) - log(60*d*g - 153*d*f - 120*d*e + 24*e*f + 13
5*d*h - 120*d*i - 48*e*h - 12*f*g - 81*f*h + 24*f*i + 24*g*h - 48*h*i + 3^(
1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*h^2*18i - 198*d^2*x - 36*f^2*x - 45
```

```
*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 + 3^(1/2)*d*e*56i - 3^(1/2)*d*f*63i - 3^
(1/2)*d*g*28i - 3^(1/2)*e*f*40i + 3^(1/2)*d*h*81i + 3^(1/2)*d*i*56i + 3^(1/
2)*e*h*32i + 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i - 3^(1/2)*f*i*40i - 3^(1/2)*
g*h*16i + 3^(1/2)*h*i*32i - 24*d*e*x + 171*d*f*x + 12*d*g*x + 48*e*f*x - 18
9*d*h*x - 24*d*i*x - 24*e*h*x - 24*f*g*x + 81*f*h*x + 48*f*i*x + 12*g*h*x -
 24*h*i*x + 3^(1/2)*d^2*x*18i + 3^(1/2)*f^2*x*18i + 3^(1/2)*h^2*x*9i - 3^(1
/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32i + 3^(1/2)*d*h*x*27i -
 3^(1/2)*d*i*x*88i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h*x*
27i + 3^(1/2)*f*i*x*32i + 3^(1/2)*g*h*x*20i - 3^(1/2)*h*i*x*40i - 3^(1/2)*d
*e*x*88i)*(d/4 - f/8 + h/8 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2
)*f*1i)/72 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/72 + (3^(1/2)*i*1i)/9) - lo
g(120*d*e - 153*d*f - 60*d*g - 24*e*f + 135*d*h + 120*d*i + 48*e*h + 12*f*g
 - 81*f*h - 24*f*i - 24*g*h + 48*h*i - 3^(1/2)*d^2*90i - 3^(1/2)*f^2*9i - 3
^(1/2)*h^2*18i + 198*d^2*x + 36*f^2*x + 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^
2 + 3^(1/2)*d*e*56i + 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i - 3^(1/2)*e*f*40i -
 3^(1/2)*d*h*81i + 3^(1/2)*d*i*56i + 3^(1/2)*e*h*32i + 3^(1/2)*f*g*20i + 3^
(1/2)*f*h*27i - 3^(1/2)*f*i*40i - 3^(1/2)*g*h*16i + 3^(1/2)*h*i*32i - 24*d*
e*x - 171*d*f*x + 12*d*g*x + 48*e*f*x + 189*d*h*x - 24*d*i*x - 24*e*h*x - 2
4*f*g*x - 81*f*h*x + 48*f*i*x + 12*g*h*x - 24*h*i*x + 3^(1/2)*d^2*x*18i + 3
^(1/2)*f^2*x*18i + 3^(1/2)*h^2*x*9i - 3^(1/2)*d*f*x*45i - 3^(1/2)*d*g*x*44i
 - 3^(1/2)*e*f*x*32i + 3^(1/2)*d*h*x*27i + 3^(1/2)*d*i*x*88i + 3^(1/2)*e*h*
x*40i + 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h*x*27i - 3^(1/2)*f*i*x*32i - 3^(1/2)
*g*h*x*20i + 3^(1/2)*h*i*x*40i + 3^(1/2)*d*e*x*88i)*(f/8 - d/4 - h/8 + (3^(
1/2)*d*1i)/18 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 + (3^(1/2)*g*1i)/18 +
(3^(1/2)*h*1i)/72 - (3^(1/2)*i*1i)/9) + log(120*d*e - 153*d*f - 60*d*g - 24
*e*f + 135*d*h + 120*d*i + 48*e*h + 12*f*g - 81*f*h - 24*f*i - 24*g*h + 48*
h*i + 3^(1/2)*d^2*90i + 3^(1/2)*f^2*9i + 3^(1/2)*h^2*18i + 198*d^2*x + 36*f
^2*x + 45*h^2*x + 126*d^2 + 45*f^2 + 36*h^2 - 3^(1/2)*d*e*56i - 3^(1/2)*d*f
*63i + 3^(1/2)*d*g*28i + 3^(1/2)*e*f*40i + 3^(1/2)*d*h*81i - 3^(1/2)*d*i*56
i - 3^(1/2)*e*h*32i - 3^(1/2)*f*g*20i - 3^(1/2)*f*h*27i + 3^(1/2)*f*i*40i +
 3^(1/2)*g*h*16i - 3^(1/2)*h*i*32i - 24*d*e*x - 171*d*f*x + 12*d*g*x + 48*e
*f*x + 189*d*h*x - 24*d*i*x - 24*e*h*x - 24*f*g*x - 81*f*h*x + 48*f*i*x + 1
2*g*h*x - 24*h*i*x - 3^(1/2)*d^2*x*18i - 3^(1/2)*f^2*x*18i - 3^(1/2)*h^2*x*
9i + 3^(1/2)*d*f*x*45i + 3^(1/2)*d*g*x*44i + 3^(1/2)*e*f*x*32i - 3^(1/2)*d*
h*x*27i - 3^(1/2)*d*i*x*88i - 3^(1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i + 3^(1/
2)*f*h*x*27i + 3^(1/2)*f*i*x*32i + 3^(1/2)*g*h*x*20i - 3^(1/2)*h*i*x*40i -
3^(1/2)*d*e*x*88i)*(d/4 - f/8 + h/8 + (3^(1/2)*d*1i)/18 - (3^(1/2)*e*1i)/9
+ (3^(1/2)*f*1i)/72 + (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/72 - (3^(1/2)*i*1i
)/9) + log(120*d*e + 153*d*f - 60*d*g - 24*e*f - 135*d*h + 120*d*i + 48*e*h
 + 12*f*g + 81*f*h - 24*f*i - 24*g*h + 48*h*i + 3^(1/2)*d^2*90i + 3^(1/2)*f
^2*9i + 3^(1/2)*h^2*18i + 198*d^2*x + 36*f^2*x + 45*h^2*x - 126*d^2 - 45*f^
2 - 36*h^2 + 3^(1/2)*d*e*56i - 3^(1/2)*d*f*63i - 3^(1/2)*d*g*28i - 3^(1/2)*
e*f*40i + 3^(1/2)*d*h*81i + 3^(1/2)*d*i*56i + 3^(1/2)*e*h*32i + 3^(1/2)*f*g
*20i - 3^(1/2)*f*h*27i - 3^(1/2)*f*i*40i - 3^(1/2)*g*h*16i + 3^(1/2)*h*i*32
i + 24*d*e*x - 171*d*f*x - 12*d*g*x - 48*e*f*x + 189*d*h*x + 24*d*i*x + 24*
```

```
e*h*x + 24*f*g*x - 81*f*h*x - 48*f*i*x - 12*g*h*x + 24*h*i*x + 3^(1/2)*d^2*
x*18i + 3^(1/2)*f^2*x*18i + 3^(1/2)*h^2*x*9i - 3^(1/2)*d*f*x*45i + 3^(1/2)*
d*g*x*44i + 3^(1/2)*e*f*x*32i + 3^(1/2)*d*h*x*27i - 3^(1/2)*d*i*x*88i - 3^(
1/2)*e*h*x*40i - 3^(1/2)*f*g*x*16i - 3^(1/2)*f*h*x*27i + 3^(1/2)*f*i*x*32i
+ 3^(1/2)*g*h*x*20i - 3^(1/2)*h*i*x*40i - 3^(1/2)*d*e*x*88i)*(f/8 - d/4 - h
/8 + (3^(1/2)*d*1i)/18 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/72 - (3^(1/2)*g*
1i)/18 + (3^(1/2)*h*1i)/72 + (3^(1/2)*i*1i)/9)
```

3.36 $\int \frac{d+ex}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=330

$$-\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{dx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)d\tan^{-1}\left(\frac{-}{\sqrt{}}\right.}{2\sqrt{2}\,a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-}}}$$

```
[Out] -1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*d*x*(b*c*x^2-2*a*c+b^2)
/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(3/2)+1/4*d*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*d*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1687, 12, 1106, 1180, 211, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\,d\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,d\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (d*x*(b^2 - 2*
a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2
- 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^2}\,dx &= \int \frac{d}{\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{ex}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= d\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx+e\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{dx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{d\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}+\frac{1}{2}e\text{Subst}\Bigg(\int \frac{}{(a+b} \\
&= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{dx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\Big(c\Big(b^2-12ac-}{} \\
&= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{dx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\,\Big(b^2-12ac-}{2\sqrt{2}} \\
&= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{dx(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\,\Big(b^2-12ac-}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 341, normalized size = 1.03

$$
\frac{1}{4}\left(\frac{2abe+4acx(d+ex)-2bdx(b+cx^2)}{a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\,\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\,\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*e + 4*a*c*x*(d + e*x) - 2*b*d*x*(b + c*x^2))/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*d*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(
```

```
3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqr
t[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*c*e*Log[-b + Sqrt[
b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e*Log[b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.14, size = 489, normalized size = 1.48

method	result
risch	$\frac{-\frac{bcd\,x^3}{2a(4ac-b^2)}+\frac{c\,x^2 e}{4ac-b^2}+\frac{d(2ac-b^2)x}{2a(4ac-b^2)}+\frac{eb}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{c\_R^2 bd}{a(4ac-b^2)}+\frac{4ce\_R}{4ac-b^2}+\frac{d(6ac-b^2)}{a(4ac-b^2)}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{-\frac{\left(-4ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+4abc-b^3\right)dx}{16a\,c^2}+\frac{e(4ac-b^2)}{8c^2}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(b+2c\,x^2+\sqrt{-4ac+b\right.}{}\right.$ ... $4(4ac-b^2)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/(4*a*c-b^2)^2*((-1/16/a/c^2*(-4*a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*
a*c+b^2)^(1/2)+4*a*b*c-b^3)*d*x+1/8*e*(4*a*c-b^2)/c^2)/(x^2+1/2/c*(-4*a*c+b
^2)^(1/2)+1/2*b/c)+1/8/a/c*(2*(-4*a*c+b^2)^(1/2)*a*e*ln(b+2*c*x^2+(-4*a*c+b
^2)^(1/2))+1/2*(12*(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d-4*a*b*
c*d+b^3*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/4/(4*a*c-b^2)^2*((1/16/a/c^2*(4*a*c*(-4*a
*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+4*a*b*c-b^3)*d*x-1/8*e*(4*a*c-b^2)/c^2
)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+1/8/a/c*(2*(-4*a*c+b^2)^(1/2)*a*e*
ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-12*(-4*a*c+b^2)^(1/2)*a*c*d+(-4*a*c
+b^2)^(1/2)*b^2*d-4*a*b*c*d+b^3*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*x^2*e + (b^2 - 2*a*c)*d*x - a*b*e)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b
*c*d*x^2 - 4*a*c*x*e + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*
a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3434 vs. $2(280) = 560$.
time = 7.23, size = 3434, normalized size = 10.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*x^2*e + b^2*d*x - 2*a*c*d*x - a*b*e)/((c*x^4 + b*x^2
 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*
c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20
```

```
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2
*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2
*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^
3*c^3)*d*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b
^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^
2 - 4*a*c)*a^4*b*c^4)*d)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqr
t((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*
b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^
2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 -
 4*a^2*c)^2*d + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c
^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 -
 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3
*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
```

```
b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*
a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d)*arctan(2*sqrt(1/2)*x/sqrt((a*b
^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*
c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b
^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*
b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/4*((b^
3*c^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 - (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)
*sqrt(b^2 - 4*a*c))*abs(a*b^2 - 4*a^2*c)*e - (a*b^5*c^2 - 8*a^2*b^3*c^3 - 2
*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 + (a*b^
4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*e)*log(
x^2 + 1/2*(a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*...
```

Mupad [B]
time = 1.50, size = 2382, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*e)/(2*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (d*x*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)) - (b*c*d*x^3)/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) +
symsum(log((5*b^3*c^4*d^3 - 96*a^2*c^5*d*e^2 - 36*a*b*c^5*d^3 + 16*a*b^2*c^
4*d*e^2)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(
1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 -
 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^
3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 432*a*b^9*c*d^2*z^2 + 24576*a^5*b^2*
c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^
4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 -
32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^2 - 672*a*b^6*c^2*d^2*e*z - 15872*a^3
*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z + 18432*a^4*c^5*d^2*e*z + 32*b^
8*c*d^2*e*z - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 - 16*b^5*c^2*d^
2*e^2 + 360*a*b^2*c^4*d^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*d^4 - 1296*a^2*c^5
*d^4, z, k)*(root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680
*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^
9*c^6*z^4 - 256*a^3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 432*a*b^9*c*d^2*z^
2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*
e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*
b^7*c^2*d^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^2 - 672*a*b^6*c^2*d
^2*e*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z + 18432*a^4*c
^5*d^2*e*z + 32*b^8*c*d^2*e*z - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e
^2 - 16*b^5*c^2*d^2*e^2 + 360*a*b^2*c^4*d^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*
d^4 - 1296*a^2*c^5*d^4, z, k)*((x*(1024*a^5*c^6*e - 16*a^2*b^6*c^3*e + 192*
```

```
a^3*b^4*c^4*e - 768*a^4*b^2*c^5*e))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2)) - (6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^
4*d - 5632*a^4*b^2*c^5*d + 16*a*b^8*c^2*d)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4
*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c
*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 4
32*a*b^9*c*d^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 +
 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^
2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^2
- 672*a*b^6*c^2*d^2*e*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*
e*z + 18432*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 960*a^2*b*c^4*d^2*e^2 + 24
0*a*b^3*c^3*d^2*e^2 - 16*b^5*c^2*d^2*e^2 + 360*a*b^2*c^4*d^4 - 256*a^3*c^4*
e^4 - 25*b^4*c^3*d^4 - 1296*a^2*c^5*d^4, z, k)*x*(4096*a^6*b*c^6 + 16*a^2*b
^9*c^2 - 256*a^3*b^7*c^3 + 1536*a^4*b^5*c^4 - 4096*a^5*b^3*c^5))/(2*(a^2*b^
6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) + (32*a*b^5*c^3*d*e + 102
4*a^3*b*c^5*d*e - 384*a^2*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2)) - (x*(288*a^3*c^6*d^2 - b^6*c^3*d^2 + 18*a*b^4*c^4*d
^2 - 64*a^3*b*c^5*e^2 - 128*a^2*b^2*c^5*d^2 + 16*a^2*b^3*c^4*e^2))/(2*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - (x*(16*a^2*c^5*e^3 -
b^3*c^4*d^2*e + 12*a*b*c^5*d^2*e))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)))*root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 +
 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 104
8576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 432*a*b^9*c
*d^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b
^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 46
08*a^2*b^7*c^2*d^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^2 - 672*a*b^
6*c^2*d^2*e*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z + 1843
2*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^
3*d^2*e^2 - 16*b^5*c^2*d^2*e^2 + 360*a*b^2*c^4*d^4 - 256*a^3*c^4*e^4 - 25*b
^4*c^3*d^4 - 1296*a^2*c^5*d^4, z, k), k, 1, 4)
```

3.37 $\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=368

$$-\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\text{ta}}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-}}$$

```
[Out] -1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(b^2*d-2*a*c*d-a*b*f+
c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*e*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*f+(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*f+(-4*a*
b*f+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d -
12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{ex}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d+6acd-abf-c(bd-2af)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)} + e\int \frac{}{(a+}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bc\right.}{}$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bc\right.}{}$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(bc\right.}{}$$

Mathematica [A]
time = 0.74, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e+fx)-2bdx(b+cx^2)+4acx(d+x(e+fx))}{a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}\,d+4af\right)-2a\left(6cd+\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}\,d-4abf-2a\sqrt{b^2-4ac}\,f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 -
4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]
*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
 (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.15, size = 579, normalized size = 1.57

method	result
risch	$\frac{\frac{c(2fa-bd)x^3}{2a\left(4ac-b^2\right)}+\frac{c x^2 e}{4ac-b^2}+\frac{\left(abf+2acd-b^2 d\right)x}{2a\left(4ac-b^2\right)}+\frac{eb}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{c(2fa-bd)\_R^2}{a\left(4ac-b^2\right)}+\frac{4ce\_R}{4ac-b^2}-\frac{abf-6acd+b^2 d}{a\left(4ac-b^2\right)}\right)}{2c\_R^3+\_R b}\right.}{4}$
default	$16c^2\left(\frac{\frac{\left(4\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2 d+8a^2cf-2a b^2 f-4abcd+b^3 d\right)x}{16ac}+\frac{e\left(4ac-b^2\right)}{8c}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(b+2c x^2+\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)^2*((1/16*(4*(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)
^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x+1/8*e*(4*a*c-b^2)/c
)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/8/a*(2*(-4*a*c+b^2)^(1/2)*a*e*ln
(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*(-4*a*c+b^2)^(1/2)*a*b*f+12*(-4*a*c+
b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3
*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))))-1/4/c/(4*a*c-b^2)^2*((-1/16*(-4*(-4*a*c+b^2)^(1/2)
*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x-
1/8*e*(4*a*c-b^2)/c)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+1/8/a*(2*(-4*a*
c+b^2)^(1/2)*a*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*(-4*a*c+b^2)^(1/2
)*a*b*f-12*(-4*a*c+b^2)^(1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*
```

```
b^2*f-4*a*b*c*d+b^3*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*c*x^2*e - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2 - 2*a*c)*
d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2) - 1/2*integrate((4*a*c*x*e - a*b*f - (b*c*d - 2*a*c*f)*x^2 - (b^2 - 6*
a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5164 vs. $2(322) = 644$.
time = 6.87, size = 5164, normalized size = 14.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*f*x^3 - 2*a*c*x^2*e + b^2*d*x - 2*a*c*d*x - a*b*f*x
- a*b*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b
```

*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b
^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f + 2*(sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a
*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 +
24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 -
 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*
d*abs(a*b^2 - 4*a^2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3
*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 +
 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*f*abs(a*b^2 - 4*a^2
*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^
2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d +
4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4

```
*a*c)*a^4*b^2*c^3)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((
a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2
*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 +
 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*
a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c
+ 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4...
```

Mupad [B]
time = 1.71, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*d
^2*f - 3*b^4*c^3*d^2*f + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + 16*a*b^2*c^4*
d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 16*a^
2*b*c^4*e^2*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) -
 root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3
*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 -
256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2 - 3072
*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f^2*z^2 +
 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 81
92*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2*c^4*e^2*z
^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4
*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32*a*b^10
*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 - 4
096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*d*e*f*z - 768
*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^2*e*z + 1536*
a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*c^4*d^2*e*z +
```

4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a^4*c^5*d^2*e*z +
32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^2*f - 192*a^3
*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d*f^3 - 960*a^2*b*
c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*e^2*f + 2016*a^2*b*c^4*
d^3*f - 496*a*b^3*c^3*d^3*f + 224*a^3*b*c^3*d*f^3 - 16*a^2*b^3*c^2*e^2*f^2
- 960*a^2*b^2*c^3*d^2*f^2 - 18*a*b^5*c*d*f^3 - 288*a^3*c^4*d^2*f^2 - 16*b^5
*c^2*d^2*e^2 - 24*a^3*b^2*c^2*f^4 + 30*b^5*c^2*d^3*f - 9*b^6*c*d^2*f^2 - 9*
a^2*b^4*c*f^4 + 360*a*b^2*c^4*d^4 - 16*a^4*c^3*f^4 - 256*a^3*c^4*e^4 - 25*b
^4*c^3*d^4 - 1296*a^2*c^5*d^4, z, k)*((32*a*b^5*c^3*d*e - 512*a^4*c^5*e*f +
1024*a^3*b*c^5*d*e - 384*a^2*b^3*c^4*d*e + 32*a^2*b^4*c^3*e*f)/(8*(a^2*b^6
- 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + root(1572864*a^8*b^2*c^5*
z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z
^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*a^2
*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 204
8*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 -
49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1
536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*
z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c
^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e
^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64
*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*
a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*
a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z
- 2048*a^5*c^4*e*f^2*z + 18432*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^
4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4
*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d*f^3 - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^
3*d^2*e^2 + 768*a^3*c^4*d*e^2*f + 2016*a^2*b*c^4*d^3*f - 496*a*b^3*c^3*d^3*
f + 224*a^3*b*c^3*d*f^3 - 16*a^2*b^3*c^2*e^2*f^2 - 960*a^2*b^2*c^3*d^2*f^2
- 18*a*b^5*c*d*f^3 - 288*a^3*c^4*d^2*f^2 - 16*b^5*c^2*d^2*e^2 - 24*a^3*b^2*
c^2*f^4 + 30*b^5*c^2*d^3*f - 9*b^6*c*d^2*f^2 - 9*a^2*b^4*c*f^4 + 360*a*b^2*
c^4*d^4 - 16*a^4*c^3*f^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*d^4 - 1296*a^2*c^5*
d^4, z, k)*((x*(1024*a^5*c^6*e - 16*a^2*b^6*c^3*e + 192*a^3*b^4*c^4*e - 768
*a^4*b^2*c^5*e))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))
- (6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*
c^5*d + 16*a^2*b^7*c^2*f - 192*a^3*b^5*c^3*f + 768*a^4*b^3*c^4*f + 16*a*b^8
*c^2*d - 1024*a^5*b*c^5*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 32768
0*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a
^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d
*f*z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*
c^4*f^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c
*d^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*
b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 6144
0*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^
2 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^1

```
1*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*
d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^
2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*
c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a^4*c
^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^
2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2...
```

3.38 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{}}}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/2*(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(-b*g+2*c
*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*f+(4*a*b*f-
12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(b*d-2*a*f+(-4*a*b*f+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2
)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = $\{1687, 1192, 1180, 211, 1261, 652, 632, 212\}$

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*
f - (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
b^2 - 4*a*c)^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^2}\,dx = \int \frac{d+fx^2}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x(e+gx^2)}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{be-2ag+(2ce-bg)x^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(c\left(bd \cdots\right.\right.}{\cdots}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{be-2ag+(2ce-bg)x^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(bd \cdots\right.}{\cdots}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{be-2ag+(2ce-bg)x^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(bd \cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.80, size = 421, normalized size = 1.09

$$\frac{1}{4}\left(\frac{-4a^2g-2bdx(b+cx^2)+4acx(d+x(e+fx))+2ab(e+x(f-gx))}{a\,(-b^2+4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}\,d+4af\right)-2a\left(6cd+\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}\,d-4abf-2a\sqrt{b^2-4ac}\,f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2(-2ce+bg)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} - \frac{2(-2ce+bg)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*g - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)) + 2*a*b*(e + x
*(f - g*x)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2
*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d +
b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2]
```

```
)/(b^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(340) = 680$.

time = 0.16, size = 714, normalized size = 1.85

method	result
risch	$\frac{\frac{c(2fa-bd)x^3}{2a(4ac-b^2)} - \frac{(bg-2ce)x^2}{2(4ac-b^2)} + \frac{(abf+2acd-b^2d)x}{2a(4ac-b^2)} - \frac{2ag-eb}{2(4ac-b^2)}}{cx^4+bx^2+a} + \frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)} \frac{\left(\frac{c(2fa-bd)\_R^2}{a(4ac-b^2)} - \frac{2(bg-2ce)\_R}{4ac-b^2} - \frac{ab}{}\right.}{2c\_R^3+\_R}\right.}{4}$
default	$16c^2 \left(\frac{\frac{\left(4\sqrt{-4ac+b^2}\,acd - \sqrt{-4ac+b^2}\,b^2d+8a^2cf-2ab^2f-4abcd+b^3d\right)x}{16ac} + \frac{-4\sqrt{-4ac+b^2}\,acg+\sqrt{-4ac+b^2}\,b}{16c^2}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}} \right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)^2*((1/16*(4*(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)
^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x+1/16/c^2*(-4*(-4*a*
c+b^2)^(1/2)*a*c*g+(-4*a*c+b^2)^(1/2)*b^2*g-4*a*b*g*c+8*a*c^2*e+b^3*g-2*b^2
*c*e))/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/8/a*(1/4*(-4*(-4*a*c+b^2)^(
1/2)*a*b*g+8*(-4*a*c+b^2)^(1/2)*a*c*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1
/2*(-4*(-4*a*c+b^2)^(1/2)*a*b*f+12*(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1
/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/4/c/(
4*a*c-b^2)^2*((-1/16*(-4*(-4*a*c+b^2)^(1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+
8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x-1/16/c^2*(4*(-4*a*c+b^2)^(1/2)*a
*c*g-(-4*a*c+b^2)^(1/2)*b^2*g-4*a*b*g*c+8*a*c^2*e+b^3*g-2*b^2*c*e))/(x^2+1/
2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+1/8/a*(-1/4*(4*(-4*a*c+b^2)^(1/2)*a*b*g-8*(
-4*a*c+b^2)^(1/2)*a*c*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*(-4*a*c
+b^2)^(1/2)*a*b*f-12*(-4*a*c+b^2)^(1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+8*a^
2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*f)*x^3 + 2*a^2*g + (a*b*g - 2*a*c*e)*x^2 - a*b*e - (a*b
*f - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (
a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(a*b*f + (b*c*d - 2*a*c*f)*x^2 + (
b^2 - 6*a*c)*d + 2*(a*b*g - 2*a*c*e)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*
a^2*c)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5580 vs. $2(341) = 682$.
time = 7.62, size = 5580, normalized size = 14.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*f*x^3 + a*b*g*x^2 - 2*a*c*x^2*e + b^2*d*x - 2*a*c*d*
x - a*b*f*x + 2*a^2*g - a*b*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) - 1/
16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
```

- 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*f*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b^

```
3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c
 - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^
5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b
*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) + 1/16*((2*
b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a
*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*
f + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*s...
```

Mupad [B]
time = 1.77, size = 2500, normalized size = 6.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*d
^2*f - 3*b^4*c^3*d^2*f + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + 16*a*b^2*c^4*
d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 4*a*b
^4*c^2*d*g^2 + 16*a^2*b*c^4*e^2*f - 24*a^2*b^2*c^3*d*g^2 + 4*a^2*b^3*c^2*f*
g^2 - 16*a*b^3*c^3*d*e*g + 96*a^2*b*c^4*d*e*g - 16*a^2*b^2*c^3*e*f*g)/(8*(a
^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^
2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8
*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 3
2768*a^6*b*c^4*e*g*z^2 - 512*a^3*b^7*c*e*g*z^2 + 576*a^2*b^8*c*d*f*z^2 - 24
576*a^5*b^3*c^3*e*g*z^2 + 6144*a^4*b^5*c^2*e*g*z^2 + 24576*a^5*b^2*c^4*d*f*
z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 - 1536*a^4*b^6*c*
g^2*z^2 + 12288*a^6*b*c^4*f^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5
*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 8192*a^6*b^2*c^3*g^2*z^2 + 6144*a^5*b^4*c^
2*g^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5
*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 614
40*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z
^2 - 32*a*b^10*d*f*z^2 + 128*a^3*b^8*g^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a
```

^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 + 384*a^2*b^6*c*d*f*g*z - 4096*a^4*b*c^4*d
*e*f*z + 64*a*b^7*c*d*e*f*z + 2048*a^4*b^2*c^3*d*f*g*z - 1536*a^3*b^4*c^2*d
*f*g*z + 3072*a^3*b^3*c^3*d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 1024*a^5*b*c^
3*f^2*g*z + 192*a^3*b^5*c*f^2*g*z - 9216*a^4*b*c^4*d^2*g*z + 32*a^2*b^6*c*e
*f^2*z - 672*a*b^6*c^2*d^2*e*z + 336*a*b^7*c*d^2*g*z - 768*a^4*b^3*c^2*f^2*
g*z + 7936*a^3*b^3*c^3*d^2*g*z - 2496*a^2*b^5*c^2*d^2*g*z + 1536*a^4*b^2*c^
3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*
b^4*c^3*d^2*e*z - 32*a*b^8*d*f*g*z - 16*a^2*b^7*f^2*g*z - 2048*a^5*c^4*e*f^
2*z + 18432*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 16*b^9*d^2*g*z - 768*a^3*b
*c^3*d*e*f*g + 32*a*b^5*c*d*e*f*g - 192*a^2*b^3*c^2*d*e*f*g + 16*a^2*b^4*c*
e*f^2*g + 48*a^2*b^4*c*d*f*g^2 - 240*a*b^4*c^2*d^2*e*g - 32*a*b^4*c^2*d*e^2
*f + 192*a^3*b^2*c^2*e*f^2*g + 192*a^3*b^2*c^2*d*f*g^2 + 960*a^2*b^2*c^3*d^
2*e*g + 192*a^2*b^2*c^3*d*e^2*f - 48*a^3*b^3*c*f^2*g^2 - 192*a^3*b*c^3*e^2*
f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d*f^3 - 960*a^2*b*c^4*d^2*e^2
 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*e^2*f + 512*a^3*b*c^3*e^3*g + 128*
a^3*b^3*c*e*g^3 + 60*a*b^5*c*d^2*g^2 + 2016*a^2*b*c^4*d^3*f - 496*a*b^3*c^3
*d^3*f + 224*a^3*b*c^3*d*f^3 - 384*a^3*b^2*c^2*e^2*g^2 - 240*a^2*b^3*c^2*d^
2*g^2 - 16*a^2*b^3*c^2*e^2*f^2 - 960*a^2*b^2*c^3*d^2*f^2 + 16*b^6*c*d^2*e*g
 - 8*a*b^6*d*f*g^2 - 18*a*b^5*c*d*f^3 - 4*a^2*b^5*f^2*g^2 - 288*a^3*c^4*d^2
*f^2 - 16*b^5*c^2*d^2*e^2 - 24*a^3*b^2*c^2*f^4 + 30*b^5*c^2*d^3*f - 9*b^6*c
*d^2*f^2 - 9*a^2*b^4*c*f^4 + 360*a*b^2*c^4*d^4 - 4*b^7*d^2*g^2 - 16*a^4*c^3
*f^4 - 16*a^3*b^4*g^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*d^4 - 1296*a^2*c^5*d^4
, z, k)*(root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6
*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^
6*z^4 - 256*a^3*b^12*z^4 + 32768*a^6*b*c^4*e*g*z^2 - 512*a^3*b^7*c*e*g*z^2
 + 576*a^2*b^8*c*d*f*z^2 - 24576*a^5*b^3*c^3*e*g*z^2 + 6144*a^4*b^5*c^2*e*g*
z^2 + 24576*a^5*b^2*c^4*d*f*z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c
^3*d*f*z^2 - 1536*a^4*b^6*c*g^2*z^2 + 12288*a^6*b*c^4*f^2*z^2 + 61440*a^5*b
*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 8192*a^6*b^2*c
^3*g^2*z^2 + 6144*a^5*b^4*c^2*g^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4
*b^5*c^2*f^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 5
12*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*
z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32*a*b^10*d*f*z^2 + 128*a^3*b^8*g^2*z^2 -
32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 + 384*a^2*b^6*
c*d*f*g*z - 4096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 2048*a^4*b^2*c^3*
d*f*g*z - 1536*a^3*b^4*c^2*d*f*g*z + 3072*a^3*b^3*c^3*d*e*f*z - 768*a^2*b^5
*c^2*d*e*f*z + 1024*a^5*b*c^3*f^2*g*z + 192*a^3*b^5*c*f^2*g*z - 9216*a^4*b*
c^4*d^2*g*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^2*e*z + 336*a*b^7*c*d^
2*g*z - 768*a^4*b^3*c^2*f^2*g*z + 7936*a^3*b^3*c^3*d^2*g*z - 2496*a^2*b^5*c
^2*d^2*g*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3
*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z - 32*a*b^8*d*f*g*z - 16*a^2*b^7
*f^2*g*z - 2048*a^5*c^4*e*f^2*z + 18432*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z
 - 16*b^9*d^2*g*z - 768*a^3*b*c^3*d*e*f*g + 32*a*b^5*c*d*e*f*g - 192*a^2*b^3
*c^2*d*e*f*g + 16*a^2*b^4*c*e*f^2*g + 48*a^2*b^4*c*d*f*g^2 - 240*a*b^4*c^2*
d^2*e*g - 32*a*b^4*c^2*d*e^2*f + 192*a^3*b^2*c^2*e*f^2*g + 192*a^3*b^2*c^2*

```
d*f*g^2 + 960*a^2*b^2*c^3*d^2*e*g + 192*a^2*b^2*c^3*d*e^2*f - 48*a^3*b^3*c*
f^2*g^2 - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d
*f^3 - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*e^2*f
+ 512*a^3*b*c^3*e^3*g + 128*a^3*b^3*c*e*g^3 + 60*a*b^5*c*d^2*g^2 + 2016*a^2
*b*c^4*d^3*f - 496*a*b^3*c^3*d^3*f + 224*a^3*b*...
```

3.39 $\int \frac{d+ex+fx^2+gx^3+hx^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=439

$$-\frac{be-2ag+(2ce-bg)x^2}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(bcd-2acf+abh-\right.}{2}$$

```
[Out] 1/2*(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(b^2*d
-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)+(-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2
)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*f
+a*b*h+(4*a*b*c*f+b^2*(-a*h+c*d)-4*a*c*(a*h+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(
-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*f+a*b*h+(-4*a*b*c*f-b
^2*(-a*h+c*d)+4*a*c*(a*h+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)
/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.31, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1687, 1692, 1180, 211, 1261, 652, 632, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2ag+x^2(2ce-bg)+be}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*
a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*f + a*b*h + (4*a*b*c
*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*
a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*f + a*b*h - (4*a*b*c*f
+ b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
```

```
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x(e+gx^2)}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+fx^2+hx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}\text{Subst}\Bigg(\int \frac{\quad}{(a+\cdots}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2\cdots}{2a\left(b^2-4ac\right)\left(a+bx^2+cx\cdots\right.}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2\cdots}{2a\left(b^2-4ac\right)\left(a+bx^2+cx\cdots\right.}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2\cdots}{2a\left(b^2-4ac\right)\left(a+bx^2+cx\cdots\right.}$$

Mathematica [A]
time = 1.16, size = 489, normalized size = 1.11

$$\frac{1}{4}\left(\frac{-4a^2(g+hx)-2bdx(b+cx^2)+4acx(d+x(e+fx))+2ab(e+x(f-x(g+hx)))}{a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2(cd-ah)-2ac\left(6cd+\sqrt{b^2-4ac}\,f+2ah\right)+b\left(c\sqrt{b^2-4ac}\,d+4acf+a\sqrt{b^2-4ac}\,h\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2(-cd+ah)+2ac\left(6cd-\sqrt{b^2-4ac}\,f+2ah\right)+b\left(c\sqrt{b^2-4ac}\,d-4acf+a\sqrt{b^2-4ac}\,h\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2(-2ce+bg)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{2(-2ce+bg)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*a^2*(g + h*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)) + 2*a*
b*(e + x*(f - x*(g + h*x))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*(b^2*(c*d - a*h) - 2*a*c*(6*c*d + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*S
qrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b
```

```
- Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[
b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 -
 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqr
t[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g)*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(-2*c*e + b*
g)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]
time = 0.09, size = 665, normalized size = 1.51

method	result
risch	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(bg-2ce)x^2}{2(4ac-b^2)}-\frac{(2a^2h-abf-2acd+b^2d)x}{2a(4ac-b^2)}-\frac{2ag-eb}{2(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(abh-2acf+bcd)\_R^2}{a(4ac-b^2)}-\right.}{}\right.}{4}$
default	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(bg-2ce)x^2}{2(4ac-b^2)}-\frac{(2a^2h-abf-2acd+b^2d)x}{2a(4ac-b^2)}-\frac{2ag-eb}{2(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(-\frac{\frac{\left(-4\sqrt{-4ac+b^2}\,abcg+8\sqrt{-4ac+b^2}\,ac^2e\right.}{4c}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(a*b*h-2*a*c*f+b*c*d)/(4*a*c-b^2)*x^3-1/2*(b*g-2*c*e)/(4*a*c-b^2)*x
^2-1/2*(2*a^2*h-a*b*f-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x-1/2*(2*a*g-b*e)/(4*a*c
-b^2))/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(1/4/c/(4*a*c-b^2)*(-1/4*(-4*(-4*a
*c+b^2)^(1/2)*a*b*c*g+8*(-4*a*c+b^2)^(1/2)*a*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c
+b^2)^(1/2))+1/2*(4*(-4*a*c+b^2)^(1/2)*a^2*c*h+(-4*a*c+b^2)^(1/2)*a*b^2*h-4
*(-4*a*c+b^2)^(1/2)*a*b*c*f+12*(-4*a*c+b^2)^(1/2)*a*c^2*d-(-4*a*c+b^2)^(1/2
)*b^2*c*d+4*a^2*b*c*h-8*a^2*c^2*f-a*b^3*h+2*a*b^2*c*f+4*a*b*c^2*d-b^3*c*d)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(-4*(-4*a*c+b^2)^(1/2)*a*b*c*
g+8*(-4*a*c+b^2)^(1/2)*a*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*(
-4*a*c+b^2)^(1/2)*a^2*c*h+(-4*a*c+b^2)^(1/2)*a*b^2*h-4*(-4*a*c+b^2)^(1/2)*a
*b*c*f+12*(-4*a*c+b^2)^(1/2)*a*c^2*d-(-4*a*c+b^2)^(1/2)*b^2*c*d-4*a^2*b*c*h
+8*a^2*c^2*f+a*b^3*h-2*a*b^2*c*f-4*a*b*c^2*d+b^3*c*d)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((b*c*d - 2*a*c*f + a*b*h)*x^3 + 2*a^2*g + (a*b*g - 2*a*c*e)*x^2 - a*b*
e - (a*b*f - 2*a^2*h - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((a*b*f - 2*a^2*h
+ (b*c*d - 2*a*c*f + a*b*h)*x^2 + (b^2 - 6*a*c)*d + 2*(a*b*g - 2*a*c*e)*x)/
(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7501 vs. 2(394) = 788.
time = 6.51, size = 7501, normalized size = 17.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*f*x^3 + a*b*h*x^3 + a*b*g*x^2 - 2*a*c*x^2*e + b^2*d*
x - 2*a*c*d*x - a*b*f*x + 2*a^2*h*x + 2*a^2*g - a*b*e)/((c*x^4 + b*x^2 + a)
```

```
*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d - 2*(
2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 -
2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*f + (2*a*b^3*c^2 - 8*a^2*b*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*
(a*b^2 - 4*a^2*c)^2*h - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
- 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 28*a^2*
b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*
a*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)*d*abs(a*b^2 - 4*a^2*c) - 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4*c^2 - 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b
^3*c^2 - 8*(b^2 - 4*a*c)*a^3*b*c^3)*f*abs(a*b^2 - 4*a^2*c) + 4*(sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2
 - 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 -
 4*a*c)*a^4*c^3)*h*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 +
 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
```

```
2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*
b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d + 4*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4
 + 32*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*f - (2*a
^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*...
```

Mupad [B]
time = 2.31, size = 2500, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*e - 2*a*g)/(2*(4*a*c - b^2)) + (x^2*(2*c*e - b*g))/(2*(4*a*c - b^2)) -
(x*(b^2*d + 2*a^2*h - 2*a*c*d - a*b*f))/(2*a*(4*a*c - b^2)) - (x^3*(b*c*d -
 2*a*c*f + a*b*h))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log((5
*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*d^2*f - 3*a^3*
b^3*c*h^3 - 4*a^4*b*c^2*h^3 - 3*b^4*c^3*d^2*f - 32*a^3*c^4*e^2*h + b^5*c^2*
d^2*h + 8*a^4*c^3*f*h^2 + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + a*b^5*c*d*h^
2 + 48*a^3*c^4*d*f*h + 16*a*b^2*c^4*d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^
3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 4*a*b^4*c^2*d*g^2 + 16*a^2*b*c^4*e^2*f - a*b
^3*c^3*d^2*h - 60*a^2*b*c^4*d^2*h - 28*a^3*b*c^3*d*h^2 + a^2*b^4*c*f*h^2 -
28*a^3*b*c^3*f^2*h - 24*a^2*b^2*c^3*d*g^2 - 9*a^2*b^3*c^2*d*h^2 + 4*a^2*b^3
*c^2*f*g^2 - 5*a^2*b^3*c^2*f^2*h + 18*a^3*b^2*c^2*f*h^2 - 8*a^3*b^2*c^2*g^2
*h - 16*a*b^3*c^3*d*e*g + 96*a^2*b*c^4*d*e*g - 4*a*b^4*c^2*d*f*h + 32*a^3*b
*c^3*e*g*h + 52*a^2*b^2*c^3*d*f*h - 16*a^2*b^2*c^3*e*f*g)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^6*z^4 -
983040*a^7*b^4*c^5*z^4 + 327680*a^6*b^6*c^4*z^4 - 61440*a^5*b^8*c^3*z^4 + 6
144*a^4*b^10*c^2*z^4 - 256*a^3*b^12*c*z^4 - 1048576*a^9*c^7*z^4 + 192*a^3*b
```

^8*c*f*h*z^2 + 57344*a^6*b*c^5*d*h*z^2 + 32768*a^6*b*c^5*e*g*z^2 + 96*a^2*b
^9*c*d*h*z^2 - 32*a*b^10*c*d*f*z^2 + 6144*a^5*b^4*c^3*f*h*z^2 - 2048*a^4*b^
6*c^2*f*h*z^2 - 49152*a^5*b^3*c^4*d*h*z^2 - 24576*a^5*b^3*c^4*e*g*z^2 + 153
60*a^4*b^5*c^3*d*h*z^2 + 6144*a^4*b^5*c^3*e*g*z^2 - 2048*a^3*b^7*c^2*d*h*z^
2 - 512*a^3*b^7*c^2*e*g*z^2 + 24576*a^5*b^2*c^5*d*f*z^2 - 3072*a^3*b^6*c^3*
d*f*z^2 + 2048*a^4*b^4*c^4*d*f*z^2 + 576*a^2*b^8*c^2*d*f*z^2 + 12288*a^7*b*
c^4*h^2*z^2 + 128*a^3*b^8*c*g^2*z^2 + 12288*a^6*b*c^5*f^2*z^2 - 16*a^2*b^9*
c*f^2*z^2 + 61440*a^5*b*c^6*d^2*z^2 + 432*a*b^9*c^2*d^2*z^2 - 16384*a^7*c^5
*f*h*z^2 - 49152*a^6*c^6*d*f*z^2 - 8192*a^6*b^3*c^3*h^2*z^2 + 1536*a^5*b^5*
c^2*h^2*z^2 - 8192*a^6*b^2*c^4*g^2*z^2 + 6144*a^5*b^4*c^3*g^2*z^2 - 1536*a^
4*b^6*c^2*g^2*z^2 - 8192*a^5*b^3*c^4*f^2*z^2 + 1536*a^4*b^5*c^3*f^2*z^2 + 2
4576*a^5*b^2*c^5*e^2*z^2 - 6144*a^4*b^4*c^4*e^2*z^2 + 512*a^3*b^6*c^3*e^2*z
^2 - 61440*a^4*b^3*c^5*d^2*z^2 + 24064*a^3*b^5*c^4*d^2*z^2 - 4608*a^2*b^7*c
^3*d^2*z^2 - 16*a^3*b^9*h^2*z^2 - 32768*a^6*c^6*e^2*z^2 - 16*b^11*c*d^2*z^2
 - 6144*a^5*b*c^4*d*g*h*z + 96*a^2*b^7*c*d*g*h*z - 4096*a^4*b*c^5*d*e*f*z +
 64*a*b^7*c^2*d*e*f*z - 32*a*b^8*c*d*f*g*z + 4608*a^4*b^3*c^3*d*g*h*z - 115
2*a^3*b^5*c^2*d*g*h*z - 9216*a^4*b^2*c^4*d*e*h*z + 2304*a^3*b^4*c^3*d*e*h*z
 + 2048*a^4*b^2*c^4*d*f*g*z - 1536*a^3*b^4*c^3*d*f*g*z + 384*a^2*b^6*c^2*d*
f*g*z - 192*a^2*b^6*c^2*d*e*h*z + 3072*a^3*b^3*c^4*d*e*f*z - 768*a^2*b^5*c^
3*d*e*f*z - 1024*a^6*b*c^3*g*h^2*z - 192*a^4*b^5*c*g*h^2*z + 1024*a^5*b*c^4
*f^2*g*z - 32*a^3*b^6*c*e*h^2*z - 16*a^2*b^7*c*f^2*g*z - 9216*a^4*b*c^5*d^2
*g*z + 336*a*b^7*c^2*d^2*g*z - 672*a*b^6*c^3*d^2*e*z + 12288*a^5*c^5*d*e*h*
z + 768*a^5*b^3*c^2*g*h^2*z - 1536*a^5*b^2*c^3*e*h^2*z - 768*a^4*b^3*c^3*f^
2*g*z + 384*a^4*b^4*c^2*e*h^2*z + 192*a^3*b^5*c^2*f^2*g*z + 7936*a^3*b^3*c^
4*d^2*g*z - 2496*a^2*b^5*c^3*d^2*g*z + 1536*a^4*b^2*c^4*e*f^2*z - 384*a^3*b
^4*c^3*e*f^2*z + 32*a^2*b^6*c^2*e*f^2*z - 15872*a^3*b^2*c^5*d^2*e*z + 4992*
a^2*b^4*c^4*d^2*e*z + 16*a^3*b^7*g*h^2*z + 2048*a^6*c^4*e*h^2*z - 2048*a^5*
c^5*e*f^2*z + 32*b^8*c^2*d^2*e*z + 18432*a^4*c^6*d^2*e*z - 16*b^9*c*d^2*g*z
 - 256*a^4*b*c^3*e*f*g*h - 768*a^3*b*c^4*d*e*f*g + 32*a*b^5*c^2*d*e*f*g - 1
92*a^3*b^3*c^2*e*f*g*h + 896*a^3*b^2*c^3*d*e*g*h - 96*a^2*b^4*c^2*d*e*g*h -
 192*a^2*b^3*c^3*d*e*f*g + 48*a^3*b^4*c*f*g^2*h + 16*a^3*b^4*c*e*g*h^2 + 24
*a^2*b^5*c*d*g^2*h + 2208*a^3*b*c^4*d^2*f*h + 800*a^4*b*c^3*d*f*h^2 - 102*a
*b^5*c^2*d^2*f*h - 30*a^2*b^5*c*d*f*h^2 - 896*a^3*b*c^4*d*e^2*h - 240*a*b^4
*c^3*d^2*e*g - 32*a*b^4*c^3*d*e^2*f + 12*a*b^6*c*d*f^2*h - 8*a*b^6*c*d*f*g^
2 + 64*a^4*b^2*c^2*f*g^2*h + 192*a^4*b^2*c^2*e*g*h^2 - 224*a^3*b^3*c^2*d*g^
2*h + 192*a^3*b^2*c^3*e^2*f*h - 864*a^3*b^2*c^3*d*f^2*h + 336*a^3*b^3*c^2*d
*f*h^2 + 192*a^3*b^2*c^3*e*f^2*g + 144*a^2*b^3*c^3*d^2*f*h + 16*a^2*b^4*c^2
*e*f^2*g - 12*a^2*b^4*c^2*d*f^2*h + 192*a^3*b^2*c^3*d*f*g^2 + 96*a^2*b^3*c^
3*d*e^2*h + 48*a^2*b^4*c^2*d*f*g^2 + 960*a^2*b^2*c^4*d^2*e*g + 192*a^2*b^2*
c^4*d*e^2*f - 48*a^4*b^3*c*g^2*h^2 + 80*a^3*b^3*c^2*f^3*h - 42*a^3*b^4*c*f^
2*h^2 - 192*a^4*b*c^3*e^2*h^2 - 4*a^2*b^5*c*f^2*g^2 - 192*a^4*b^2*c^2*d*h^3
 - 192*a^2*b^2*c^4*d^3*h + 128*a^3*b^3*c^2*e*g^3 - 192*a^3*b*c^4*e^2*f^2 +
60*a*b^5*c^2*d^2*g^2 + 198*a*b^4*c^3*d^2*f^2 + 144*a^2*b^3*c^3*d*f^3 - 960*
a^2*b*c^5*d^2*e^2 + 240*a*b^3*c^4*d^2*e^2 + 256*a^4*c^4*e^2*f*h - 192*a^4*c
^4*d*f^2*h + 16*b^6*c^2*d^2*e*g + 96*a^5*b*c^2*f*h^3 + 96*a^4*b*c^3*f^3*h +

```
 80*a^4*b^3*c*f*h^3 + 6*a^2*b^5*c*f^3*h + 768*a^3*c^5*d*e^2*f + 512*a^3*b*c
^4*e^3*g + 132*a*b^4*c^3*d^3*h - 28*a^3*b^4*c*d*h^3 + 12*a*b^6*c*d^2*h^2 +
2016*a^2*b*c^5*d^3*f - 496*a*b^3*c^4*d^3*f + 224*a^3*b*c^4*d*f^3 - 18*a*b^5
*c^2*d*f^3 - 192*a^4*b^2*c^2*f^2*h^2 - 48*a^3*b^3*c^2*f^2*g^2 - 16*a^3*b^3*
c^2*e^2*h^2 - 464*a^3*b^2*c^3*d^2*h^2 - 384*a^3*b^2*c^3*e^2*g^2 + 42*a^2*b^
4*c^2*d^2*h^2 - 240*a^2*b^3*c^3*d^2*g^2 - 16*a^...
```

3.40 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=468

$$\frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2acg-b(ce+ai)-\left(2c^2e-bcg+b^2i-2aci\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\Big(bc}{}$$

```
[Out] 1/2*x*(b^2*d-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+1/2*(2*a*c*g-b*(a*i+c*e)-(-2*a*c*i+b^2*i-b*c*g+2*c^2*e)*x^
2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(2*a*i-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*f+a*b*h+(4*a*b*c*f+b^2*(-a*h+c*d)-4*a*c*(a
*h+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*
(b*c*d-2*a*c*f+a*b*h+(-4*a*b*c*f-b^2*(-a*h+c*d)+4*a*c*(a*h+3*c*d))/(-4*a*c+
b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {1687, 1692, 1180, 211, 1677, 1674, 12, 632, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd-ah)+4abcf-4ac(ah+3cd)}{\sqrt{b^2-4ac}}+abh-2acf+bcd\right)}{2\sqrt{2}\,a\sqrt{c}\,\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-\left(x^2\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2ai-bg+2ce)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e -
b*c*g + b^2*i - 2*a*c*i)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b
*c*d - 2*a*c*f + a*b*h + (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*
d - 2*a*c*f + a*b*h - (4*a*b*c*f + b^2*(c*d - a*h) - 4*a*c*(3*c*d + a*h))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*c*e -
 b*g + 2*a*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+40x^5}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x(e+gx^2+40x^4)}{(a+bx^2+cx^4)^2}\,dx + \int \frac{d+fx^2+hx^4}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{1}{2}\mathrm{Subst}\Big($$

$$= -\frac{40ab+bce-2acg+(40b^2-2c(40a-ce)-bcg)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(b^2d-}{}$$

$$= -\frac{40ab+bce-2acg+(40b^2-2c(40a-ce)-bcg)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(b^2d-}{}$$

$$= -\frac{40ab+bce-2acg+(40b^2-2c(40a-ce)-bcg)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(b^2d-}{}$$

$$= -\frac{40ab+bce-2acg+(40b^2-2c(40a-ce)-bcg)x^2}{2c(b^2-4ac)(a+bx^2+cx^4)} + \frac{x(b^2d-}{}$$

Mathematica [A]

time = 1.27, size = 524, normalized size = 1.12

$$\frac{1}{4}\left(\frac{2(-bcdx(b+cx^2)+a^2(bi-2c(g+x(h+ix)))+a(b^2ix^2+2c^2x(d+x(e+fx))+bc(e+x(f-x(g+hx)))))}{ac(-b^2+4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(b^2(cd-ah)-2ac\left(6cd+\sqrt{b^2-4ac}\,f+2ah\right)+b\left(c\sqrt{b^2-4ac}\,d+4acf+a\sqrt{b^2-4ac}\,h\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2(-cd+ah)+2ac\left(6cd-\sqrt{b^2-4ac}\,f+2ah\right)+b\left(c\sqrt{b^2-4ac}\,d-4acf+a\sqrt{b^2-4ac}\,h\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{2(-2ce+bg-2ai)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{2(2ce-bg+2ai)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x
]
```

```
[Out] ((2*(-(b*c*d*x*(b + c*x^2)) + a^2*(b*i - 2*c*(g + x*(h + i*x))) + a*(b^2*i*
x^2 + 2*c^2*x*(d + x*(e + f*x)) + b*c*(e + x*(f - x*(g + h*x))))))/(a*c*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*h) - 2*a*c*(6*c*d
 + Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*f + a*Sq
rt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b
^2*(-(c*d) + a*h) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*f + 2*a*h) + b*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]) + (2*(-2*c*e + b*g - 2*a*i)*Log[-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (2*(2*c*e - b*g + 2*a*i)*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]

time = 0.09, size = 724, normalized size = 1.55

method	result
risch	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(2aci-b^2i+bcg-2c^2e)x^2}{2c(4ac-b^2)}-\frac{(2a^2h-abf-2acd+b^2d)x}{2a(4ac-b^2)}+\frac{abi-2acg+bce}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{(abh-}{}\right.}{}\right.$
default	$\frac{-\frac{(abh-2acf+bcd)x^3}{2a(4ac-b^2)}-\frac{(2aci-b^2i+bcg-2c^2e)x^2}{2c(4ac-b^2)}-\frac{(2a^2h-abf-2acd+b^2d)x}{2a(4ac-b^2)}+\frac{abi-2acg+bce}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{2c\left(-\frac{\left(8\sqrt{-4ac+b^2}\,a^2ci-4\sqrt{-4\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBO
SE)
```

```
[Out] (-1/2/a*(a*b*h-2*a*c*f+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a*c*i-b^2*i+b*c*g-2*c^
2*e)/c/(4*a*c-b^2)*x^2-1/2*(2*a^2*h-a*b*f-2*a*c*d+b^2*d)/a/(4*a*c-b^2)*x+1/
2/c*(a*b*i-2*a*c*g+b*c*e)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(1
/4/c/(4*a*c-b^2)*(-1/4*(8*(-4*a*c+b^2)^(1/2)*a^2*c*i-4*(-4*a*c+b^2)^(1/2)*a
*b*c*g+8*(-4*a*c+b^2)^(1/2)*a*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/
2*(4*(-4*a*c+b^2)^(1/2)*a^2*c*h+(-4*a*c+b^2)^(1/2)*a*b^2*h-4*(-4*a*c+b^2)^(
1/2)*a*b*c*f+12*(-4*a*c+b^2)^(1/2)*a*c^2*d-(-4*a*c+b^2)^(1/2)*b^2*c*d+4*a^2
*b*c*h-8*a^2*c^2*f-a*b^3*h+2*a*b^2*c*f+4*a*b*c^2*d-b^3*c*d)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(8*(-4*a*c+b^2)^(1/2)*a^2*c*i-4*(-4*a*c+b^2)
^(1/2)*a*b*c*g+8*(-4*a*c+b^2)^(1/2)*a*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1
/2))+1/2*(4*(-4*a*c+b^2)^(1/2)*a^2*c*h+(-4*a*c+b^2)^(1/2)*a*b^2*h-4*(-4*a*c
+b^2)^(1/2)*a*b*c*f+12*(-4*a*c+b^2)^(1/2)*a*c^2*d-(-4*a*c+b^2)^(1/2)*b^2*c*
d-4*a^2*b*c*h+8*a^2*c^2*f+a*b^3*h-2*a*b^2*c*f-4*a*b*c^2*d+b^3*c*d)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="m
axima")
```

```
[Out] 1/2*(2*a^2*c*g + (b*c^2*d - 2*a*c^2*f + a*b*c*h)*x^3 - a*b*c*e - I*a^2*b +
(a*b*c*g - 2*a*c^2*e - I*a*b^2 + 2*I*a^2*c)*x^2 - (a*b*c*f - 2*a^2*c*h - (b
^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4
+ (a*b^3*c - 4*a^2*b*c^2)*x^2) + 1/2*integrate((a*b*f - 2*a^2*h + (b*c*d -
2*a*c*f + a*b*h)*x^2 + (b^2 - 6*a*c)*d + 2*(a*b*g - 2*a*c*e - 2*I*a^2)*x)/(
c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19735 vs. $2(419) = 838$.
time = 7.14, size = 19735, normalized size = 42.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="g
iac")
```

```
[Out] 1/128*(-32*I*a^2*b^5*c + 256*I*a^3*b^3*c^2 - 64*I*a^2*b^4*c^2 - 512*I*a^4*b
*c^3 + 256*I*a^3*b^2*c^3 - 16*I*a^2*b^4*c + 128*I*a^3*b^2*c^2 - 64*I*a^2*b^
3*c^2 - 256*I*a^4*c^3 + 256*I*a^3*b*c^3 - 16*I*a^2*b^2*c^2 + 64*I*a^3*c^3 +
 (-4*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 80*I*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^5*c^2 - 448*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 +
 128*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 768*I*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 384*I*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^5*c + 64*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^3*c^2 - 4*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^4*c^2 - 8*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^4*c^2 + 4*sqrt(b^2 - 4*a*c)*b^5*c^2 - 192*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 64*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 96*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 64*sqrt(b^2 - 4*a*c)*a*b^3*c^3 + 8*sq
rt(b^2 - 4*a*c)*b^4*c^3 - 192*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*c^4 + 192*sqrt(b^2 - 4*a*c)*a^2*b*c^4 - 96*sqrt(b^2 - 4*
a*c)*a*b^2*c^4 + 2*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*I*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 12*I*sqrt(b^2 - 4*a*c)*s
qrt(-b^2 + 4*a*c)*b^4*c^2 - 2*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
*c^2 + 2*b^5*c^2 + 32*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
 192*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a*b^2*c^3 + 80*I*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 32*a*b^3*c^3 + 4*b^4*c^3 - 576*I*sqrt
(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a^2*c^4 - 288*I*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^4 + 96*a^2*b*c^4 - 48*a*b^2*c^4 + 2*sqrt(2)*sqrt(-b^2 +
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(-b^2 + 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*I*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*sqrt(b^2 - 4*a*c)*b^4*c^2 - 6*
I*sqrt(-b^2 + 4*a*c)*b^4*c^2 - 8*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
```

```
t(b^2 - 4*a*c)*c)*a*b*c^3 + 48*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 + 8*sqrt(b^2 - 4*a*c)*a*b^2*c^3 + 96*I*sqrt(-b^2 + 4
*a*c)*a*b^2*c^3 - 288*I*sqrt(-b^2 + 4*a*c)*a^2*c^4 - 48*sqrt(b^2 - 4*a*c)*a
*b*c^4 + 6*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*b^3*c^2 + 3*I*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - b^4*c^2 - 24*I*sqrt(b^2 - 4*a*c)*sqr
t(-b^2 + 4*a*c)*a*b*c^3 - 12*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^3 + 4*a*b^2*c^3 - 24*a*b*c^4 + 2*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c^2 + 3*I*sqrt(-b^2 + 4*a*c)*b^3*c^2 - 12*I*sqrt(-b^
2 + 4*a*c)*a*b*c^3 - 2*sqrt(b^2 - 4*a*c)*b^2*c^3 - b^2*c^3)*d - 2*(8*I*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 64*I*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c^2 + 128*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*I*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 8*sqrt(2)*sqrt(-b^2 +
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 32*sqrt(2)*sqrt(-b^2 + 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 8*I*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*sqrt(2)*sqrt(-b^2 + 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 8*sqrt(b^2 - 4*a*c)*a*b^4*
c^2 - 32*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 32*sqrt(b^2 - 4*a*c)*a^2*b^2*c^3 - 16*sqrt(b^2 - 4*a*c)*a*b^3*c^3 + 2
*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*I*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 24*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)*a*b^3*c^2 + 16*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 4*
a*b^4*c^2 + 32*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 96*I*sqr
t(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a^2*b*c^3 - 64*I*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 + 16*a^2*b^2*c^3 - 8*a*b^3*c^3 + 2*sqrt(2)*sqrt(-
b^2 + 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(-b^2
+ 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*I*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 12*sqrt(2)*sqrt(-b^2 +
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(b^2 - 4*a*c)*a*b^
3*c^2 + 12*I*sqrt(-b^2 + 4*a*c)*a*b^3*c^2 - 8*I*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(b^2 - 4*a*c)*a^2*b*c^3 - 48
*I*sqrt(-b^2 + 4*a*c)*a^2*b*c^3 - 12*sqrt(b^2 - 4*a*c)*a*b^2*c^3 + 6*I*sqrt
(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a*b^2*c^2 + 3*I*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - a*b^3*c^2 - 24*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)*a^2*c^3 - 12*I*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 4*a^2
*b*c^3 - 6*a*b^2*c^3 + 2*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b*c^2 + 3*I*sqrt(-b^2 + 4*a*c)*a*b^2...
```

Mupad [B]
time = 3.12, size = 2500, normalized size = 5.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((b*c*e - 2*a*c*g + a*b*i)/(2*c*(4*a*c - b^2)) - (x*(b^2*d + 2*a^2*h - 2*a*
c*d - a*b*f))/(2*a*(4*a*c - b^2)) + (x^2*(2*c^2*e + b^2*i - b*c*g - 2*a*c*i
))/(2*c*(4*a*c - b^2)) - (x^3*(b*c*d - 2*a*c*f + a*b*h))/(2*a*(4*a*c - b^2)
))/(a + b*x^2 + c*x^4) + symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2
*c^5*d*e^2 + 72*a^2*c^5*d^2*f - 3*a^3*b^3*c*h^3 - 4*a^4*b*c^2*h^3 - 3*b^4*c
^3*d^2*f - 32*a^3*c^4*e^2*h - 96*a^4*c^3*d*i^2 + b^5*c^2*d^2*h + 8*a^4*c^3*
f*h^2 - 32*a^5*c^2*h*i^2 + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + a*b^5*c*d*h
^2 - 192*a^3*c^4*d*e*i + 48*a^3*c^4*d*f*h - 64*a^4*c^3*e*h*i + 16*a*b^2*c^4
*d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 4*a*
b^4*c^2*d*g^2 + 16*a^2*b*c^4*e^2*f - a*b^3*c^3*d^2*h - 60*a^2*b*c^4*d^2*h -
 28*a^3*b*c^3*d*h^2 + a^2*b^4*c*f*h^2 - 28*a^3*b*c^3*f^2*h + 16*a^4*b*c^2*f
*i^2 - 24*a^2*b^2*c^3*d*g^2 - 9*a^2*b^3*c^2*d*h^2 + 4*a^2*b^3*c^2*f*g^2 + 1
6*a^3*b^2*c^2*d*i^2 - 5*a^2*b^3*c^2*f^2*h + 18*a^3*b^2*c^2*f*h^2 - 8*a^3*b^
2*c^2*g^2*h - 16*a*b^3*c^3*d*e*g + 96*a^2*b*c^4*d*e*g - 4*a*b^4*c^2*d*f*h +
 96*a^3*b*c^3*d*g*i + 32*a^3*b*c^3*e*f*i + 32*a^3*b*c^3*e*g*h + 32*a^4*b*c^
2*g*h*i + 32*a^2*b^2*c^3*d*e*i + 52*a^2*b^2*c^3*d*f*h - 16*a^2*b^2*c^3*e*f*
g - 16*a^2*b^3*c^2*d*g*i - 16*a^3*b^2*c^2*f*g*i)/(8*(a^2*b^6 - 64*a^5*c^3 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^6*z^4 - 983040*a^
7*b^4*c^5*z^4 + 327680*a^6*b^6*c^4*z^4 - 61440*a^5*b^8*c^3*z^4 + 6144*a^4*b
^10*c^2*z^4 - 256*a^3*b^12*c*z^4 - 1048576*a^9*c^7*z^4 + 32768*a^7*b*c^4*g*
i*z^2 - 512*a^4*b^7*c*g*i*z^2 + 192*a^3*b^8*c*f*h*z^2 + 57344*a^6*b*c^5*d*h
*z^2 + 32768*a^6*b*c^5*e*g*z^2 + 96*a^2*b^9*c*d*h*z^2 - 32*a*b^10*c*d*f*z^2
 - 24576*a^6*b^3*c^3*g*i*z^2 + 6144*a^5*b^5*c^2*g*i*z^2 + 49152*a^6*b^2*c^4
*e*i*z^2 - 12288*a^5*b^4*c^3*e*i*z^2 + 6144*a^5*b^4*c^3*f*h*z^2 - 2048*a^4*
b^6*c^2*f*h*z^2 + 1024*a^4*b^6*c^2*e*i*z^2 - 49152*a^5*b^3*c^4*d*h*z^2 - 24
576*a^5*b^3*c^4*e*g*z^2 + 15360*a^4*b^5*c^3*d*h*z^2 + 6144*a^4*b^5*c^3*e*g*
z^2 - 2048*a^3*b^7*c^2*d*h*z^2 - 512*a^3*b^7*c^2*e*g*z^2 + 24576*a^5*b^2*c^
5*d*f*z^2 - 3072*a^3*b^6*c^3*d*f*z^2 + 2048*a^4*b^4*c^4*d*f*z^2 + 576*a^2*b
^8*c^2*d*f*z^2 + 512*a^5*b^6*c*i^2*z^2 + 12288*a^7*b*c^4*h^2*z^2 + 128*a^3*
b^8*c*g^2*z^2 + 12288*a^6*b*c^5*f^2*z^2 - 16*a^2*b^9*c*f^2*z^2 + 61440*a^5*
b*c^6*d^2*z^2 + 432*a*b^9*c^2*d^2*z^2 - 65536*a^7*c^5*e*i*z^2 - 16384*a^7*c
^5*f*h*z^2 - 49152*a^6*c^6*d*f*z^2 + 24576*a^7*b^2*c^3*i^2*z^2 - 6144*a^6*b
^4*c^2*i^2*z^2 - 8192*a^6*b^3*c^3*h^2*z^2 + 1536*a^5*b^5*c^2*h^2*z^2 - 8192
*a^6*b^2*c^4*g^2*z^2 + 6144*a^5*b^4*c^3*g^2*z^2 - 1536*a^4*b^6*c^2*g^2*z^2
- 8192*a^5*b^3*c^4*f^2*z^2 + 1536*a^4*b^5*c^3*f^2*z^2 + 24576*a^5*b^2*c^5*e
^2*z^2 - 6144*a^4*b^4*c^4*e^2*z^2 + 512*a^3*b^6*c^3*e^2*z^2 - 61440*a^4*b^3
*c^5*d^2*z^2 + 24064*a^3*b^5*c^4*d^2*z^2 - 4608*a^2*b^7*c^3*d^2*z^2 - 32768
*a^8*c^4*i^2*z^2 - 16*a^3*b^9*h^2*z^2 - 32768*a^6*c^6*e^2*z^2 - 16*b^11*c*d
^2*z^2 - 192*a^3*b^6*c*d*h*i*z - 6144*a^5*b*c^4*d*g*h*z - 4096*a^5*b*c^4*d*
f*i*z + 96*a^2*b^7*c*d*g*h*z + 64*a^2*b^7*c*d*f*i*z - 4096*a^4*b*c^5*d*e*f*
z + 64*a*b^7*c^2*d*e*f*z - 32*a*b^8*c*d*f*g*z - 9216*a^5*b^2*c^3*d*h*i*z +
2304*a^4*b^4*c^2*d*h*i*z + 4608*a^4*b^3*c^3*d*g*h*z + 3072*a^4*b^3*c^3*d*f*
i*z - 1152*a^3*b^5*c^2*d*g*h*z - 768*a^3*b^5*c^2*d*f*i*z - 9216*a^4*b^2*c^4
*d*e*h*z + 2304*a^3*b^4*c^3*d*e*h*z + 2048*a^4*b^2*c^4*d*f*g*z - 1536*a^3*b
^4*c^3*d*f*g*z + 384*a^2*b^6*c^2*d*f*g*z - 192*a^2*b^6*c^2*d*e*h*z + 3072*a
```

```
^3*b^3*c^4*d*e*f*z - 768*a^2*b^5*c^3*d*e*f*z + 384*a^5*b^4*c*h^2*i*z - 1024
*a^6*b*c^3*g*h^2*z - 192*a^4*b^5*c*g*h^2*z + 32*a^3*b^6*c*f^2*i*z + 1024*a^
5*b*c^4*f^2*g*z - 32*a^3*b^6*c*e*h^2*z - 16*a^2*b^7*c*f^2*g*z - 9216*a^4*b*
c^5*d^2*g*z + 336*a*b^7*c^2*d^2*g*z - 672*a*b^6*c^3*d^2*e*z + 12288*a^6*c^4
*d*h*i*z + 12288*a^5*c^5*d*e*h*z + 32*a*b^8*c*d^2*i*z - 1536*a^6*b^2*c^2*h^
2*i*z + 1536*a^5*b^2*c^3*f^2*i*z + 768*a^5*b^3*c^2*g*h^2*z - 384*a^4*b^4*c^
2*f^2*i*z - 15872*a^4*b^2*c^4*d^2*i*z + 4992*a^3*b^4*c^3*d^2*i*z - 1536*a^5
*b^2*c^3*e*h^2*z - 768*a^4*b^3*c^3*f^2*g*z - 672*a^2*b^6*c^2*d^2*i*z + 384*
a^4*b^4*c^2*e*h^2*z + 192*a^3*b^5*c^2*f^2*g*z + 7936*a^3*b^3*c^4*d^2*g*z -
2496*a^2*b^5*c^3*d^2*g*z + 1536*a^4*b^2*c^4*e*f^2*z - 384*a^3*b^4*c^3*e*f^2
*z + 32*a^2*b^6*c^2*e*f^2*z - 15872*a^3*b^2*c^5*d^2*e*z + 4992*a^2*b^4*c^4*
d^2*e*z + 2048*a^7*c^3*h^2*i*z - 32*a^4*b^6*h^2*i*z - 2048*a^6*c^4*f^2*i*z
+ 16*a^3*b^7*g*h^2*z + 18432*a^5*c^5*d^2*i*z + 2048*a^6*c^4*e*h^2*z - 2048*
a^5*c^5*e*f^2*z + 32*b^8*c^2*d^2*e*z + 18432*a^4*c^6*d^2*e*z - 16*b^9*c*d^2
*g*z - 256*a^5*b*c^2*f*g*h*i - 192*a^4*b^3*c*f*g*h*i - 96*a^3*b^4*c*d*g*h*i
 - 1792*a^4*b*c^3*d*e*h*i - 768*a^4*b*c^3*d*f*g*i - 256*a^4*b*c^3*e*f*g*h +
 32*a^2*b^5*c*d*f*g*i - 768*a^3*b*c^4*d*e*f*g + 32*a*b^5*c^2*d*e*f*g + 896*
a^4*b^2*c^2*d*g*h*i + 384*a^4*b^2*c^2*e*f*h*i - 192*a^3*b^3*c^2*e*f*g*h - 1
92*a^3*b^3*c^2*d*f*g*i + 192*a^3*b^3*c^2*d*e*h*i + 896*a^3*b^2*c^3*d*e*g*h
+ 384*a^3*b^2*c^3*d*e*f*i - 96*a^2*b^4*c^2*d*e*g*h - 64*a^2*b^4*c^2*d*e*f*i
 - 192*a^2*b^3*c^3*d*e*f*g + 192*a^5*b^2*c*g*h^2*i + 192*a^5*b^2*c*f*h*i^2
- 384*a^5*b*c^2*e*h^2*i - 32*a^4*b^3*c*e*h^2*i ...
```

3.41 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=770

$$\frac{mx}{c^2}-\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l+bc(bj+3al)\right)x^2}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x(abc(cf+ak)-$$

```
[Out] m*x/c^2+1/2*(-b*c*(a*j+c*e)+a*b^2*l+2*a*c*(-a*l+c*g)-(2*c^3*e-c^2*(2*a*j+b*
g)-b^3*l+b*c*(3*a*l+b*j))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(a*b*
c*(a*k+c*f)-b^2*(a^2*m+c^2*d)+2*a*c*(a^2*m-a*c*h+c^2*d)+(a*b^2*c*k+2*a*c^2*
(-a*k+c*f)-a*b^3*m-b*c*(-3*a^2*m+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x
^4+b*x^2+a)+1/2*(4*c^3*e-c^2*(-4*a*j+2*b*g)+b^3*l-6*a*b*c*l)*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*l*ln(c*x^4+b*x^2+a)/c^
2+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(a*b^2*c*k-2*a
*c^2*(3*a*k+c*f)-3*a*b^3*m+b*c*(13*a^2*m+a*c*h+c^2*d)+(-a*b^3*c*k+4*a*b*c^2
*(2*a*k+c*f)+3*a*b^4*m+b^2*c*(-19*a^2*m-a*c*h+c^2*d)-4*a*c^2*(-5*a^2*m+a*c*
h+3*c^2*d))/(-4*a*c+b^2)^(1/2))/a/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(a*b^2*c*k-2*a*c^2*(3*a*k+c*f)-3*a*b^3*m+b*c*(13*a^2*m+a*c*h+c^2*d)+(a*b^3
*c*k-4*a*b*c^2*(2*a*k+c*f)-3*a*b^4*m-b^2*c*(-19*a^2*m-a*c*h+c^2*d)+4*a*c^2*
(-5*a^2*m+a*c*h+3*c^2*d))/(-4*a*c+b^2)^(1/2))/a/c^(5/2)/(-4*a*c+b^2)*2^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 5.65, antiderivative size = 770, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1687, 1692, 1690, 1180, 211, 1677, 1674, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(-19a^2m-ach+c^2d)+4ac^2(-5a^2m+ach+3c^2d)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc(13a^2m+ach+c^2d)-3ab^3m+ab^2ck-2ac^2(3ak+cf)\right)}{2\sqrt{2}\,ac^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-b^2c(-19a^2m-ach+c^2d)+4ac^2(-5a^2m+ach+3c^2d)-3ab^4m+ab^3ck-4abc^2(2ak+cf)}{\sqrt{b^2-4ac}}+bc(13a^2m+ach+c^2d)-3ab^3m+ab^2ck-2ac^2(3ak+cf)\right)}{2\sqrt{2}\,ac^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x\left(-\left(b^2(a^2m+c^2d)\right)+x^2\left(-bc(-3a^2m+ach+c^2d)-ab^3m+ab^2ck+2ac^2(cf-ak)\right)+2ac(a^2m-ach+c^2d)+abc(ak+cf)\right)}{2ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4aj)-6abcl+b^3l+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(-c^2(2aj+bg)+bc(3al+bj)+b^3(-l)+2c^3e\right)-ab^2l+bc(aj+ce)-2ac(cg-al)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{l\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{mx}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4)^2,x]
```

```
[Out] (m*x)/c^2 - (b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2
*(b*g + 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*
d - a*c*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*
d + a*c*h - 3*a^2*m))*x^2))/(2*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(a*b^2*c*k - 2*a*c^2*(c*f + 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^
2*m) - (a*b^3*c*k - 4*a*b*c^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*
```

```
c*h - 19*a^2*m) + 4*a*c^2*(3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2
)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*k - 2*a*c^2*(c*f +
 3*a*k) - 3*a*b^3*m + b*c*(c^2*d + a*c*h + 13*a^2*m) + (a*b^3*c*k - 4*a*b*c
^2*(c*f + 2*a*k) - 3*a*b^4*m - b^2*c*(c^2*d - a*c*h - 19*a^2*m) + 4*a*c^2*(
3*c^2*d + a*c*h - 5*a^2*m))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*j) + b^3*l - 6*a*b*c*l)*Ar
cTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (l*Lo
g[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1690

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^2}\,dx = \int \frac{x(e+gx^2+jx^4+lx^6)}{(a+bx^2+cx^4)^2}\,dx + \int \frac{d+fx^2+h\cdots}{(a+b\cdots}$$

$$= -\frac{x(abc(cf+ak)-b^2(c^2d+a^2m)+2ac(c^2d-\cdots}{\cdots}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+(2c^3e-\cdots}{2c^2(b^2-4ac)(a+\cdots}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+(2c^3\cdots}{2c^2(b^2-4ac)\cdots}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+(2c^3\cdots}{2c^2(b^2-4ac)\cdots}$$

$$= \frac{mx}{c^2} - \frac{bc(ce+aj)-ab^2l-2ac(cg-al)+(2c^3\cdots}{2c^2(b^2-4ac)\cdots}$$

Mathematica [**A**]
time = 3.49, size = 935, normalized size = 1.21

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*m*x + (2*Sqrt[c]*(2*a^3*c*(l + m*x) - b*c^2*d*x*(b + c*x^2) + a*
(b^2*c*x^2*(j + k*x) - b^3*x^2*(l + m*x) + 2*c^3*x*(d + x*(e + f*x)) + b*c^
2*(e + x*(f - x*(g + h*x)))) - a^2*(b^2*(l + m*x) + 2*c^2*(g + x*(h + x*(j
+ k*x))) - b*c*(j + x*(k + 3*x*(l + m*x))))))/(a*(-b^2 + 4*a*c)*(a + b*x^2
+ c*x^4)) - (Sqrt[2]*(-3*a*b^4*m + 2*a*c^2*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f
 + 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*k - 10*a^2*m) + a*b^3*(c*k + 3*Sqrt[b^2
- 4*a*c]*m) - b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d + 4*a*f) + a*c*(Sqrt[b^2 - 4*a*
c]*h + 8*a*k) + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(-(c^2*d) + a*c*h + a*(
-(Sqrt[b^2 - 4*a*c]*k) + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
```

```
t[2]*(3*a*b^4*m + 2*a*c^2*(-6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f - 2*a*c*h + 3*a
*Sqrt[b^2 - 4*a*c]*k + 10*a^2*m) + a*b^3*(-(c*k) + 3*Sqrt[b^2 - 4*a*c]*m) -
 b*c*(c^2*(Sqrt[b^2 - 4*a*c]*d - 4*a*f) + a*c*(Sqrt[b^2 - 4*a*c]*h - 8*a*k)
 + 13*a^2*Sqrt[b^2 - 4*a*c]*m) + b^2*c*(c^2*d - a*c*h - a*(Sqrt[b^2 - 4*a*c
]*k + 19*a*m)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a
*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(-4*c^3*e + 2*
c^2*(b*g - 2*a*j) + b^2*(-b + Sqrt[b^2 - 4*a*c])*l + a*c*(6*b*l - 4*Sqrt[b^
2 - 4*a*c]*l))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) +
 (Sqrt[c]*(4*c^3*e + c^2*(-2*b*g + 4*a*j) + b^2*(b + Sqrt[b^2 - 4*a*c])*l -
 2*a*c*(3*b + 2*Sqrt[b^2 - 4*a*c])*l)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])
/(b^2 - 4*a*c)^(3/2))/(4*c^(5/2))
```

Maple [A]
time = 1.32, size = 1317, normalized size = 1.71

method	result
risch	$\frac{mx}{c^2} + \frac{\frac{\left(3a^2bcm-2a^2c^2k-a\,b^3m+a\,b^2ck-ab\,c^2h+2a\,c^3f-b\,c^3d\right)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(3abcl-2a\,c^2j-b^3l+b^2cj-b\,c^2g+2c^3e\right)x^2}{8ac-2b^2}+\frac{\left(2a^3cm-a^2b^2m+a^2bck-2a^2 \ldots\right.}{2a\left(4a \ldots\right.}}{c^2(c\,x^4+b\,x^2+a)}$
default	$\frac{mx}{c^2} - \frac{-\frac{\left(3a^2bcm-2a^2c^2k-a\,b^3m+a\,b^2ck-ab\,c^2h+2a\,c^3f-b\,c^3d\right)x^3}{2a\left(4ac-b^2\right)}-\frac{\left(3abcl-2a\,c^2j-b^3l+b^2cj-b\,c^2g+2c^3e\right)x^2}{2\left(4ac-b^2\right)}-\frac{\left(2a^3cm-a^2b^2m+a^2bck-2 \ldots\right.}{2a\left(\ldots\right.}}{c\,x^4+b\,x^2+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] m*x/c^2-1/c^2*((-1/2/a*(3*a^2*b*c*m-2*a^2*c^2*k-a*b^3*m+a*b^2*c*k-a*b*c^2*h
+2*a*c^3*f-b*c^3*d)/(4*a*c-b^2)*x^3-1/2*(3*a*b*c*l-2*a*c^2*j-b^3*l+b^2*c*j-
b*c^2*g+2*c^3*e)/(4*a*c-b^2)*x^2-1/2*(2*a^3*c*m-a^2*b^2*m+a^2*b*c*k-2*a^2*c
^2*h+a*b*c^2*f+2*a*c^3*d-b^2*c^2*d)/a/(4*a*c-b^2)*x-1/2*(2*a^2*c*l-a*b^2*l+
a*b*c*j-2*a*c^2*g+b*c^2*e)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(
1/4/c/(4*a*c-b^2)*(-1/4*(12*(-4*a*c+b^2)^(1/2)*a^2*b*c^2*l-8*(-4*a*c+b^2)^(
1/2)*a^2*c^3*j-2*(-4*a*c+b^2)^(1/2)*a*b^3*c*l+4*(-4*a*c+b^2)^(1/2)*a*b*c^3*
g-8*(-4*a*c+b^2)^(1/2)*a*c^4*e+32*a^3*c^3*l-16*a^2*b^2*c^2*l+2*a*b^4*c*l)/c
*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(20*(-4*a*c+b^2)^(1/2)*a^3*c^2*m-19*
```

```
(-4*a*c+b^2)^(1/2)*a^2*b^2*c*m+8*(-4*a*c+b^2)^(1/2)*a^2*b*c^2*k-4*(-4*a*c+b
^2)^(1/2)*a^2*c^3*h+3*(-4*a*c+b^2)^(1/2)*a*b^4*m-(-4*a*c+b^2)^(1/2)*a*b^3*c
*k-(-4*a*c+b^2)^(1/2)*a*b^2*c^2*h+4*(-4*a*c+b^2)^(1/2)*a*b*c^3*f-12*(-4*a*c
+b^2)^(1/2)*a*c^4*d+(-4*a*c+b^2)^(1/2)*b^2*c^3*d-52*a^3*b*c^2*m+24*a^3*c^3*
k+25*a^2*b^3*c*m-10*a^2*b^2*c^2*k-4*a^2*b*c^3*h+8*a^2*c^4*f-3*a*b^5*m+a*b^4
*c*k+a*b^3*c^2*h-2*a*b^2*c^3*f-4*a*b*c^4*d+b^3*c^3*d)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
)+1/4/c/(4*a*c-b^2)*(1/4*(12*(-4*a*c+b^2)^(1/2)*a^2*b*c^2*l-8*(-4*a*c+b^2)^
(1/2)*a^2*c^3*j-2*(-4*a*c+b^2)^(1/2)*a*b^3*c*l+4*(-4*a*c+b^2)^(1/2)*a*b*c^3
*g-8*(-4*a*c+b^2)^(1/2)*a*c^4*e-32*a^3*c^3*l+16*a^2*b^2*c^2*l-2*a*b^4*c*l)/
c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(20*(-4*a*c+b^2)^(1/2)*a^3*c^2*m-19*
(-4*a*c+b^2)^(1/2)*a^2*b^2*c*m+8*(-4*a*c+b^2)^(1/2)*a^2*b*c^2*k-4*(-4*a*c+b
^2)^(1/2)*a^2*c^3*h+3*(-4*a*c+b^2)^(1/2)*a*b^4*m-(-4*a*c+b^2)^(1/2)*a*b^3*c
*k-(-4*a*c+b^2)^(1/2)*a*b^2*c^2*h+4*(-4*a*c+b^2)^(1/2)*a*b*c^3*f-12*(-4*a*c
+b^2)^(1/2)*a*c^4*d+(-4*a*c+b^2)^(1/2)*b^2*c^3*d+52*a^3*b*c^2*m-24*a^3*c^3*
k-25*a^2*b^3*c*m+10*a^2*b^2*c^2*k+4*a^2*b*c^3*h-8*a^2*c^4*f+3*a*b^5*m-a*b^4
*c*k-a*b^3*c^2*h+2*a*b^2*c^3*f+4*a*b*c^4*d-b^3*c^3*d)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*c^2*g - a^2*b*c*j - a*b*c^2*e + (b*c^3*d - 2*a*c^3*f + a*b*c^2*h
 - (a*b^2*c - 2*a^2*c^2)*k + (a*b^3 - 3*a^2*b*c)*m)*x^3 + (a*b*c^2*g - 2*a*
c^3*e - (a*b^2*c - 2*a^2*c^2)*j + (a*b^3 - 3*a^2*b*c)*l)*x^2 + (a^2*b^2 - 2
*a^3*c)*l - (a*b*c^2*f - 2*a^2*c^2*h + a^2*b*c*k - (b^2*c^2 - 2*a*c^3)*d -
(a^2*b^2 - 2*a^3*c)*m)*x)/(a^2*b^2*c^2 - 4*a^3*c^3 + (a*b^2*c^3 - 4*a^2*c^4
)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) + m*x/c^2 - 1/2*integrate(-(a*b*c^2*
f - 2*a^2*c^2*h + a^2*b*c*k + 2*(a*b^2*c - 4*a^2*c^2)*l*x^3 + (b*c^3*d - 2*
a*c^3*f + a*b*c^2*h + (a*b^2*c - 6*a^2*c^2)*k - (3*a*b^3 - 13*a^2*b*c)*m)*x
^2 + (b^2*c^2 - 6*a*c^3)*d - (3*a^2*b^2 - 10*a^3*c)*m + 2*(a*b*c^2*g - 2*a^
2*c^2*j + a^2*b*c*l - 2*a*c^3*e)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c^2 - 4*
a^2*c^3)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 20187 vs. 2(719) = 1438.
time = 8.85, size = 20187, normalized size = 26.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^2,x, algorithm="giac")
```

```
[Out] m*x/c^2 + 1/4*l*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/16*((a^2*b^4*c^5 - 8*a^
3*b^2*c^6 + 16*a^4*c^7)^2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d - 2*(a^2*b^4*c^5 - 8*a^3*
b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^2*c^5 - 8*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*a*c^5)*f + (a^2*b^4*c^5 - 8*a^3*
b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*a*b*c^4)*h + (a^2*b^4*c^
5 - 8*a^3*b^2*c^6 + 16*a^4*c^7)^2*(2*a*b^4*c^3 - 20*a^2*b^2*c^4 + 48*a^3*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 24*sqr
```

```
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 12*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*a*b^2*
c^3 + 12*(b^2 - 4*a*c)*a^2*c^4)*k - (a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c
^7)^2*(6*a*b^5*c^2 - 50*a^2*b^3*c^3 + 104*a^3*b*c^4 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 6*(b^2 - 4*a*c)*a*b^3*c^2 + 26*(b^2 - 4*a*
c)*a^2*b*c^3)*m + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 -
18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 + 2*a^2*b^8*c^9 + 120*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 + 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c^10 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 -
 36*a^3*b^6*c^10 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11
 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 - 14*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^11 + 240*a^4*b^4*c^11 + 384*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^12 + 192*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*b*c^12 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
2*c^12 - 704*a^5*b^2*c^12 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
c^13 + 768*a^6*c^13 - 2*(b^2 - 4*a*c)*a^2*b^6*c^9 + 28*(b^2 - 4*a*c)*a^3*b^
4*c^10 - 128*(b^2 - 4*a*c)*a^4*b^2*c^11 + 192*(b^2 - 4*a*c)*a^5*c^12)*d*abs
(-a^2*b^4*c^5 + 8*a^3*b^2*c^6 - 16*a^4*c^7) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b^7*c^8 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^5*c^9 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 2*a^3*b^7
*c^9 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 + 16*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 + sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^10 - 24*a^4*b^5*c^10 - 64*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^6*b*c^11 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^11 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 + 96*a^5*b
^3*c^11 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12 - 128*a^6*b
*c^12 - 2*(b^2 - 4*a*c)*a^3*b^5*c^9 + 16*(b^2 - 4*a*c)*a^4*b^3*c^10 - 32*(b
^2 - 4*a*c)*a^5*b*c^11)*f*abs(-a^2*b^4*c^5 + 8*a^3*b^2*c^6 - 16*a^4*c^7) -
4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^8 - 12*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^5*c^9 + 2*a^4*b^6*c^9 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^2*c^10 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10
+ sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 - 24*a^5*b^4*c^10 -
64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^11 - 32*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^6*b*c^11 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b^2*c^11 + 96*a^6*b^2*c^11 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
```

```
c)*a^6*c^12 - 128*a^7*c^12 - 2*(b^2 - 4*a*c)*a^4*b^4*c^9 + 16*(b^2 - 4*a*c)
*a^5*b^2*c^10 - 32*(b^2 - 4*a*c)*a^6*c^11)*h*ab...
```

Mupad [B]
time = 13.91, size = 2500, normalized size = 3.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(root(1572864*a^8*b^2*c^10*z^4 - 983040*a^7*b^4*c^9*z^4 + 327680*
a^6*b^6*c^8*z^4 - 61440*a^5*b^8*c^7*z^4 + 6144*a^4*b^10*c^6*z^4 - 256*a^3*b
^12*c^5*z^4 - 1048576*a^9*c^11*z^4 - 1572864*a^8*b^2*c^8*l*z^3 + 983040*a^7
*b^4*c^7*l*z^3 - 327680*a^6*b^6*c^6*l*z^3 + 61440*a^5*b^8*c^5*l*z^3 - 6144*
a^4*b^10*c^4*l*z^3 + 256*a^3*b^12*c^3*l*z^3 + 1048576*a^9*c^9*l*z^3 + 96*a^
3*b^12*c*k*m*z^2 + 98304*a^8*b*c^7*j*l*z^2 + 24576*a^8*b*c^7*h*m*z^2 + 1556
48*a^7*b*c^8*d*m*z^2 + 98304*a^7*b*c^8*e*l*z^2 + 57344*a^7*b*c^8*f*k*z^2 +
32768*a^7*b*c^8*g*j*z^2 + 57344*a^6*b*c^9*d*h*z^2 + 32768*a^6*b*c^9*e*g*z^2
 - 32*a*b^10*c^5*d*f*z^2 - 491520*a^8*b^2*c^6*k*m*z^2 + 358400*a^7*b^4*c^5*
k*m*z^2 - 129024*a^6*b^6*c^4*k*m*z^2 + 24768*a^5*b^8*c^3*k*m*z^2 - 2432*a^4
*b^10*c^2*k*m*z^2 - 90112*a^7*b^3*c^6*j*l*z^2 + 30720*a^6*b^5*c^5*j*l*z^2 -
 4608*a^5*b^7*c^4*j*l*z^2 + 256*a^4*b^9*c^3*j*l*z^2 - 21504*a^6*b^5*c^5*h*m
*z^2 + 9216*a^5*b^7*c^4*h*m*z^2 + 8192*a^7*b^3*c^6*h*m*z^2 - 1568*a^4*b^9*c
^3*h*m*z^2 + 96*a^3*b^11*c^2*h*m*z^2 - 172032*a^7*b^2*c^7*f*m*z^2 + 116736*
a^6*b^4*c^6*f*m*z^2 - 49152*a^7*b^2*c^7*g*l*z^2 + 45056*a^6*b^4*c^6*g*l*z^2
 - 35840*a^5*b^6*c^5*f*m*z^2 + 24576*a^7*b^2*c^7*h*k*z^2 - 15360*a^5*b^6*c^
5*g*l*z^2 + 5184*a^4*b^8*c^4*f*m*z^2 - 3072*a^5*b^6*c^5*h*k*z^2 + 2304*a^4*
b^8*c^4*g*l*z^2 + 2048*a^6*b^4*c^6*h*k*z^2 + 576*a^4*b^8*c^4*h*k*z^2 - 288*
a^3*b^10*c^3*f*m*z^2 - 128*a^3*b^10*c^3*g*l*z^2 - 32*a^3*b^10*c^3*h*k*z^2 -
 147456*a^6*b^3*c^7*d*m*z^2 - 90112*a^6*b^3*c^7*e*l*z^2 + 52224*a^5*b^5*c^6
*d*m*z^2 - 49152*a^6*b^3*c^7*f*k*z^2 + 30720*a^5*b^5*c^6*e*l*z^2 - 24576*a^
6*b^3*c^7*g*j*z^2 + 15360*a^5*b^5*c^6*f*k*z^2 - 8192*a^4*b^7*c^5*d*m*z^2 +
6144*a^5*b^5*c^6*g*j*z^2 - 4608*a^4*b^7*c^5*e*l*z^2 - 2048*a^4*b^7*c^5*f*k*
z^2 - 512*a^4*b^7*c^5*g*j*z^2 + 480*a^3*b^9*c^4*d*m*z^2 + 256*a^3*b^9*c^4*e
*l*z^2 + 96*a^3*b^9*c^4*f*k*z^2 + 131072*a^6*b^2*c^8*d*k*z^2 + 49152*a^6*b^
2*c^8*e*j*z^2 - 43008*a^5*b^4*c^7*d*k*z^2 - 12288*a^5*b^4*c^7*e*j*z^2 + 614
4*a^4*b^6*c^6*d*k*z^2 + 1024*a^4*b^6*c^6*e*j*z^2 - 320*a^3*b^8*c^5*d*k*z^2
+ 6144*a^5*b^4*c^7*f*h*z^2 - 2048*a^4*b^6*c^6*f*h*z^2 + 192*a^3*b^8*c^5*f*h
*z^2 - 49152*a^5*b^3*c^8*d*h*z^2 - 24576*a^5*b^3*c^8*e*g*z^2 + 15360*a^4*b^
5*c^7*d*h*z^2 + 6144*a^4*b^5*c^7*e*g*z^2 - 2048*a^3*b^7*c^6*d*h*z^2 - 512*a
^3*b^7*c^6*e*g*z^2 + 96*a^2*b^9*c^5*d*h*z^2 + 24576*a^5*b^2*c^9*d*f*z^2 - 3
072*a^3*b^6*c^7*d*f*z^2 + 2048*a^4*b^4*c^8*d*f*z^2 + 576*a^2*b^8*c^6*d*f*z^
2 - 430080*a^9*b*c^6*m^2*z^2 + 3408*a^4*b^11*c*m^2*z^2 - 64*a^3*b^12*c*l^2*
```

```
z^2 + 61440*a^8*b*c^7*k^2*z^2 + 12288*a^7*b*c^8*h^2*z^2 + 12288*a^6*b*c^9*f
^2*z^2 + 61440*a^5*b*c^10*d^2*z^2 + 432*a*b^9*c^6*d^2*z^2 + 245760*a^9*c^7*
k*m*z^2 + 81920*a^8*c^8*f*m*z^2 - 49152*a^8*c^8*h*k*z^2 - 147456*a^7*c^9*d*
k*z^2 - 65536*a^7*c^9*e*j*z^2 - 16384*a^7*c^9*f*h*z^2 - 49152*a^6*c^10*d*f*
z^2 + 716800*a^8*b^3*c^5*m^2*z^2 - 483840*a^7*b^5*c^4*m^2*z^2 + 170496*a^6*
b^7*c^3*m^2*z^2 - 33232*a^5*b^9*c^2*m^2*z^2 + 516096*a^8*b^2*c^6*l^2*z^2 -
288768*a^7*b^4*c^5*l^2*z^2 + 88576*a^6*b^6*c^4*l^2*z^2 - 15744*a^5*b^8*c^3*
l^2*z^2 + 1536*a^4*b^10*c^2*l^2*z^2 - 61440*a^7*b^3*c^6*k^2*z^2 + 24064*a^6
*b^5*c^5*k^2*z^2 - 4608*a^5*b^7*c^4*k^2*z^2 + 432*a^4*b^9*c^3*k^2*z^2 - 16*
a^3*b^11*c^2*k^2*z^2 + 24576*a^7*b^2*c^7*j^2*z^2 - 6144*a^6*b^4*c^6*j^2*z^2
 + 512*a^5*b^6*c^5*j^2*z^2 - 8192*a^6*b^3*c^7*h^2*z^2 + 1536*a^5*b^5*c^6*h^
2*z^2 - 16*a^3*b^9*c^4*h^2*z^2 - 8192*a^6*b^2*c^8*g^2*z^2 + 6144*a^5*b^4*c^
7*g^2*z^2 - 1536*a^4*b^6*c^6*g^2*z^2 + 128*a^3*b^8*c^5*g^2*z^2 - 8192*a^5*b
^3*c^8*f^2*z^2 + 1536*a^4*b^5*c^7*f^2*z^2 - 16*a^2*b^9*c^5*f^2*z^2 + 24576*
a^5*b^2*c^9*e^2*z^2 - 6144*a^4*b^4*c^8*e^2*z^2 + 512*a^3*b^6*c^7*e^2*z^2 -
61440*a^4*b^3*c^9*d^2*z^2 + 24064*a^3*b^5*c^8*d^2*z^2 - 4608*a^2*b^7*c^7*d^
2*z^2 - 393216*a^9*c^7*l^2*z^2 - 144*a^3*b^13*m^2*z^2 - 32768*a^8*c^8*j^2*z
^2 - 32768*a^6*c^10*e^2*z^2 - 16*b^11*c^5*d^2*z^2 + 18432*a^8*b*c^5*h*l*m*z
 - 96*a^3*b^10*c*g*k*m*z + 90112*a^7*b*c^6*e*k*m*z + 36864*a^7*b*c^6*f*j*m*
z - 16384*a^7*b*c^6*g*j*l*z + 14336*a^7*b*c^6*d*l*m*z - 10240*a^7*b*c^6*f*k
*l*z + 4096*a^7*b*c^6*h*j*k*z + 10240*a^7*b*c^6*g*h*m*z - 47104*a^6*b*c^7*d
*h*l*z + 36864*a^6*b*c^7*e*f*m*z + 30720*a^6*b*c^7*d*g*m*z - 16384*a^6*b*c^
7*e*g*l*z + 6144*a^6*b*c^7*f*g*k*z + 4096*a^6*b*c^7*e*h*k*z + 32*a*b^10*c^3
*d*f*l*z - 4096*a^5*b*c^8*d*f*j*z - 6144*a^5*b*c^8*d*g*h*z - 32*a*b^8*c^5*d
*f*g*z - 4096*a^4*b*c^9*d*e*f*z + 64*a*b^7*c^6*d*e*f*z + 110592*a^8*b^2*c^4
*k*l*m*z - 36864*a^7*b^4*c^3*k*l*m*z + 5376*a^6*b^6*c^2*k*l*m*z - 79872*a^7
*b^3*c^4*j*k*m*z + 26112*a^6*b^5*c^3*j*k*m*z - 3712*a^5*b^7*c^2*j*k*m*z - 1
3824*a^7*b^3*c^4*h*l*m*z + 3456*a^6*b^5*c^3*h*l*m*z - 288*a^5*b^7*c^2*h*l*m
*z - 45056*a^7*b^2*c^5*g*k*m*z + 39936*a^6*b^4*c^4*g*k*m*z + 30720*a^7*b^2*
c^5*f*l*m*z - 18432*a^7*b^2*c^5*h*k*l*z - 13056*a^5*b^6*c^3*g*k*m*z - 7680*
a^6*b^4*c^4*f*l*m*z + 5376*a^6*b^4*c^4*h*j*m*z + 4608*a^6*b^4*c^4*h*k*l*z +
 3072*a^7*b^2*c^5*h*j*m*z - 1984*a^5*b^6*c^3*h*j*m*z + 1856*a^4*b^8*c^2*g*k
*m*z + 640*a^5*b^6*c^3*f*l*m*z - 384*a^5*b^6*c^3*h*k*l*z + 192*a^4*b^8*c^2*
h*j*m*z - 79872*a^6*b^3*c^5*e*k*m*z - 27648*a^6...
```

3.42 $\int \frac{d+ex}{(4-5x^2+x^4)^3} \, dx$

Optimal. Leaf size=143

$$\frac{dx(17-5x^2)}{144(4-5x^2+x^4)^2}+\frac{e(5-2x^2)}{36(4-5x^2+x^4)^2}-\frac{dx(59-35x^2)}{3456(4-5x^2+x^4)}-\frac{e(5-2x^2)}{54(4-5x^2+x^4)}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}$$

```
[Out] 1/144*d*x*(-5*x^2+17)/(x^4-5*x^2+4)^2+1/36*e*(-2*x^2+5)/(x^4-5*x^2+4)^2-1/3
456*d*x*(-35*x^2+59)/(x^4-5*x^2+4)-1/54*e*(-2*x^2+5)/(x^4-5*x^2+4)-313/2073
6*d*arctanh(1/2*x)+13/648*d*arctanh(x)-1/81*e*ln(-x^2+1)+1/81*e*ln(-x^2+4)
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {1687, 12, 1106, 1192, 1180, 213, 1121, 628, 630, 31}

$$-\frac{dx(59-35x^2)}{3456(x^4-5x^2+4)}+\frac{dx(17-5x^2)}{144(x^4-5x^2+4)^2}-\frac{313d\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{13}{648}d\tanh^{-1}(x)-\frac{1}{81}e\log(1-x^2)+\frac{1}{81}e\log(4-x^2)-\frac{e(5-2x^2)}{54(x^4-5x^2+4)}+\frac{e(5-2x^2)}{36(x^4-5x^2+4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (d*x*(17 - 5*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (e*(5 - 2*x^2))/(36*(4 - 5*x
^2 + x^4)^2) - (d*x*(59 - 35*x^2))/(3456*(4 - 5*x^2 + x^4)) - (e*(5 - 2*x^2
))/(54*(4 - 5*x^2 + x^4)) - (313*d*ArcTanh[x/2])/20736 + (13*d*ArcTanh[x])/
648 - (e*Log[1 - x^2])/81 + (e*Log[4 - x^2])/81
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex}{\left(4-5x^2+x^4\right)^3}\,dx &= \int \frac{d}{\left(4-5x^2+x^4\right)^3}\,dx+\int \frac{ex}{\left(4-5x^2+x^4\right)^3}\,dx\\
&= d\int \frac{1}{\left(4-5x^2+x^4\right)^3}\,dx+e\int \frac{x}{\left(4-5x^2+x^4\right)^3}\,dx\\
&= \frac{dx(17-5x^2)}{144\left(4-5x^2+x^4\right)^2}-\frac{1}{144}d\int \frac{-19+25x^2}{\left(4-5x^2+x^4\right)^2}\,dx+\frac{1}{2}e\text{Subst}\left(\int \frac{1}{\left(4-5x+x^2\right)^3}\right.\\
&= \frac{dx(17-5x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{e(5-2x^2)}{36\left(4-5x^2+x^4\right)^2}-\frac{dx(59-35x^2)}{3456\left(4-5x^2+x^4\right)}+\frac{d\int \frac{519+105x^2}{4-5x^2+x^4}\,d}{10368}\\
&= \frac{dx(17-5x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{e(5-2x^2)}{36\left(4-5x^2+x^4\right)^2}-\frac{dx(59-35x^2)}{3456\left(4-5x^2+x^4\right)}-\frac{e(5-2x^2)}{54\left(4-5x^2+\right.}\\
&= \frac{dx(17-5x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{e(5-2x^2)}{36\left(4-5x^2+x^4\right)^2}-\frac{dx(59-35x^2)}{3456\left(4-5x^2+x^4\right)}-\frac{e(5-2x^2)}{54\left(4-5x^2+\right.}\\
&= \frac{dx(17-5x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{e(5-2x^2)}{36\left(4-5x^2+x^4\right)^2}-\frac{dx(59-35x^2)}{3456\left(4-5x^2+x^4\right)}-\frac{e(5-2x^2)}{54\left(4-5x^2+\right.}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 128, normalized size = 0.90

$$\frac{\frac{288\left(e(20-8x^2)+dx(17-5x^2)\right)}{\left(4-5x^2+x^4\right)^2}+\frac{12\left(64e(-5+2x^2)+dx(-59+35x^2)\right)}{4-5x^2+x^4}-32(13d+16e)\log(1-x)+(313d+512e)\log(2-x)+32(13d-16e)\log(1+x)+(-313d+512e)\log(2+x)}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(e*(20 - 8*x^2) + d*x*(17 - 5*x^2)))/(4 - 5*x^2 + x^4)^2 + (12*(64*e*
(-5 + 2*x^2) + d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e)*Lo
g[1 - x] + (313*d + 512*e)*Log[2 - x] + 32*(13*d - 16*e)*Log[1 + x] + (-313
*d + 512*e)*Log[2 + x])/41472
```

Maple [A]
time = 0.05, size = 162, normalized size = 1.13

method	result

norman	$\frac{\frac{5}{9}e\,x^2+\frac{1}{27}e\,x^6-\frac{5}{18}e\,x^4+\frac{43}{864}dx+\frac{35}{384}d\,x^3-\frac{13}{192}d\,x^5+\frac{35}{3456}x^7d-\frac{25}{108}e}{(x^4-5x^2+4)^2}+\left(-\frac{313d}{41472}+\frac{e}{81}\right)\ln(x+2)+\left(-\frac{13d}{1296}-\frac{e}{81}\right)\ln(-$
risch	$\frac{\frac{5}{9}e\,x^2+\frac{1}{27}e\,x^6-\frac{5}{18}e\,x^4+\frac{43}{864}dx+\frac{35}{384}d\,x^3-\frac{13}{192}d\,x^5+\frac{35}{3456}x^7d-\frac{25}{108}e}{(x^4-5x^2+4)^2}+\frac{13\ln(1+x)d}{1296}-\frac{\ln(1+x)e}{81}+\frac{313\ln(2-x)d}{41472}+\frac{\ln(2-x)e}{81}-$
default	$\left(-\frac{313d}{41472}+\frac{e}{81}\right)\ln(x+2)-\frac{-\frac{19d}{6912}+\frac{17e}{3456}}{x+2}-\frac{-\frac{d}{1728}+\frac{e}{864}}{2(x+2)^2}-\frac{-\frac{19d}{6912}-\frac{17e}{3456}}{x-2}-\frac{\frac{d}{1728}+\frac{e}{864}}{2(x-2)^2}+\left(\frac{313d}{41472}+\frac{e}{81}\right)\ln(x-2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-313/41472*d+1/81*e)*ln(x+2)-(-19/6912*d+17/3456*e)/(x+2)-1/2*(-1/1728*d+1
/864*e)/(x+2)^2-(-19/6912*d-17/3456*e)/(x-2)-1/2*(1/1728*d+1/864*e)/(x-2)^2
+(313/41472*d+1/81*e)*ln(x-2)+(-13/1296*d-1/81*e)*ln(-1+x)-(-1/432*d-1/144*
e)/(-1+x)-1/2*(-1/216*d-1/216*e)/(-1+x)^2-(-1/432*d+1/144*e)/(1+x)-1/2*(1/2
16*d-1/216*e)/(1+x)^2+(13/1296*d-1/81*e)*ln(1+x)
```

Maxima [A]
time = 0.27, size = 129, normalized size = 0.90

$$-\frac{1}{41472}(313\,d-512\,e)\log(x+2)+\frac{1}{1296}(13\,d-16\,e)\log(x+1)-\frac{1}{1296}(13\,d+16\,e)\log(x-1)+\frac{1}{41472}(313\,d+512\,e)\log(x-2)+\frac{35\,dx^7+128\,x^6e-234\,dx^5-960\,x^4e+315\,dx^3+1920\,x^2e+172\,dx-800\,e}{3456\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d - 512*e)*log(x + 2) + 1/1296*(13*d - 16*e)*log(x + 1) - 1/1
296*(13*d + 16*e)*log(x - 1) + 1/41472*(313*d + 512*e)*log(x - 2) + 1/3456*
(35*d*x^7 + 128*x^6*e - 234*d*x^5 - 960*x^4*e + 315*d*x^3 + 1920*x^2*e + 17
2*d*x - 800*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(125) = 250.
time = 0.61, size = 307, normalized size = 2.15

$$\frac{420\,dx^7+1536\,ex^6-2808\,dx^5-11520\,ex^4+3780\,dx^3+23040\,ex^2+2064\,dx-((313\,d-512\,e)x^8-10\,(313\,d-512\,e)x^6+33\,(313\,d-512\,e)x^4-40\,(313\,d-512\,e)x^2+5008\,d-8192\,e)\log(x+2)+32\,((13\,d-16\,e)x^8-10\,(13\,d-16\,e)x^6+33\,(13\,d-16\,e)x^4-40\,(13\,d-16\,e)x^2+208\,d-256\,e)\log(x+1)-32\,((13\,d+16\,e)x^8-10\,(13\,d+16\,e)x^6+33\,(13\,d+16\,e)x^4-40\,(13\,d+16\,e)x^2+208\,d+256\,e)\log(x-1)+((313\,d+512\,e)x^8-10\,(313\,d+512\,e)x^6+33\,(313\,d+512\,e)x^4-40\,(313\,d+512\,e)x^2+5008\,d+8192\,e)\log(x-2)-9600\,e}{41472\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*d*x^7 + 1536*e*x^6 - 2808*d*x^5 - 11520*e*x^4 + 3780*d*x^3 + 2
3040*e*x^2 + 2064*d*x - ((313*d - 512*e)*x^8 - 10*(313*d - 512*e)*x^6 + 33*
(313*d - 512*e)*x^4 - 40*(313*d - 512*e)*x^2 + 5008*d - 8192*e)*log(x + 2)
+ 32*((13*d - 16*e)*x^8 - 10*(13*d - 16*e)*x^6 + 33*(13*d - 16*e)*x^4 - 40*
(13*d - 16*e)*x^2 + 208*d - 256*e)*log(x + 1) - 32*((13*d + 16*e)*x^8 - 10*
(13*d + 16*e)*x^6 + 33*(13*d + 16*e)*x^4 - 40*(13*d + 16*e)*x^2 + 208*d + 2
56*e)*log(x - 1) + ((313*d + 512*e)*x^8 - 10*(313*d + 512*e)*x^6 + 33*(313*
```

```
d + 512*e)*x^4 - 40*(313*d + 512*e)*x^2 + 5008*d + 8192*e)*log(x - 2) - 960
0*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(126) = 252$.
time = 2.21, size = 668, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] (13*d - 16*e)*log(x + (-1106258459719280*d**4*e - 13113710954343*d**4*(13*d
 - 16*e) - 817263343042560*d**2*e**3 + 153628968222720*d**2*e**2*(13*d - 16
*e) + 9530197557248*d**2*e*(13*d - 16*e)**2 + 88038005760*d**2*(13*d - 16*e
)**3 + 5035763255214080*e**5 + 142661633703936*e**4*(13*d - 16*e) - 1967095
0215680*e**3*(13*d - 16*e)**2 - 557272006656*e**2*(13*d - 16*e)**3)/(229412
56248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1296
 - (13*d + 16*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d**4*(1
3*d + 16*e) - 817263343042560*d**2*e**3 - 153628968222720*d**2*e**2*(13*d +
 16*e) + 9530197557248*d**2*e*(13*d + 16*e)**2 - 88038005760*d**2*(13*d + 1
6*e)**3 + 5035763255214080*e**5 - 142661633703936*e**4*(13*d + 16*e) - 1967
0950215680*e**3*(13*d + 16*e)**2 + 557272006656*e**2*(13*d + 16*e)**3)/(229
41256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*e**4))/1
296 - (313*d - 512*e)*log(x + (-1106258459719280*d**4*e + 13113710954343*d*
*4*(313*d - 512*e)/32 - 817263343042560*d**2*e**3 - 4800905256960*d**2*e**2
*(313*d - 512*e) + 9306833552*d**2*e*(313*d - 512*e)**2 - 85974615*d**2*(31
3*d - 512*e)**3/32 + 5035763255214080*e**5 - 4458176053248*e**4*(313*d - 51
2*e) - 19209912320*e**3*(313*d - 512*e)**2 + 17006592*e**2*(313*d - 512*e)*
*3)/(22941256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813420544*d*
e**4))/41472 + (313*d + 512*e)*log(x + (-1106258459719280*d**4*e - 13113710
954343*d**4*(313*d + 512*e)/32 - 817263343042560*d**2*e**3 + 4800905256960*
d**2*e**2*(313*d + 512*e) + 9306833552*d**2*e*(313*d + 512*e)**2 + 85974615
*d**2*(313*d + 512*e)**3/32 + 5035763255214080*e**5 + 4458176053248*e**4*(3
13*d + 512*e) - 19209912320*e**3*(313*d + 512*e)**2 - 17006592*e**2*(313*d
+ 512*e)**3)/(22941256248261*d**5 - 2312740746035200*d**3*e**2 + 4473912813
420544*d*e**4))/41472 + (35*d*x**7 - 234*d*x**5 + 315*d*x**3 + 172*d*x + 12
8*e*x**6 - 960*e*x**4 + 1920*e*x**2 - 800*e)/(3456*x**8 - 34560*x**6 + 1140
48*x**4 - 138240*x**2 + 55296)
```

Giac [A]
time = 3.80, size = 123, normalized size = 0.86

$$-\frac{1}{41472}(313\,d-512\,e)\log\left(|x+2|\right)+\frac{1}{1296}(13\,d-16\,e)\log\left(|x+1|\right)-\frac{1}{1296}(13\,d+16\,e)\log\left(|x-1|\right)+\frac{1}{41472}(313\,d+512\,e)\log\left(|x-2|\right)+\frac{35\,dx^7+128\,x^6e-234\,dx^5-960\,x^4e+315\,dx^3+1920\,x^2e+172\,dx-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d - 512*e)*log(abs(x + 2)) + 1/1296*(13*d - 16*e)*log(abs(x +
 1)) - 1/1296*(13*d + 16*e)*log(abs(x - 1)) + 1/41472*(313*d + 512*e)*log(a
bs(x - 2)) + 1/3456*(35*d*x^7 + 128*x^6*e - 234*d*x^5 - 960*x^4*e + 315*d*x
^3 + 1920*x^2*e + 172*d*x - 800*e)/(x^4 - 5*x^2 + 4)^2
```

Mupad [B]
time = 0.09, size = 118, normalized size = 0.83

$$\ln(x+1)\left(\frac{13\,d}{1296}-\frac{e}{81}\right)-\ln(x-1)\left(\frac{13\,d}{1296}+\frac{e}{81}\right)+\ln(x-2)\left(\frac{313\,d}{41472}+\frac{e}{81}\right)-\ln(x+2)\left(\frac{313\,d}{41472}-\frac{e}{81}\right)+\frac{\frac{35\,d\,x^7}{3456}+\frac{e\,x^6}{27}-\frac{13\,d\,x^5}{192}-\frac{5\,e\,x^4}{18}+\frac{35\,d\,x^3}{384}+\frac{5\,e\,x^2}{9}+\frac{43\,d\,x}{864}-\frac{25\,e}{108}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^4 - 5*x^2 + 4)^3,x)
```

```
[Out] log(x + 1)*((13*d)/1296 - e/81) - log(x - 1)*((13*d)/1296 + e/81) + log(x -
 2)*((313*d)/41472 + e/81) - log(x + 2)*((313*d)/41472 - e/81) + ((43*d*x)/
864 - (25*e)/108 + (35*d*x^3)/384 - (13*d*x^5)/192 + (35*d*x^7)/3456 + (5*e
*x^2)/9 - (5*e*x^4)/18 + (e*x^6)/27)/(33*x^4 - 40*x^2 - 10*x^6 + x^8 + 16)
```

3.43 $\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3} \, dx$

Optimal. Leaf size=175

$$\frac{e(5-2x^2)}{36(4-5x^2+x^4)^2}+\frac{x(17d+20f-(5d+8f)x^2)}{144(4-5x^2+x^4)^2}-\frac{e(5-2x^2)}{54(4-5x^2+x^4)}-\frac{x(59d+380f-35(d+4f)x^2)}{3456(4-5x^2+x^4)}-\frac{(31$$

```
[Out] 1/36*e*(-2*x^2+5)/(x^4-5*x^2+4)^2+1/144*x*(17*d+20*f-(5*d+8*f)*x^2)/(x^4-5*
x^2+4)^2-1/54*e*(-2*x^2+5)/(x^4-5*x^2+4)-1/3456*x*(59*d+380*f-35*(d+4*f)*x^
2)/(x^4-5*x^2+4)-1/20736*(313*d+820*f)*arctanh(1/2*x)+1/648*(13*d+25*f)*arc
tanh(x)-1/81*e*ln(-x^2+1)+1/81*e*ln(-x^2+4)
```

Rubi [A]
time = 0.15, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1687, 1192, 1180, 213, 12, 1121, 628, 630, 31}

$$-\frac{x(-35x^2(d+4f)+59d+380f)}{3456(x^4-5x^2+4)}+\frac{x(-(x^2(5d+8f))+17d+20f)}{144(x^4-5x^2+4)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{81}e\log\left(1-x^2\right)+\frac{1}{81}e\log\left(4-x^2\right)-\frac{e(5-2x^2)}{54(x^4-5x^2+4)}+\frac{e(5-2x^2)}{36(x^4-5x^2+4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (e*(5 - 2*x^2))/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f - (5*d + 8*f)*x^
2))/(144*(4 - 5*x^2 + x^4)^2) - (e*(5 - 2*x^2))/(54*(4 - 5*x^2 + x^4)) - (x
*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 82
0*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[x])/648 - (e*Log[1 - x^2]
)/81 + (e*Log[4 - x^2])/81
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(4-5x^2+x^4)^3}\,dx = \int \frac{ex}{(4-5x^2+x^4)^3}\,dx + \int \frac{d+fx^2}{(4-5x^2+x^4)^3}\,dx$$

$$= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{1}{144}\int \frac{-19d+20f+5(5d+8f)x^2}{(4-5x^2+x^4)^2}\,dx + e\int \frac{\quad}{(4-}$$

$$= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{x(59d+380f-35(d+4f)x^2)}{3456\,(4-5x^2+x^4)} + \frac{\int \frac{3(173d+260f)+105}{4-5x^2+x}}{10368}$$

$$= \frac{e(5-2x^2)}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{x(59d+380f-35(d+4f)x}{3456\,(4-5x^2+x^4)}$$

$$= \frac{e(5-2x^2)}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{e(5-2x^2)}{54\,(4-5x^2+x^4)} - \frac{x(59d+}{34}$$

$$= \frac{e(5-2x^2)}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{e(5-2x^2)}{54\,(4-5x^2+x^4)} - \frac{x(59d+}{34}$$

$$= \frac{e(5-2x^2)}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{e(5-2x^2)}{54\,(4-5x^2+x^4)} - \frac{x(59d+}{34}$$

Mathematica [A]

time = 0.08, size = 161, normalized size = 0.92

$$\frac{\frac{288(17dx+20fx-5dx^3-8fx^3+e(20-8x^2))}{(4-5x^2+x^4)^2} + \frac{12(64e(-5+2x^2)+20fx(-19+7x^2)+dx(-59+35x^2))}{4-5x^2+x^4} - 32(13d+16e+25f)\log(1-x) + (313d+512e+820f)\log(2-x) + 32(13d-16e+25f)\log(1+x) + (-313d+512e-820f)\log(2+x)}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2)))/(4 - 5*x^2 +
x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 20*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^2)
))/(4 - 5*x^2 + x^4) - 32*(13*d + 16*e + 25*f)*Log[1 - x] + (313*d + 512*e
+ 820*f)*Log[2 - x] + 32*(13*d - 16*e + 25*f)*Log[1 + x] + (-313*d + 512*e
- 820*f)*Log[2 + x])/41472
```

Maple [A]

time = 0.06, size = 198, normalized size = 1.13

method	result
norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x+\frac{5e\,x^2}{9}+\frac{e\,x^6}{27}-\frac{5e\,x^4}{18}-\frac{25e}{108}}{(x^4-5x^2+4)^2}+\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}\right)\ln(x$
risch	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x+\frac{5e\,x^2}{9}+\frac{e\,x^6}{27}-\frac{5e\,x^4}{18}-\frac{25e}{108}}{(x^4-5x^2+4)^2}-\frac{313\ln(x+2)d}{41472}+\frac{\ln(x+2)e}{81}-205$

default	$\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}\right)\ln\left(x+2\right)-\frac{-\frac{19d}{6912}+\frac{17e}{3456}-\frac{5f}{576}}{x+2}-\frac{-\frac{d}{1728}+\frac{e}{864}-\frac{f}{432}}{2(x+2)^2}-\frac{-\frac{19d}{6912}-\frac{17e}{3456}-\frac{5f}{576}}{x-2}-\frac{\frac{d}{1728}+\frac{e}{864}+\frac{f}{432}}{2(x-2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-313/41472*d+1/81*e-205/10368*f)*ln(x+2)-(-19/6912*d+17/3456*e-5/576*f)/(x
+2)-1/2*(-1/1728*d+1/864*e-1/432*f)/(x+2)^2-(-19/6912*d-17/3456*e-5/576*f)/
(x-2)-1/2*(1/1728*d+1/864*e+1/432*f)/(x-2)^2+(313/41472*d+1/81*e+205/10368*
f)*ln(x-2)+(-13/1296*d-1/81*e-25/1296*f)*ln(-1+x)-(-1/432*d-1/144*e-5/432*f
)/(-1+x)-1/2*(-1/216*d-1/216*e-1/216*f)/(-1+x)^2-(-1/432*d+1/144*e-5/432*f)
/(1+x)-1/2*(1/216*d-1/216*e+1/216*f)/(1+x)^2+(13/1296*d-1/81*e+25/1296*f)*l
n(1+x)
```

Maxima [A]
time = 0.27, size = 163, normalized size = 0.93

$$-\frac{1}{41472}\,(313\,d+820\,f-512\,e)\log\left(x+2\right)+\frac{1}{1296}\,(13\,d+25\,f-16\,e)\log\left(x+1\right)-\frac{1}{1296}\,(13\,d+25\,f+16\,e)\log\left(x-1\right)+\frac{1}{41472}\,(313\,d+820\,f+512\,e)\log\left(x-2\right)+\frac{35\,(d+4\,f)x^7+128\,x^6e-18\,(13\,d+60\,f)x^5-960\,x^4e+63\,(5\,d+36\,f)x^3+1920\,x^2e+4\,(43\,d-260\,f)x-800\,e}{3456\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d + 820*f - 512*e)*log(x + 2) + 1/1296*(13*d + 25*f - 16*e)*l
og(x + 1) - 1/1296*(13*d + 25*f + 16*e)*log(x - 1) + 1/41472*(313*d + 820*f
 + 512*e)*log(x - 2) + 1/3456*(35*(d + 4*f)*x^7 + 128*x^6*e - 18*(13*d + 60
*f)*x^5 - 960*x^4*e + 63*(5*d + 36*f)*x^3 + 1920*x^2*e + 4*(43*d - 260*f)*x
 - 800*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(156) = 312$.
time = 0.59, size = 389, normalized size = 2.22

$$\frac{420\,(d+4\,f)x^7+1536\,ex^6-216\,(13\,d+60\,f)x^5-11520\,ex^4+756\,(5\,d+36\,f)x^3+23040\,ex^2+48\,(43\,d-260\,f)x-((313\,d-512\,e+820\,f)x^8-10\,(313\,d-512\,e+820\,f)x^6+33\,(313\,d-512\,e+820\,f)x^4-40\,(313\,d-512\,e+820\,f)x^2+5008\,d-8192\,e+13120\,f)\log\left(x+2\right)+32\,((13\,d-16\,e+25\,f)x^8-10\,(13\,d-16\,e+25\,f)x^6+33\,(13\,d-16\,e+25\,f)x^4-40\,(13\,d-16\,e+25\,f)x^2+208\,d-256\,e+400\,f)\log\left(x+1\right)-32\,((13\,d+16\,e+25\,f)x^8-10\,(13\,d+16\,e+25\,f)x^6+33\,(13\,d+16\,e+25\,f)x^4-40\,(13\,d+16\,e+25\,f)x^2+208\,d+256\,e+400\,f)\log\left(x-1\right)+((313\,d+512\,e+820\,f)x^8-10\,(313\,d+512\,e+820\,f)x^6+33\,(313\,d+512\,e+820\,f)x^4-40\,(313\,d+512\,e+820\,f)x^2+5008\,d+8192\,e+13120\,f)\log\left(x-2\right)-9600\,e}{41472\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 1536*e*x^6 - 216*(13*d + 60*f)*x^5 - 11520*e*x
^4 + 756*(5*d + 36*f)*x^3 + 23040*e*x^2 + 48*(43*d - 260*f)*x - ((313*d - 5
12*e + 820*f)*x^8 - 10*(313*d - 512*e + 820*f)*x^6 + 33*(313*d - 512*e + 82
0*f)*x^4 - 40*(313*d - 512*e + 820*f)*x^2 + 5008*d - 8192*e + 13120*f)*log(
x + 2) + 32*((13*d - 16*e + 25*f)*x^8 - 10*(13*d - 16*e + 25*f)*x^6 + 33*(1
3*d - 16*e + 25*f)*x^4 - 40*(13*d - 16*e + 25*f)*x^2 + 208*d - 256*e + 400*
f)*log(x + 1) - 32*((13*d + 16*e + 25*f)*x^8 - 10*(13*d + 16*e + 25*f)*x^6
 + 33*(13*d + 16*e + 25*f)*x^4 - 40*(13*d + 16*e + 25*f)*x^2 + 208*d + 256*e
```

```
 + 400*f)*log(x - 1) + ((313*d + 512*e + 820*f)*x^8 - 10*(313*d + 512*e + 8
20*f)*x^6 + 33*(313*d + 512*e + 820*f)*x^4 - 40*(313*d + 512*e + 820*f)*x^2
 + 5008*d + 8192*e + 13120*f)*log(x - 2) - 9600*e)/(x^8 - 10*x^6 + 33*x^4 -
 40*x^2 + 16)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.78, size = 157, normalized size = 0.90

$$-\frac{1}{41472}(313\,d+820\,f-512\,e)\log\left(|x+2|\right)+\frac{1}{1296}(13\,d+25\,f-16\,e)\log\left(|x+1|\right)-\frac{1}{1296}(13\,d+25\,f+16\,e)\log\left(|x-1|\right)+\frac{1}{41472}(313\,d+820\,f+512\,e)\log\left(|x-2|\right)+\frac{35\,dx^7+140\,fx^7+128\,x^6e-234\,dx^5-1080\,fx^5-960\,x^4e+315\,dx^3+2268\,fx^3+1920\,x^2e+172\,dx-1040\,fx-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 512*e)*log(abs(x + 2)) + 1/1296*(13*d + 25*f - 16
*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f + 16*e)*log(abs(x - 1)) + 1/41472
*(313*d + 820*f + 512*e)*log(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 1
28*x^6*e - 234*d*x^5 - 1080*f*x^5 - 960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 19
20*x^2*e + 172*d*x - 1040*f*x - 800*e)/(x^4 - 5*x^2 + 4)^2
```

Mupad [**B**]
time = 0.11, size = 151, normalized size = 0.86

$$\ln(x+1)\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}\right)-\ln(x-1)\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}\right)+\ln(x-2)\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}\right)-\ln(x+2)\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}\right)+\frac{\left(\frac{35\,d}{3456}+\frac{35\,f}{864}\right)x^7+\frac{e\,x^6}{27}+\left(-\frac{13\,d}{192}-\frac{5\,f}{16}\right)x^5-\frac{5\,e\,x^4}{18}+\left(\frac{35\,d}{384}+\frac{21\,f}{32}\right)x^3+\frac{5\,e\,x^2}{9}+\left(\frac{43\,d}{864}-\frac{65\,f}{216}\right)x-\frac{25\,e}{108}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^4 - 5*x^2 + 4)^3,x)
```

```
[Out] log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296) - log(x - 1)*((13*d)/1296 + e
/81 + (25*f)/1296) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368) - lo
g(x + 2)*((313*d)/41472 - e/81 + (205*f)/10368) + (x^3*((35*d)/384 + (21*f)
/32) - x^5*((13*d)/192 + (5*f)/16) - (25*e)/108 + x^7*((35*d)/3456 + (35*f)
/864) + (5*e*x^2)/9 - (5*e*x^4)/18 + (e*x^6)/27 + x*((43*d)/864 - (65*f)/21
6))/(33*x^4 - 40*x^2 - 10*x^6 + x^8 + 16)
```

3.44 $\int \frac{d+ex+fx^2+gx^3}{\left(4-5x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=204

$$\frac{x(17d+20f-(5d+8f)x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2}-\frac{(2e+5g)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)}-\frac{x(59d+380f-35(d+4f)x^2)}{3456\left(4-5x^2+x^4\right)}$$

```
[Out] 1/144*x*(17*d+20*f-(5*d+8*f)*x^2)/(x^4-5*x^2+4)^2+1/36*(5*e+8*g-(2*e+5*g)*x
^2)/(x^4-5*x^2+4)^2-1/108*(2*e+5*g)*(-2*x^2+5)/(x^4-5*x^2+4)-1/3456*x*(59*d
+380*f-35*(d+4*f)*x^2)/(x^4-5*x^2+4)-1/20736*(313*d+820*f)*arctanh(1/2*x)+1
/648*(13*d+25*f)*arctanh(x)-1/162*(2*e+5*g)*ln(-x^2+1)+1/162*(2*e+5*g)*ln(-
x^2+4)
```

Rubi [A]
time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1687, 1192, 1180, 213, 1261, 652, 628, 630, 31}

$$-\frac{x(-35x^2(d+4f)+59d+380f)}{3456\left(x^4-5x^2+4\right)}+\frac{x(-(x^2(5d+8f))+17d+20f)}{144\left(x^4-5x^2+4\right)^2}-\frac{(313d+820f)\tanh^{-1}\left(\frac{x}{2}\right)}{20736}+\frac{1}{648}(13d+25f)\tanh^{-1}(x)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{-(x^2(2e+5g))+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (x*(17*d + 20*f - (5*d + 8*f)*x^2))/(144*(4 - 5*x^2 + x^4)^2) + (5*e + 8*g
- (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5 - 2*x^2))/(10
8*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f - 35*(d + 4*f)*x^2))/(3456*(4 - 5*x
^2 + x^4)) - ((313*d + 820*f)*ArcTanh[x/2])/20736 + ((13*d + 25*f)*ArcTanh[
x])/648 - ((2*e + 5*g)*Log[1 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{(4-5x^2+x^4)^3}\,dx &= \int \frac{d+fx^2}{(4-5x^2+x^4)^3}\,dx + \int \frac{x(e+gx^2)}{(4-5x^2+x^4)^3}\,dx \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{1}{144}\int \frac{-19d+20f+5(5d+8f)x^2}{(4-5x^2+x^4)^2}\,dx + \frac{1}{2}\text{Subs} \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} + \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2} - \frac{x(59d+380f-35(d}{3456\,(4-5x^2+} \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} + \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2} - \frac{(2e+5g)\,(5-2x^2)}{108\,(4-5x^2+x^4)} - \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} + \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2} - \frac{(2e+5g)\,(5-2x^2)}{108\,(4-5x^2+x^4)} - \\
&= \frac{x(17d+20f-(5d+8f)x^2)}{144\,(4-5x^2+x^4)^2} + \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2} - \frac{(2e+5g)\,(5-2x^2)}{108\,(4-5x^2+x^4)} -
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 193, normalized size = 0.95

$$\frac{\frac{288(17dx+20fx-5dx^3-8fx^3+e(20-8x^2)-4g(-8+5x^2))}{(4-5x^2+x^4)^2}+\frac{12(64e(-5+2x^2)+160g(-5+2x^2)+20fx(-19+7x^2)+dx(-59+35x^2))}{4-5x^2+x^4}-32(13d+16e+25f+40g)\log(1-x)+(313d+512e+820f+1280g)\log(2-x)+32(13d-16e+25f-40g)\log(1+x)+(-313d+512e-820f+1280g)\log(2+x)}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] ((288*(17*d*x + 20*f*x - 5*d*x^3 - 8*f*x^3 + e*(20 - 8*x^2) - 4*g*(-8 + 5*x
^2)))/(4 - 5*x^2 + x^4)^2 + (12*(64*e*(-5 + 2*x^2) + 160*g*(-5 + 2*x^2) + 2
0*f*x*(-19 + 7*x^2) + d*x*(-59 + 35*x^2)))/(4 - 5*x^2 + x^4) - 32*(13*d + 1
6*e + 25*f + 40*g)*Log[1 - x] + (313*d + 512*e + 820*f + 1280*g)*Log[2 - x]
 + 32*(13*d - 16*e + 25*f - 40*g)*Log[1 + x] + (-313*d + 512*e - 820*f + 12
80*g)*Log[2 + x])/41472
```

Maple [A]
time = 0.05, size = 234, normalized size = 1.15

method	result

norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x+\left(\frac{5e}{9}+\frac{25g}{18}\right)x^2+\left(\frac{e}{27}+\frac{5g}{54}\right)x^6+\left(-\frac{5e}{18}-\frac{25g}{36}\right)x^4-\frac{25e}{108}-\frac{19g}{27}}{(x^4-5x^2+4)^2}+\big(-$
risch	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}\right)x+\left(\frac{5e}{9}+\frac{25g}{18}\right)x^2+\left(\frac{e}{27}+\frac{5g}{54}\right)x^6+\left(-\frac{5e}{18}-\frac{25g}{36}\right)x^4-\frac{25e}{108}-\frac{19g}{27}}{(x^4-5x^2+4)^2}-313$
default	$\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}+\frac{5g}{162}\right)\ln(x+2)-\frac{-\frac{19d}{6912}+\frac{17e}{3456}-\frac{5f}{576}+\frac{13g}{864}}{x+2}-\frac{-\frac{d}{1728}+\frac{e}{864}-\frac{f}{432}+\frac{g}{216}}{2(x+2)^2}-\frac{-\frac{19d}{6912}-\frac{17e}{3456}-\frac{5f}{576}}{x-2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-313/41472*d+1/81*e-205/10368*f+5/162*g)*ln(x+2)-(-19/6912*d+17/3456*e-5/5
76*f+13/864*g)/(x+2)-1/2*(-1/1728*d+1/864*e-1/432*f+1/216*g)/(x+2)^2-(-19/6
912*d-17/3456*e-5/576*f-13/864*g)/(x-2)-1/2*(1/1728*d+1/864*e+1/432*f+1/216
*g)/(x-2)^2+(313/41472*d+1/81*e+205/10368*f+5/162*g)*ln(x-2)+(-13/1296*d-1/
81*e-25/1296*f-5/162*g)*ln(-1+x)-(-1/432*d-1/144*e-5/432*f-7/432*g)/(-1+x)-
1/2*(-1/216*d-1/216*e-1/216*f-1/216*g)/(-1+x)^2-(-1/432*d+1/144*e-5/432*f+7
/432*g)/(1+x)-1/2*(1/216*d-1/216*e+1/216*f-1/216*g)/(1+x)^2+(13/1296*d-1/81
*e+25/1296*f-5/162*g)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 196, normalized size = 0.96

$$-\frac{1}{41472}(313\,d+820\,f-1280\,g-512\,e)\log(x+2)+\frac{1}{1296}(13\,d+25\,f-40\,g-16\,e)\log(x+1)-\frac{1}{1296}(13\,d+25\,f+40\,g+16\,e)\log(x-1)+\frac{1}{41472}(313\,d+820\,f+1280\,g+512\,e)\log(x-2)+\frac{35\,(d+4\,f)x^7+64\,(5\,g+2\,e)x^6-18\,(13\,d+60\,f)x^5-480\,(5\,g+2\,e)x^4+63\,(5\,d+36\,f)x^3+960\,(5\,g+2\,e)x^2+4\,(43\,d-260\,f)x-2432\,g-800\,e}{3456\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g - 512*e)*log(x + 2) + 1/1296*(13*d + 25*f
- 40*g - 16*e)*log(x + 1) - 1/1296*(13*d + 25*f + 40*g + 16*e)*log(x - 1) +
 1/41472*(313*d + 820*f + 1280*g + 512*e)*log(x - 2) + 1/3456*(35*(d + 4*f)
*x^7 + 64*(5*g + 2*e)*x^6 - 18*(13*d + 60*f)*x^5 - 480*(5*g + 2*e)*x^4 + 63
*(5*d + 36*f)*x^3 + 960*(5*g + 2*e)*x^2 + 4*(43*d - 260*f)*x - 2432*g - 800
*e)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. 2(184) = 368.
time = 0.85, size = 470, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(420*(d + 4*f)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(13*d + 60*f)*x^5 -
5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f)*x^3 + 11520*(2*e + 5*g)*x^2 + 48*(4
```

```
3*d - 260*f)*x - ((313*d - 512*e + 820*f - 1280*g)*x^8 - 10*(313*d - 512*e
+ 820*f - 1280*g)*x^6 + 33*(313*d - 512*e + 820*f - 1280*g)*x^4 - 40*(313*d
 - 512*e + 820*f - 1280*g)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g)*log(x
 + 2) + 32*((13*d - 16*e + 25*f - 40*g)*x^8 - 10*(13*d - 16*e + 25*f - 40*g
)*x^6 + 33*(13*d - 16*e + 25*f - 40*g)*x^4 - 40*(13*d - 16*e + 25*f - 40*g)
*x^2 + 208*d - 256*e + 400*f - 640*g)*log(x + 1) - 32*((13*d + 16*e + 25*f
+ 40*g)*x^8 - 10*(13*d + 16*e + 25*f + 40*g)*x^6 + 33*(13*d + 16*e + 25*f +
 40*g)*x^4 - 40*(13*d + 16*e + 25*f + 40*g)*x^2 + 208*d + 256*e + 400*f + 6
40*g)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g)*x^8 - 10*(313*d + 512*
e + 820*f + 1280*g)*x^6 + 33*(313*d + 512*e + 820*f + 1280*g)*x^4 - 40*(313
*d + 512*e + 820*f + 1280*g)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g)*log
(x - 2) - 9600*e - 29184*g)/(x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.23, size = 190, normalized size = 0.93

$$-\frac{1}{41472}\,(313\,d+820\,f-1280\,g-512\,e)\log\left(|x+2|\right)+\frac{1}{1296}\,(13\,d+25\,f-40\,g-16\,e)\log\left(|x+1|\right)-\frac{1}{1296}\,(13\,d+25\,f+40\,g+16\,e)\log\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+820\,f+1280\,g+512\,e)\log\left(|x-2|\right)+\frac{35\,dx^7+140\,fx^7+320\,gx^6+128\,x^6e-234\,dx^5-1080\,fx^5-2400\,gx^4-960\,x^4e+315\,dx^3+2268\,fx^3+4800\,gx^2+1920\,x^2e+172\,dx-1040\,fx-2432\,g-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g - 512*e)*log(abs(x + 2)) + 1/1296*(13*d +
25*f - 40*g - 16*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f + 40*g + 16*e)*lo
g(abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g + 512*e)*log(abs(x - 2)) +
1/3456*(35*d*x^7 + 140*f*x^7 + 320*g*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*x
^5 - 2400*g*x^4 - 960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 4800*g*x^2 + 1920*x^
2*e + 172*d*x - 1040*f*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

Mupad [B]
time = 0.85, size = 182, normalized size = 0.89

$$\frac{\left(\frac{35\,d}{3456}+\frac{35\,f}{864}\right)\,x^7+\left(\frac{e}{27}+\frac{5\,g}{54}\right)\,x^6+\left(-\frac{13\,d}{192}-\frac{5\,f}{16}\right)\,x^5+\left(-\frac{5\,e}{18}-\frac{25\,g}{36}\right)\,x^4+\left(\frac{35\,d}{384}+\frac{21\,f}{32}\right)\,x^3+\left(\frac{5\,e}{9}+\frac{25\,g}{18}\right)\,x^2+\left(\frac{43\,d}{864}-\frac{65\,f}{216}\right)\,x-\frac{25\,e}{108}-\frac{19\,g}{27}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}-\ln\left(x-1\right)\,\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}+\frac{5\,g}{162}\right)+\ln\left(x+1\right)\,\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}-\frac{5\,g}{162}\right)+\ln\left(x-2\right)\,\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}+\frac{5\,g}{162}\right)-\ln\left(x+2\right)\,\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}-\frac{5\,g}{162}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4 - 5*x^2 + 4)^3,x)
```

```
[Out] (x^3*((35*d)/384 + (21*f)/32) - (19*g)/27 - x^5*((13*d)/192 + (5*f)/16) - (
25*e)/108 + x^7*((35*d)/3456 + (35*f)/864) + x^2*((5*e)/9 + (25*g)/18) - x^
4*((5*e)/18 + (25*g)/36) + x^6*(e/27 + (5*g)/54) + x*((43*d)/864 - (65*f)/2
16))/(33*x^4 - 40*x^2 - 10*x^6 + x^8 + 16) - log(x - 1)*((13*d)/1296 + e/81
 + (25*f)/1296 + (5*g)/162) + log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296
- (5*g)/162) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368 + (5*g)/162
) - log(x + 2)*((313*d)/41472 - e/81 + (205*f)/10368 - (5*g)/162)
```

3.45 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(4-5x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=224

$$\frac{5e+8g-(2e+5g)x^2}{36\left(4-5x^2+x^4\right)^2}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\left(4-5x^2+x^4\right)^2}-\frac{(2e+5g)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)}-\frac{x(59d+380f+84}{3456}$$

```
[Out] 1/36*(5*e+8*g-(2*e+5*g)*x^2)/(x^4-5*x^2+4)^2+1/144*x*(17*d+20*f+32*h-(5*d+8
*f+20*h)*x^2)/(x^4-5*x^2+4)^2-1/108*(2*e+5*g)*(-2*x^2+5)/(x^4-5*x^2+4)-1/34
56*x*(59*d+380*f+848*h-5*(7*d+28*f+64*h)*x^2)/(x^4-5*x^2+4)-1/20736*(313*d+
820*f+1936*h)*arctanh(1/2*x)+1/648*(13*d+25*f+61*h)*arctanh(x)-1/162*(2*e+5
*g)*ln(-x^2+1)+1/162*(2*e+5*g)*ln(-x^2+4)
```

Rubi [A]
time = 0.20, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {1687, 1692, 1192, 1180, 213, 1261, 652, 628, 630, 31}

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}(2e+5g)\log\left(1-x^2\right)+\frac{1}{162}(2e+5g)\log\left(4-x^2\right)-\frac{\left(5-2x^2\right)(2e+5g)}{108\left(x^4-5x^2+4\right)}+\frac{-\left(x^2(2e+5g)\right)+5e+8g}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (5*e + 8*g - (2*e + 5*g)*x^2)/(36*(4 - 5*x^2 + x^4)^2) + (x*(17*d + 20*f +
32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2) - ((2*e + 5*g)*(5
 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f + 848*h - 5*(7*d + 28
*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d + 820*f + 1936*h)*ArcTa
nh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648 - ((2*e + 5*g)*Log[1
 - x^2])/162 + ((2*e + 5*g)*Log[4 - x^2])/162
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3+hx^4}{(4-5x^2+x^4)^3}\,dx &= \int \frac{x(e+gx^2)}{(4-5x^2+x^4)^3}\,dx+\int \frac{d+fx^2+hx^4}{(4-5x^2+x^4)^3}\,dx\\
&= \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2}-\frac{1}{144}\int \frac{-19d+20f+32h+\cdots}{(4-5x\cdots}\\
&= \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2}-\frac{x(\cdots}{\cdots}\\
&= \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2}-\frac{(2\cdots}{10\cdots}\\
&= \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2}-\frac{(2\cdots}{10\cdots}\\
&= \frac{5e+8g-(2e+5g)x^2}{36\,(4-5x^2+x^4)^2}+\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2}-\frac{(2\cdots}{10\cdots}
\end{aligned}$$

Mathematica [**A**]
time = 0.08, size = 231, normalized size = 1.03

$$\frac{20e+32g+17dx+20fx+32hx-8ex^2-20gx^2-5dx^3-8fx^3-20hx^3}{144\,(4-5x^2+x^4)^2}+\frac{-320e-800g-59dx-380fx-848hx+128ex^2+320gx^2+35dx^3+140fx^3+320hx^3}{3456\,(4-5x^2+x^4)}+\frac{(-13d-16e-25f-40g-61h)\log(1-x)}{1296}+\frac{(313d+512e+820f+1280g+1936h)\log(2-x)}{41472}+\frac{(13d-16e+25f-40g+61h)\log(1+x)}{1296}+\frac{(-313d+512e-820f+1280g-1936h)\log(2+x)}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 5*d*x^3 - 8*
f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e - 800*g - 59*d*x - 38
0*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*x^3)
/(3456*(4 - 5*x^2 + x^4)) + ((-13*d - 16*e - 25*f - 40*g - 61*h)*Log[1 - x]
)/1296 + ((313*d + 512*e + 820*f + 1280*g + 1936*h)*Log[2 - x])/41472 + ((1
3*d - 16*e + 25*f - 40*g + 61*h)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*
f + 1280*g - 1936*h)*Log[2 + x])/41472
```

Maple [A]
time = 0.06, size = 270, normalized size = 1.21

method	result
norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}-\frac{17h}{24}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}+\frac{35h}{24}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}+\frac{5h}{54}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}-\frac{41h}{54}\right)x+\left(\frac{5e}{9}+\frac{25g}{18}\right)x^2+\left(\frac{e}{27}+\frac{5g}{54}\right)x^6+\left(-\frac{5e}{18}-\frac{25g}{36}\right)}{\left(x^4-5x^2+4\right)^2}$
risch	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}-\frac{17h}{24}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}+\frac{35h}{24}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}+\frac{5h}{54}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}-\frac{41h}{54}\right)x+\left(\frac{5e}{9}+\frac{25g}{18}\right)x^2+\left(\frac{e}{27}+\frac{5g}{54}\right)x^6+\left(-\frac{5e}{18}-\frac{25g}{36}\right)}{\left(x^4-5x^2+4\right)^2}$
default	$\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}+\frac{5g}{162}-\frac{121h}{2592}\right)\ln\left(x+2\right)-\frac{-\frac{19d}{6912}+\frac{17e}{3456}-\frac{5f}{576}+\frac{13g}{864}-\frac{11h}{432}}{x+2}-\frac{-\frac{d}{1728}+\frac{e}{864}-\frac{f}{432}+\frac{g}{216}-\frac{h}{108}}{2(x+2)^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-313/41472*d+1/81*e-205/10368*f+5/162*g-121/2592*h)*ln(x+2)-(-19/6912*d+17
/3456*e-5/576*f+13/864*g-11/432*h)/(x+2)-1/2*(-1/1728*d+1/864*e-1/432*f+1/2
16*g-1/108*h)/(x+2)^2-(-19/6912*d-17/3456*e-5/576*f-13/864*g-11/432*h)/(x-2
)-1/2*(1/1728*d+1/864*e+1/432*f+1/216*g+1/108*h)/(x-2)^2+(313/41472*d+1/81*
e+205/10368*f+5/162*g+121/2592*h)*ln(x-2)+(-13/1296*d-1/81*e-25/1296*f-5/16
2*g-61/1296*h)*ln(-1+x)-(-1/432*d-1/144*e-5/432*f-7/432*g-1/48*h)/(-1+x)-1/
2*(-1/216*d-1/216*e-1/216*f-1/216*g-1/216*h)/(-1+x)^2-(-1/432*d+1/144*e-5/4
32*f+7/432*g-1/48*h)/(1+x)-1/2*(1/216*d-1/216*e+1/216*f-1/216*g+1/216*h)/(1
+x)^2+(13/1296*d-1/81*e+25/1296*f-5/162*g+61/1296*h)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 222, normalized size = 0.99

$$-\frac{1}{41472}\,(313\,d+820\,f-1280\,g+1936\,h-512\,e)\log\left(x+2\right)+\frac{1}{1296}\,(13\,d+25\,f-40\,g+61\,h-16\,e)\log\left(x+1\right)-\frac{1}{1296}\,(13\,d+25\,f+40\,g+61\,h+16\,e)\log\left(x-1\right)+\frac{1}{41472}\,(313\,d+820\,f+1280\,g+1936\,h+512\,e)\log\left(x-2\right)+\frac{5\,(7\,d+28\,f+64\,h)x^7+64\,(5\,g+2\,e)x^6-18\,(13\,d+60\,f+136\,h)x^5-480\,(5\,g+2\,e)x^4+63\,(5\,d+36\,f+80\,h)x^3+960\,(5\,g+2\,e)x^2+4\,(43\,d-260\,f-656\,h)x-2432\,g-800\,e}{3456\,(x^8-10\,x^6+33\,x^4-40\,x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="maxima")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e)*log(x + 2) + 1/1296*(13*
d + 25*f - 40*g + 61*h - 16*e)*log(x + 1) - 1/1296*(13*d + 25*f + 40*g + 61
*h + 16*e)*log(x - 1) + 1/41472*(313*d + 820*f + 1280*g + 1936*h + 512*e)*l
og(x - 2) + 1/3456*(5*(7*d + 28*f + 64*h)*x^7 + 64*(5*g + 2*e)*x^6 - 18*(13
*d + 60*f + 136*h)*x^5 - 480*(5*g + 2*e)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 +
```

```
960*(5*g + 2*e)*x^2 + 4*(43*d - 260*f - 656*h)*x - 2432*g - 800*e)/(x^8 -
10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(204) = 408$.
time = 1.66, size = 544, normalized size = 2.43

$$\frac{60\,(7\,d+28\,f+64\,h)x^7+768\,(2\,e+5\,g)x^6-216\,(13\,d+60\,f+136\,h)x^5-5760\,(2\,e+5\,g)x^4+756\,(5\,d+36\,f+80\,h)x^3+11520\,(2\,e+5\,g)x^2+48\,(43\,d-260\,f-656\,h)x-\left((313\,d-512\,e+820\,f-1280\,g+1936\,h)x^8-10\,(313\,d-512\,e+820\,f-1280\,g+1936\,h)x^6+33\,(313\,d-512\,e+820\,f-1280\,g+1936\,h)x^4-40\,(313\,d-512\,e+820\,f-1280\,g+1936\,h)x^2+5008\,d-8192\,e+13120\,f-20480\,g+30976\,h\right)\log\left(x+2\right)+32\,\left((13\,d-16\,e+25\,f-40\,g+61\,h)x^8-10\,(13\,d-16\,e+25\,f-40\,g+61\,h)x^6+33\,(13\,d-16\,e+25\,f-40\,g+61\,h)x^4-40\,(13\,d-16\,e+25\,f-40\,g+61\,h)x^2+208\,d-256\,e+400\,f-640\,g+976\,h\right)\log\left(x+1\right)-32\,\left((13\,d+16\,e+25\,f+40\,g+61\,h)x^8-10\,(13\,d+16\,e+25\,f+40\,g+61\,h)x^6+33\,(13\,d+16\,e+25\,f+40\,g+61\,h)x^4-40\,(13\,d+16\,e+25\,f+40\,g+61\,h)x^2+208\,d+256\,e+400\,f+640\,g+976\,h\right)\log\left(x-1\right)+\left((313\,d+512\,e+820\,f+1280\,g+1936\,h)x^8-10\,(313\,d+512\,e+820\,f+1280\,g+1936\,h)x^6+33\,(313\,d+512\,e+820\,f+1280\,g+1936\,h)x^4-40\,(313\,d+512\,e+820\,f+1280\,g+1936\,h)x^2+5008\,d+8192\,e+13120\,f+20480\,g+30976\,h\right)\log\left(x-2\right)-9600\,e-29184\,g}{41472\,\left(x^8-10\,x^6+33\,x^4-40\,x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fricas")
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g)*x^6 - 216*(13*d + 60*
f + 136*h)*x^5 - 5760*(2*e + 5*g)*x^4 + 756*(5*d + 36*f + 80*h)*x^3 + 11520
*(2*e + 5*g)*x^2 + 48*(43*d - 260*f - 656*h)*x - ((313*d - 512*e + 820*f -
1280*g + 1936*h)*x^8 - 10*(313*d - 512*e + 820*f - 1280*g + 1936*h)*x^6 + 3
3*(313*d - 512*e + 820*f - 1280*g + 1936*h)*x^4 - 40*(313*d - 512*e + 820*f
 - 1280*g + 1936*h)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h)*lo
g(x + 2) + 32*((13*d - 16*e + 25*f - 40*g + 61*h)*x^8 - 10*(13*d - 16*e + 2
5*f - 40*g + 61*h)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61*h)*x^4 - 40*(13
*d - 16*e + 25*f - 40*g + 61*h)*x^2 + 208*d - 256*e + 400*f - 640*g + 976*h
)*log(x + 1) - 32*((13*d + 16*e + 25*f + 40*g + 61*h)*x^8 - 10*(13*d + 16*e
 + 25*f + 40*g + 61*h)*x^6 + 33*(13*d + 16*e + 25*f + 40*g + 61*h)*x^4 - 40
*(13*d + 16*e + 25*f + 40*g + 61*h)*x^2 + 208*d + 256*e + 400*f + 640*g + 9
76*h)*log(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h)*x^8 - 10*(313
*d + 512*e + 820*f + 1280*g + 1936*h)*x^6 + 33*(313*d + 512*e + 820*f + 128
0*g + 1936*h)*x^4 - 40*(313*d + 512*e + 820*f + 1280*g + 1936*h)*x^2 + 5008
*d + 8192*e + 13120*f + 20480*g + 30976*h)*log(x - 2) - 9600*e - 29184*g)/(
x^8 - 10*x^6 + 33*x^4 - 40*x^2 + 16)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.23, size = 224, normalized size = 1.00

$$-\frac{1}{41472}\,(313\,d+820\,f-1280\,g+1936\,h-512\,e)\log\left(|x+2|\right)+\frac{1}{1296}\,(13\,d+25\,f-40\,g+61\,h-16\,e)\log\left(|x+1|\right)-\frac{1}{1296}\,(13\,d+25\,f+40\,g+61\,h+16\,e)\log\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+820\,f+1280\,g+1936\,h+512\,e)\log\left(|x-2|\right)+\frac{35\,dx^7+140\,fx^7+320\,hx^7+320\,gx^6+128\,x^6e-234\,dx^5-1080\,fx^5-2448\,hx^5-2400\,gx^4-960\,x^4e+315\,dx^3+2268\,fx^3+5040\,hx^3+4800\,gx^2+1920\,x^2e+172\,dx-1040\,fx-2624\,hx-2432\,g-800\,e}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="giac")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e)*log(abs(x + 2)) + 1/1296
*(13*d + 25*f - 40*g + 61*h - 16*e)*log(abs(x + 1)) - 1/1296*(13*d + 25*f +
 40*g + 61*h + 16*e)*log(abs(x - 1)) + 1/41472*(313*d + 820*f + 1280*g + 19
36*h + 512*e)*log(abs(x - 2)) + 1/3456*(35*d*x^7 + 140*f*x^7 + 320*h*x^7 +
320*g*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*x^5 - 2448*h*x^5 - 2400*g*x^4 -
960*x^4*e + 315*d*x^3 + 2268*f*x^3 + 5040*h*x^3 + 4800*g*x^2 + 1920*x^2*e +
 172*d*x - 1040*f*x - 2624*h*x - 2432*g - 800*e)/(x^4 - 5*x^2 + 4)^2
```

Mupad [B]
time = 0.25, size = 209, normalized size = 0.93

$$\ln\left(x+1\right)\,\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}-\frac{5\,g}{162}+\frac{61\,h}{1296}\right)-\ln\left(x-1\right)\,\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}+\frac{5\,g}{162}+\frac{61\,h}{1296}\right)-\frac{\left(-\frac{35\,d}{3456}-\frac{35\,f}{864}-\frac{5\,h}{54}\right)\,x^7+\left(-\frac{e}{27}-\frac{5\,g}{54}\right)\,x^6+\left(\frac{13\,d}{192}+\frac{5\,f}{16}+\frac{17\,h}{24}\right)\,x^5+\left(\frac{5\,e}{18}+\frac{25\,g}{36}\right)\,x^4+\left(-\frac{35\,d}{384}-\frac{21\,f}{32}-\frac{35\,h}{24}\right)\,x^3+\left(-\frac{5\,e}{9}-\frac{25\,g}{18}\right)\,x^2+\left(\frac{65\,f}{216}-\frac{43\,d}{864}+\frac{41\,h}{54}\right)\,x+\frac{25\,e}{108}+\frac{19\,g}{27}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}+\ln\left(x-2\right)\,\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}+\frac{5\,g}{162}+\frac{121\,h}{2592}\right)-\ln\left(x+2\right)\,\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}-\frac{5\,g}{162}+\frac{121\,h}{2592}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^4 - 5*x^2 + 4)^3,x)
```

```
[Out] log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296 - (5*g)/162 + (61*h)/1296) - l
og(x - 1)*((13*d)/1296 + e/81 + (25*f)/1296 + (5*g)/162 + (61*h)/1296) - ((
25*e)/108 + (19*g)/27 - x^2*((5*e)/9 + (25*g)/18) + x^4*((5*e)/18 + (25*g)/
36) - x^6*(e/27 + (5*g)/54) + x*((65*f)/216 - (43*d)/864 + (41*h)/54) + x^5
*((13*d)/192 + (5*f)/16 + (17*h)/24) - x^3*((35*d)/384 + (21*f)/32 + (35*h)
/24) - x^7*((35*d)/3456 + (35*f)/864 + (5*h)/54))/(33*x^4 - 40*x^2 - 10*x^6
 + x^8 + 16) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368 + (5*g)/162
 + (121*h)/2592) - log(x + 2)*((313*d)/41472 - e/81 + (205*f)/10368 - (5*g)
/162 + (121*h)/2592)
```

3.46 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(4-5x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=239

$$\frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\left(4-5x^2+x^4\right)^2}+\frac{5e+8g+20i-(2e+5g+17i)x^2}{36\left(4-5x^2+x^4\right)^2}-\frac{(2e+5g+11i)\left(5-2x^2\right)}{108\left(4-5x^2+x^4\right)}-$$

```
[Out] 1/144*x*(17*d+20*f+32*h-(5*d+8*f+20*h)*x^2)/(x^4-5*x^2+4)^2+1/36*(5*e+8*g+2
0*i-(2*e+5*g+17*i)*x^2)/(x^4-5*x^2+4)^2-1/108*(2*e+5*g+11*i)*(-2*x^2+5)/(x^
4-5*x^2+4)-1/3456*x*(59*d+380*f+848*h-5*(7*d+28*f+64*h)*x^2)/(x^4-5*x^2+4)-
1/20736*(313*d+820*f+1936*h)*arctanh(1/2*x)+1/648*(13*d+25*f+61*h)*arctanh(
x)-1/162*(2*e+5*g+11*i)*ln(-x^2+1)+1/162*(2*e+5*g+11*i)*ln(-x^2+4)
```

Rubi [A]

time = 0.22, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {1687, 1692, 1192, 1180, 213, 1677, 1674, 12, 628, 630, 31}

$$-\frac{x\left(-5x^2(7d+28f+64h)+59d+380f+848h\right)}{3456\left(x^4-5x^2+4\right)}+\frac{x\left(-\left(x^2(5d+8f+20h)\right)+17d+20f+32h\right)}{144\left(x^4-5x^2+4\right)^2}-\frac{\tanh^{-1}\left(\frac{x}{2}\right)(313d+820f+1936h)}{20736}+\frac{1}{648}\tanh^{-1}(x)(13d+25f+61h)-\frac{1}{162}\log\left(1-x^2\right)(2e+5g+11i)+\frac{1}{162}\log\left(4-x^2\right)(2e+5g+11i)-\frac{\left(5-2x^2\right)(2e+5g+11i)}{108\left(x^4-5x^2+4\right)}+\frac{-\left(x^2(2e+5g+17i)\right)+5e+8g+20i}{36\left(x^4-5x^2+4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (x*(17*d + 20*f + 32*h - (5*d + 8*f + 20*h)*x^2))/(144*(4 - 5*x^2 + x^4)^2)
 + (5*e + 8*g + 20*i - (2*e + 5*g + 17*i)*x^2)/(36*(4 - 5*x^2 + x^4)^2) - (
(2*e + 5*g + 11*i)*(5 - 2*x^2))/(108*(4 - 5*x^2 + x^4)) - (x*(59*d + 380*f
+ 848*h - 5*(7*d + 28*f + 64*h)*x^2))/(3456*(4 - 5*x^2 + x^4)) - ((313*d +
820*f + 1936*h)*ArcTanh[x/2])/20736 + ((13*d + 25*f + 61*h)*ArcTanh[x])/648
 - ((2*e + 5*g + 11*i)*Log[1 - x^2])/162 + ((2*e + 5*g + 11*i)*Log[4 - x^2]
)/162
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+46x^5}{(4-5x^2+x^4)^3}\,dx = \int \frac{x(e+gx^2+46x^4)}{(4-5x^2+x^4)^3}\,dx + \int \frac{d+fx^2+hx^4}{(4-5x^2+x^4)^3}\,dx$$

$$= \frac{x(17d+20f+32h-(5d+8f+20h)x^2)}{144\,(4-5x^2+x^4)^2} - \frac{1}{144}\int \frac{-19d+20f}{}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f+32h-(5d}{144\,(4-5x^2+}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f+32h-(5d}{144\,(4-5x^2+}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f+32h-(5d}{144\,(4-5x^2+}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f+32h-(5d}{144\,(4-5x^2+}$$

$$= \frac{920+5e+8g-(782+2e+5g)x^2}{36\,(4-5x^2+x^4)^2} + \frac{x(17d+20f+32h-(5d}{144\,(4-5x^2+}$$

Mathematica [A]
time = 0.08, size = 261, normalized size = 1.09

$$\frac{20e+32g+80i+17dx+20fx+32hx-8ex^2-20gx^2-68ix^2-5dx^3-8fx^3-20hx^3}{144\,(4-5x^2+x^4)^2}+\frac{-320e-800g-1760i-59dx-380fx-848hx+128ex^2+320gx^2+704ix^2+35dx^3+140fx^3+320hx^3}{3456\,(4-5x^2+x^4)}+\frac{(-13d-16e-25f-40g-61h-88i)\log(1-x)}{1296}+\frac{(313d+512e+820f+1280g+1936h+2816i)\log(2-x)}{41472}+\frac{(13d-16e+25f-40g+61h-88i)\log(1+x)}{1296}+\frac{(-313d+512e-820f+1280g-1936h+2816i)\log(2+x)}{41472}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(4 - 5*x^2 + x^4)^3,x]
```

```
[Out] (20*e + 32*g + 80*i + 17*d*x + 20*f*x + 32*h*x - 8*e*x^2 - 20*g*x^2 - 68*i*
x^2 - 5*d*x^3 - 8*f*x^3 - 20*h*x^3)/(144*(4 - 5*x^2 + x^4)^2) + (-320*e - 8
00*g - 1760*i - 59*d*x - 380*f*x - 848*h*x + 128*e*x^2 + 320*g*x^2 + 704*i*
x^2 + 35*d*x^3 + 140*f*x^3 + 320*h*x^3)/(3456*(4 - 5*x^2 + x^4)) + ((-13*d
- 16*e - 25*f - 40*g - 61*h - 88*i)*Log[1 - x])/1296 + ((313*d + 512*e + 82
0*f + 1280*g + 1936*h + 2816*i)*Log[2 - x])/41472 + ((13*d - 16*e + 25*f -
40*g + 61*h - 88*i)*Log[1 + x])/1296 + ((-313*d + 512*e - 820*f + 1280*g -
1936*h + 2816*i)*Log[2 + x])/41472
```

Maple [A]
time = 0.07, size = 306, normalized size = 1.28

method	result
norman	$\frac{\left(-\frac{13d}{192}-\frac{5f}{16}-\frac{17h}{24}\right)x^5+\left(\frac{35d}{384}+\frac{21f}{32}+\frac{35h}{24}\right)x^3+\left(\frac{35d}{3456}+\frac{35f}{864}+\frac{5h}{54}\right)x^7+\left(\frac{43d}{864}-\frac{65f}{216}-\frac{41h}{54}\right)x+\left(\frac{5e}{9}+\frac{25g}{18}+\frac{26i}{9}\right)x^2+\left(\frac{e}{27}+\frac{5g}{54}+\frac{11i}{54}\right)x^6+\left(-\right.}{(x^4-5x^2+4)^2}$

default	$\left(-\frac{313d}{41472}+\frac{e}{81}-\frac{205f}{10368}+\frac{5g}{162}-\frac{121h}{2592}+\frac{11i}{162}\right)\ln\left(x+2\right)-\frac{-\frac{19d}{6912}+\frac{17e}{3456}-\frac{5f}{576}+\frac{13g}{864}-\frac{11h}{432}+\frac{i}{24}}{x+2}-\frac{-\frac{d}{1728}+\frac{e}{864}-\frac{f}{432}+2}{2(x+2)^2}$
risch	$-\frac{121\ln(x+2)h}{2592}+\frac{121\ln(2-x)h}{2592}+\frac{61\ln(1+x)h}{1296}-\frac{61\ln(1-x)h}{1296}+\frac{11\ln(x+2)i}{162}+\frac{11\ln(2-x)i}{162}+\frac{5\ln(x+2)g}{162}+\frac{5\ln(2-x)g}{162}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x,method=_RETURNVERBOSE
)
```

```
[Out] (-313/41472*d+1/81*e-205/10368*f+5/162*g-121/2592*h+11/162*i)*ln(x+2)-(-19/
6912*d+17/3456*e-5/576*f+13/864*g-11/432*h+1/24*i)/(x+2)-1/2*(-1/1728*d+1/8
64*e-1/432*f+1/216*g-1/108*h+1/54*i)/(x+2)^2-(-19/6912*d-17/3456*e-5/576*f-
13/864*g-11/432*h-1/24*i)/(x-2)-1/2*(1/1728*d+1/864*e+1/432*f+1/216*g+1/108
*h+1/54*i)/(x-2)^2+(313/41472*d+1/81*e+205/10368*f+5/162*g+121/2592*h+11/16
2*i)*ln(x-2)+(-13/1296*d-1/81*e-25/1296*f-5/162*g-61/1296*h-11/162*i)*ln(-1
+x)-(-1/432*d-1/144*e-5/432*f-7/432*g-1/48*h-11/432*i)/(-1+x)-1/2*(-1/216*d
-1/216*e-1/216*f-1/216*g-1/216*h-1/216*i)/(-1+x)^2-(-1/432*d+1/144*e-5/432*
f+7/432*g-1/48*h+11/432*i)/(1+x)-1/2*(1/216*d-1/216*e+1/216*f-1/216*g+1/216
*h-1/216*i)/(1+x)^2+(13/1296*d-1/81*e+25/1296*f-5/162*g+61/1296*h-11/162*i)
*ln(1+x)
```

Maxima [A]
time = 0.28, size = 230, normalized size = 0.96

$$-\frac{1}{41472}(313d+820f-1280g+1936h-512e-2816i)\log(x+2)+\frac{1}{1296}(13d+25f-40g+61h-16e-88i)\log(x+1)-\frac{1}{1296}(13d+25f+40g+61h+16e+88i)\log(x-1)+\frac{1}{41472}(313d+820f+1280g+1936h+512e+2816i)\log(x-2)+\frac{5(7d+28f+64h)x^7+64(5g+2e+11i)x^6-18(13d+60f+136h)x^5-480(5g+2e+11i)x^4+63(5d+36f+80h)x^3+192(25g+10e+52i)x^2+4(43d-260f-656h)x-2432g-800e-5120i}{3456(x^8-10x^6+33x^4-40x^2+16)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="max
ima")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e - 2816*I)*log(x + 2) + 1/
1296*(13*d + 25*f - 40*g + 61*h - 16*e - 88*I)*log(x + 1) - 1/1296*(13*d +
25*f + 40*g + 61*h + 16*e + 88*I)*log(x - 1) + 1/41472*(313*d + 820*f + 128
0*g + 1936*h + 512*e + 2816*I)*log(x - 2) + 1/3456*(5*(7*d + 28*f + 64*h)*x
^7 + 64*(5*g + 2*e + 11*I)*x^6 - 18*(13*d + 60*f + 136*h)*x^5 - 480*(5*g +
2*e + 11*I)*x^4 + 63*(5*d + 36*f + 80*h)*x^3 + 192*(25*g + 10*e + 52*I)*x^2
 + 4*(43*d - 260*f - 656*h)*x - 2432*g - 800*e - 5120*I)/(x^8 - 10*x^6 + 33
*x^4 - 40*x^2 + 16)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(219) = 438.
time = 6.68, size = 616, normalized size = 2.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="fri
cas")
```

```
[Out] 1/41472*(60*(7*d + 28*f + 64*h)*x^7 + 768*(2*e + 5*g + 11*i)*x^6 - 216*(13*
d + 60*f + 136*h)*x^5 - 5760*(2*e + 5*g + 11*i)*x^4 + 756*(5*d + 36*f + 80*
h)*x^3 + 2304*(10*e + 25*g + 52*i)*x^2 + 48*(43*d - 260*f - 656*h)*x - ((31
3*d - 512*e + 820*f - 1280*g + 1936*h - 2816*i)*x^8 - 10*(313*d - 512*e + 8
20*f - 1280*g + 1936*h - 2816*i)*x^6 + 33*(313*d - 512*e + 820*f - 1280*g +
 1936*h - 2816*i)*x^4 - 40*(313*d - 512*e + 820*f - 1280*g + 1936*h - 2816*
i)*x^2 + 5008*d - 8192*e + 13120*f - 20480*g + 30976*h - 45056*i)*log(x + 2
) + 32*((13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^8 - 10*(13*d - 16*e + 2
5*f - 40*g + 61*h - 88*i)*x^6 + 33*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i
)*x^4 - 40*(13*d - 16*e + 25*f - 40*g + 61*h - 88*i)*x^2 + 208*d - 256*e +
400*f - 640*g + 976*h - 1408*i)*log(x + 1) - 32*((13*d + 16*e + 25*f + 40*g
 + 61*h + 88*i)*x^8 - 10*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^6 + 33
*(13*d + 16*e + 25*f + 40*g + 61*h + 88*i)*x^4 - 40*(13*d + 16*e + 25*f + 4
0*g + 61*h + 88*i)*x^2 + 208*d + 256*e + 400*f + 640*g + 976*h + 1408*i)*lo
g(x - 1) + ((313*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^8 - 10*(31
3*d + 512*e + 820*f + 1280*g + 1936*h + 2816*i)*x^6 + 33*(313*d + 512*e + 8
20*f + 1280*g + 1936*h + 2816*i)*x^4 - 40*(313*d + 512*e + 820*f + 1280*g +
 1936*h + 2816*i)*x^2 + 5008*d + 8192*e + 13120*f + 20480*g + 30976*h + 450
56*i)*log(x - 2) - 9600*e - 29184*g - 61440*i)/(x^8 - 10*x^6 + 33*x^4 - 40*
x^2 + 16)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.04, size = 244, normalized size = 1.02

$$-\frac{1}{41472}\,(313\,d+820\,f-1280\,g+1936\,h-512\,e-2816i)\log\left(|x+2|\right)+\frac{1}{1296}\,(13\,d+25\,f-40\,g+61\,h-16\,e-88i)\log\left(|x+1|\right)-\frac{1}{1296}\,(13\,d+25\,f+40\,g+61\,h+16\,e+88i)\log\left(|x-1|\right)+\frac{1}{41472}\,(313\,d+820\,f+1280\,g+1936\,h+512\,e+2816i)\log\left(|x-2|\right)+\frac{35\,dx^7+140\,fx^7+320\,hx^7+320\,gx^6+128\,x^6e-234\,dx^5-1080\,fx^5-2448\,hx^5+704i\,x^6-2400\,gx^4-960\,x^4e+315\,dx^3+2268\,fx^3+5040\,hx^3-5280i\,x^4+4800\,gx^2+1920\,x^2e+172\,dx-1040\,fx-2624\,hx+9984i\,x^2-2432\,g-800\,e-5120i}{3456\,(x^4-5\,x^2+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^3,x, algorithm="gia
c")
```

```
[Out] -1/41472*(313*d + 820*f - 1280*g + 1936*h - 512*e - 2816*I)*log(abs(x + 2))
 + 1/1296*(13*d + 25*f - 40*g + 61*h - 16*e - 88*I)*log(abs(x + 1)) - 1/129
6*(13*d + 25*f + 40*g + 61*h + 16*e + 88*I)*log(abs(x - 1)) + 1/41472*(313*
```

```
d + 820*f + 1280*g + 1936*h + 512*e + 2816*I)*log(abs(x - 2)) + 1/3456*(35*
d*x^7 + 140*f*x^7 + 320*h*x^7 + 320*g*x^6 + 128*x^6*e - 234*d*x^5 - 1080*f*
x^5 - 2448*h*x^5 + 704*I*x^6 - 2400*g*x^4 - 960*x^4*e + 315*d*x^3 + 2268*f*
x^3 + 5040*h*x^3 - 5280*I*x^4 + 4800*g*x^2 + 1920*x^2*e + 172*d*x - 1040*f*
x - 2624*h*x + 9984*I*x^2 - 2432*g - 800*e - 5120*I)/(x^4 - 5*x^2 + 4)^2
```

Mupad [B]
time = 0.62, size = 233, normalized size = 0.97

$$\ln(x+1)\left(\frac{13\,d}{1296}-\frac{e}{81}+\frac{25\,f}{1296}-\frac{5\,g}{162}+\frac{61\,h}{1296}-\frac{11\,i}{162}\right)-\ln(x-1)\left(\frac{13\,d}{1296}+\frac{e}{81}+\frac{25\,f}{1296}+\frac{5\,g}{162}+\frac{61\,h}{1296}+\frac{11\,i}{162}\right)-\frac{\left(-\frac{35\,d}{3456}-\frac{35\,f}{864}-\frac{5\,h}{54}\right)x^7+\left(-\frac{e}{27}-\frac{5\,g}{54}-\frac{11\,i}{54}\right)x^6+\left(\frac{13\,d}{192}+\frac{5\,f}{16}+\frac{17\,h}{24}\right)x^5+\left(\frac{5\,e}{18}+\frac{25\,g}{36}+\frac{55\,i}{36}\right)x^4+\left(-\frac{35\,d}{384}-\frac{21\,f}{32}-\frac{35\,h}{24}\right)x^3+\left(-\frac{5\,e}{9}-\frac{25\,g}{18}-\frac{26\,i}{9}\right)x^2+\left(\frac{65\,f}{216}-\frac{43\,d}{864}+\frac{41\,h}{54}\right)x+\frac{25\,e}{108}+\frac{19\,g}{27}+\frac{40\,i}{27}}{x^8-10\,x^6+33\,x^4-40\,x^2+16}+\ln(x-2)\left(\frac{313\,d}{41472}+\frac{e}{81}+\frac{205\,f}{10368}+\frac{5\,g}{162}+\frac{121\,h}{2592}+\frac{11\,i}{162}\right)-\ln(x+2)\left(\frac{313\,d}{41472}-\frac{e}{81}+\frac{205\,f}{10368}-\frac{5\,g}{162}+\frac{121\,h}{2592}-\frac{11\,i}{162}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^4 - 5*x^2 + 4)^3,x)
```

```
[Out] log(x + 1)*((13*d)/1296 - e/81 + (25*f)/1296 - (5*g)/162 + (61*h)/1296 - (1
1*i)/162) - log(x - 1)*((13*d)/1296 + e/81 + (25*f)/1296 + (5*g)/162 + (61*
h)/1296 + (11*i)/162) - ((25*e)/108 + (19*g)/27 + (40*i)/27 + x*((65*f)/216
 - (43*d)/864 + (41*h)/54) + x^5*((13*d)/192 + (5*f)/16 + (17*h)/24) - x^3*
((35*d)/384 + (21*f)/32 + (35*h)/24) - x^7*((35*d)/3456 + (35*f)/864 + (5*h
)/54) - x^2*((5*e)/9 + (25*g)/18 + (26*i)/9) - x^6*(e/27 + (5*g)/54 + (11*i
)/54) + x^4*((5*e)/18 + (25*g)/36 + (55*i)/36))/(33*x^4 - 40*x^2 - 10*x^6 +
 x^8 + 16) + log(x - 2)*((313*d)/41472 + e/81 + (205*f)/10368 + (5*g)/162 +
 (121*h)/2592 + (11*i)/162) - log(x + 2)*((313*d)/41472 - e/81 + (205*f)/10
368 - (5*g)/162 + (121*h)/2592 - (11*i)/162)
```

3.47 $\int \frac{d+ex}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=185

$$\frac{dx(1-x^2)}{12(1+x^2+x^4)^2}+\frac{e(1+2x^2)}{12(1+x^2+x^4)^2}+\frac{dx(2-7x^2)}{24(1+x^2+x^4)}+\frac{e(1+2x^2)}{6(1+x^2+x^4)}-\frac{13d\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\tan^{-1}\left(\right.}{48\sqrt{3}}$$

```
[Out] 1/12*d*x*(-x^2+1)/(x^4+x^2+1)^2+1/12*e*(2*x^2+1)/(x^4+x^2+1)^2+1/24*d*x*(-7
*x^2+2)/(x^4+x^2+1)+1/6*e*(2*x^2+1)/(x^4+x^2+1)-9/32*d*ln(x^2-x+1)+9/32*d*l
n(x^2+x+1)-13/144*d*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+13/144*d*arctan(1/3
*(1+2*x)*3^(1/2))*3^(1/2)+2/9*e*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {1687, 12, 1106, 1192, 1183, 648, 632, 210, 642, 1121, 628}

$$-\frac{13d\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{13d\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{48\sqrt{3}}+\frac{2e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{9}{32}d\log\left(x^2-x+1\right)+\frac{9}{32}d\log\left(x^2+x+1\right)+\frac{dx(2-7x^2)}{24(x^4+x^2+1)}+\frac{dx(1-x^2)}{12(x^4+x^2+1)^2}+\frac{e(2x^2+1)}{6(x^4+x^2+1)}+\frac{e(2x^2+1)}{12(x^4+x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (d*x*(1 - x^2))/(12*(1 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4
)^2) + (d*x*(2 - 7*x^2))/(24*(1 + x^2 + x^4)) + (e*(1 + 2*x^2))/(6*(1 + x^2
 + x^4)) - (13*d*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (13*d*ArcTan[(1
+ 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3
]) - (9*d*Log[1 - x + x^2])/32 + (9*d*Log[1 + x + x^2])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
```

```
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex}{(1+x^2+x^4)^3}\,dx = \int \frac{d}{(1+x^2+x^4)^3}\,dx + \int \frac{ex}{(1+x^2+x^4)^3}\,dx$$

$$= d\int \frac{1}{(1+x^2+x^4)^3}\,dx + e\int \frac{x}{(1+x^2+x^4)^3}\,dx$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{1}{12}d\int \frac{11-5x^2}{(1+x^2+x^4)^2}\,dx + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{(1+x+x^2)^3}\,dx, x, x\right.$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{dx(2-7x^2)}{24(1+x^2+x^4)} + \frac{1}{72}d\int \frac{60-21x^2}{1+x^2+x^4}\,dx$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{dx(2-7x^2)}{24(1+x^2+x^4)} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{1}{144}$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{dx(2-7x^2)}{24(1+x^2+x^4)} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{1}{96}(1$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{dx(2-7x^2)}{24(1+x^2+x^4)} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{2e\,\text{ta}}{}$$

$$= \frac{dx(1-x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{dx(2-7x^2)}{24(1+x^2+x^4)} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} - \frac{13d\,\text{t}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 186, normalized size = 1.01

$$\frac{1}{144}\left(\frac{6(dx(2-7x^2)+e(4+8x^2))}{1+x^2+x^4} + \frac{12(e+2ex^2+d(x-x^3))}{(1+x^2+x^4)^2} - \frac{\left(-47i+7\sqrt{3}\right)d\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}} - \frac{\left(47i+7\sqrt{3}\right)d\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}} - 32\sqrt{3}\,e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(d*x*(2 - 7*x^2) + e*(4 + 8*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 +
 d*(x - x^3)))/(1 + x^2 + x^4)^2 - ((-47*I + 7*Sqrt[3])*d*ArcTan[((-I + Sqr
t[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - ((47*I + 7*Sqrt[3])*d*ArcTan[((I + S
qrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[3]/(1 + 2
*x^2)])/144
```

Maple [A]
time = 0.08, size = 158, normalized size = 0.85

method	result
default	$\frac{\left(-\frac{7d}{3}-\frac{4e}{3}\right)x^3-6d\,x^2+\left(-\frac{20d}{3}+\frac{e}{3}\right)x-4d+2e}{16(x^2+x+1)^2}+\frac{9d\ln(x^2+x+1)}{32}+\frac{\left(\frac{13d}{2}-16e\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{72}-\frac{\left(\frac{7d}{3}-\frac{4e}{3}\right)x^3-6d\,x^{}}{16(x^{}}$
risch	$\frac{13\sqrt{3}\,d\arctan\left(\frac{1458d^2x\sqrt{3}}{2187d^2+1024e^2}+\frac{2048e^2x\sqrt{3}}{3\left(2187d^2+1024e^2\right)}+\frac{729\sqrt{3}\,d^2}{2187d^2+1024e^2}+\frac{1024e^2\sqrt{3}}{3\left(2187d^2+1024e^2\right)}\right)}{144}+\frac{-\frac{7}{24}x^7d+\frac{1}{3}e\,x^6-\frac{5}{24}d\,x^5+\frac{1}{2}e\,x^4-\frac{7}{24}d}{(x^4+x^2+1)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-7/3*d-4/3*e)*x^3-6*d*x^2+(-20/3*d+1/3*e)*x-4*d+2*e)/(x^2+x+1)^2+9/3
2*d*ln(x^2+x+1)+1/72*(13/2*d-16*e)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/16
*((7/3*d-4/3*e)*x^3-6*d*x^2+(20/3*d+1/3*e)*x-4*d-2*e)/(x^2-x+1)^2-9/32*d*ln
(x^2-x+1)-1/72*(-13/2*d-16*e)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 143, normalized size = 0.77

$$\frac{1}{144}\sqrt{3}\,(13\,d-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\,(13\,d+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{9}{32}\,d\log\left(x^2+x+1\right)-\frac{9}{32}\,d\log\left(x^2-x+1\right)-\frac{7\,dx^7-8\,x^6e+5\,dx^5-12\,x^4e+7\,dx^3-16\,x^2e-4\,dx-6\,e}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqrt(3)*(
13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*log(x^2 + x + 1) - 9/32
*d*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*x^6*e + 5*d*x^5 - 12*x^4*e + 7*d*x^
3 - 16*x^2*e - 4*d*x - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A]
time = 0.44, size = 278, normalized size = 1.50

$$-\frac{84\,dx^7-96\,ex^6+60\,dx^5-144\,ex^4+84\,dx^3-192\,ex^2-2\,\sqrt{3}\,((13\,d-32\,e)x^8+2\,(13\,d-32\,e)x^6+3\,(13\,d-32\,e)x^4+2\,(13\,d-32\,e)x^2+13\,d-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-2\,\sqrt{3}\,((13\,d+32\,e)x^8+2\,(13\,d+32\,e)x^6+3\,(13\,d+32\,e)x^4+2\,(13\,d+32\,e)x^2+13\,d+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-48\,dx-81\,(dx^8+2\,dx^6+3\,dx^4+2\,dx^2+d)\log\left(x^2+x+1\right)+81\,(dx^8+2\,dx^6+3\,dx^4+2\,dx^2+d)\log\left(x^2-x+1\right)-72\,e}{288\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*d*x^7 - 96*e*x^6 + 60*d*x^5 - 144*e*x^4 + 84*d*x^3 - 192*e*x^2 -
 2*sqrt(3)*((13*d - 32*e)*x^8 + 2*(13*d - 32*e)*x^6 + 3*(13*d - 32*e)*x^4 +
 2*(13*d - 32*e)*x^2 + 13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(
3)*((13*d + 32*e)*x^8 + 2*(13*d + 32*e)*x^6 + 3*(13*d + 32*e)*x^4 + 2*(13*d
 + 32*e)*x^2 + 13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) - 48*d*x - 81*(d*
x^8 + 2*d*x^6 + 3*d*x^4 + 2*d*x^2 + d)*log(x^2 + x + 1) + 81*(d*x^8 + 2*d*x
^6 + 3*d*x^4 + 2*d*x^2 + d)*log(x^2 - x + 1) - 72*e)/(x^8 + 2*x^6 + 3*x^4 +
 2*x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.16, size = 1103, normalized size = 5.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-1025428432*d**4*e - 33475
2912*d**4*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288) - 431308800*d**2*e**3 - 3
143688192*d**2*e**2*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288) + 9917005824*d*
*2*e*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/288)**2 + 11878244352*d**2*(-9*d/32
 - sqrt(3)*I*(13*d + 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/
32 - sqrt(3)*I*(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 - sqrt(3)*I*(1
3*d + 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 - sqrt(3)*I*(13*d + 32*e)/2
88)**3)/(217696167*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (-9*d
/32 + sqrt(3)*I*(13*d + 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*
d**4*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288) - 431308800*d**2*e**3 - 314368
8192*d**2*e**2*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288) + 9917005824*d**2*e*
(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288)**2 + 11878244352*d**2*(-9*d/32 + sq
rt(3)*I*(13*d + 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(-9*d/32 +
sqrt(3)*I*(13*d + 32*e)/288) + 3850371072*e**3*(-9*d/32 + sqrt(3)*I*(13*d +
 32*e)/288)**2 + 20384317440*e**2*(-9*d/32 + sqrt(3)*I*(13*d + 32*e)/288)**
3)/(217696167*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (9*d/32 -
sqrt(3)*I*(13*d - 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*d**4*(
9*d/32 - sqrt(3)*I*(13*d - 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d*
*2*e**2*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32
- sqrt(3)*I*(13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 - sqrt(3)*I*(1
3*d - 32*e)/288)**3 + 142606336*e**5 + 754974720*e**4*(9*d/32 - sqrt(3)*I*(
13*d - 32*e)/288) + 3850371072*e**3*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288)*
*2 + 20384317440*e**2*(9*d/32 - sqrt(3)*I*(13*d - 32*e)/288)**3)/(217696167
*d**5 - 1217128448*d**3*e**2 - 617611264*d*e**4)) + (9*d/32 + sqrt(3)*I*(13
*d - 32*e)/288)*log(x + (-1025428432*d**4*e - 334752912*d**4*(9*d/32 + sqrt
```

```
(3)*I*(13*d - 32*e)/288) - 431308800*d**2*e**3 - 3143688192*d**2*e**2*(9*d/
32 + sqrt(3)*I*(13*d - 32*e)/288) + 9917005824*d**2*e*(9*d/32 + sqrt(3)*I*(
13*d - 32*e)/288)**2 + 11878244352*d**2*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/2
88)**3 + 142606336*e**5 + 754974720*e**4*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/
288) + 3850371072*e**3*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/288)**2 + 20384317
440*e**2*(9*d/32 + sqrt(3)*I*(13*d - 32*e)/288)**3)/(217696167*d**5 - 12171
28448*d**3*e**2 - 617611264*d*e**4)) + (-7*d*x**7 - 5*d*x**5 - 7*d*x**3 + 4
*d*x + 8*e*x**6 + 12*e*x**4 + 16*e*x**2 + 6*e)/(24*x**8 + 48*x**6 + 72*x**4
 + 48*x**2 + 24)
```

Giac [A]
time = 5.78, size = 131, normalized size = 0.71

$$\frac{1}{144}\sqrt{3}\,(13\,d-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\,(13\,d+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{9}{32}\,d\log\left(x^2+x+1\right)-\frac{9}{32}\,d\log\left(x^2-x+1\right)-\frac{7\,dx^7-8\,x^6e+5\,dx^5-12\,x^4e+7\,dx^3-16\,x^2e-4\,dx-6\,e}{24\,\left(x^4+x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqrt(3)*(
13*d + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/32*d*log(x^2 + x + 1) - 9/32
*d*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 8*x^6*e + 5*d*x^5 - 12*x^4*e + 7*d*x^
3 - 16*x^2*e - 4*d*x - 6*e)/(x^4 + x^2 + 1)^2
```

Mupad [B]
time = 0.26, size = 185, normalized size = 1.00

$$\frac{-\frac{7\,d\,x^7}{24}+\frac{e\,x^6}{3}-\frac{5\,d\,x^5}{24}+\frac{e\,x^4}{2}-\frac{7\,d\,x^3}{24}+\frac{2\,e\,x^2}{3}+\frac{d\,x}{6}+\frac{e}{4}}{x^8+2\,x^6+3\,x^4+2\,x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9\,d}{32}-\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(x^2 + x^4 + 1)^3,x)
```

```
[Out] (e/4 + (d*x)/6 - (7*d*x^3)/24 - (5*d*x^5)/24 - (7*d*x^7)/24 + (2*e*x^2)/3 +
 (e*x^4)/2 + (e*x^6)/3)/(2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(x - (3^(1/2
)*1i)/2 - 1/2)*((9*d)/32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9) + log(x
- (3^(1/2)*1i)/2 + 1/2)*((9*d)/32 - (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9)
 + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*d*13i)/288 - (9*d)/32 + (3^(1/2)
*e*1i)/9) + log(x + (3^(1/2)*1i)/2 + 1/2)*((9*d)/32 + (3^(1/2)*d*13i)/288 -
 (3^(1/2)*e*1i)/9)
```

3.48 $\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3} dx$

Optimal. Leaf size=223

$$\frac{e(1+2x^2)}{12(1+x^2+x^4)^2}+\frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2}+\frac{e(1+2x^2)}{6(1+x^2+x^4)}+\frac{x(2d+3f-7(d-f)x^2)}{24(1+x^2+x^4)}-\frac{(13d+2f)\tan^{-1}\left(\right.}{48\sqrt{3}}$$

```
[Out] 1/12*e*(2*x^2+1)/(x^4+x^2+1)^2+1/12*x*(d+f-(d-2*f)*x^2)/(x^4+x^2+1)^2+1/6*e
*(2*x^2+1)/(x^4+x^2+1)+1/24*x*(2*d+3*f-7*(d-f)*x^2)/(x^4+x^2+1)-1/32*(9*d-4
*f)*ln(x^2-x+1)+1/32*(9*d-4*f)*ln(x^2+x+1)-1/144*(13*d+2*f)*arctan(1/3*(1-2
*x)*3^(1/2))*3^(1/2)+1/144*(13*d+2*f)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+2
/9*e*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1687, 1192, 1183, 648, 632, 210, 642, 12, 1121, 628}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f)}{48\sqrt{3}}+\frac{2e\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x(-7x^2(d-f)+2d+3f)}{24(x^4+x^2+1)}+\frac{x(-(x^2(d-2f))+d+f)}{12(x^4+x^2+1)^2}+\frac{e(2x^2+1)}{6(x^4+x^2+1)}+\frac{e(2x^2+1)}{12(x^4+x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (e*(1 + 2*x^2))/(12*(1 + x^2 + x^4)^2) + (x*(d + f - (d - 2*f)*x^2))/(12*(1
 + x^2 + x^4)^2) + (e*(1 + 2*x^2))/(6*(1 + x^2 + x^4)) + (x*(2*d + 3*f - 7*
(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTan[(1 - 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + (
2*e*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f)*Log[1 - x + x^2
])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
```

```
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{(1+x^2+x^4)^3}\,dx &= \int \frac{ex}{(1+x^2+x^4)^3}\,dx + \int \frac{d+fx^2}{(1+x^2+x^4)^3}\,dx \\
&= \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{1}{12}\int \frac{11d-f-5(d-2f)x^2}{(1+x^2+x^4)^2}\,dx + e\int \frac{x}{(1+x^2+x^4)^3}\,dx \\
&= \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{x(2d+3f-7(d-f)x^2)}{24(1+x^2+x^4)} + \frac{1}{72}\int \frac{15(4d-f)-21(d-f}{1+x^2+x^4} \\
&= \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{x(2d+3f-7(d-f)x^2)}{24(1+x^2+x^4)} + \frac{1}{144}\int \frac{1}{} \\
&= \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{x(2d+3f-7(d-}{24(1+x^2+x} \\
&= \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{x(2d+3f-7(d-}{24(1+x^2+x} \\
&= \frac{e(1+2x^2)}{12(1+x^2+x^4)^2} + \frac{x(d+f-(d-2f)x^2)}{12(1+x^2+x^4)^2} + \frac{e(1+2x^2)}{6(1+x^2+x^4)} + \frac{x(2d+3f-7(d-}{24(1+x^2+x}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 235, normalized size = 1.05

$$\frac{1}{144}\left(\frac{6(2dx+3fx-7dx^3+7fx^3+e(4+8x^2))}{1+x^2+x^4}+\frac{12(e+2ex^2+x(d+f-dx^2+2fx^2))}{(1+x^2+x^4)^2}-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-32\sqrt{3}\,e\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 + e*(4 + 8*x^2)))/(1 + x^2 + x^4) +
(12*(e + 2*e*x^2 + x*(d + f - d*x^2 + 2*f*x^2)))/(1 + x^2 + x^4)^2 - (((-47
*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqr
t[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f)*ArcTa
n[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 32*Sqrt[3]*e*ArcTan[Sqrt[
3]/(1 + 2*x^2)])/144
```

Maple [A]
time = 0.12, size = 202, normalized size = 0.91

method	result
default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}\right)x^3+(-6d+4f)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}\right)x-4d+\frac{4f}{3}+2e}{16(x^2+x+1)^2}+\frac{(27d-12f)\ln(x^2+x+1)}{96}+\frac{\left(\frac{13d}{2}-16e+f\right)\arctan\left(\frac{(2x+1)\sqrt{}}{3}\right.}{72}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e)*x^3+(-6*d+4*f)*x^2+(-20/3*d+13/3*f+1/3*e)*x-4*d+
4/3*f+2*e)/(x^2+x+1)^2+1/96*(27*d-12*f)*ln(x^2+x+1)+1/72*(13/2*d-16*e+f)*ar
ctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/16*((7/3*d-7/3*f-4/3*e)*x^3+(-6*d+4*f)*
x^2+(20/3*d-13/3*f+1/3*e)*x-4*d+4/3*f-2*e)/(x^2-x+1)^2-1/96*(27*d-12*f)*ln(
x^2-x+1)-1/72*(-13/2*d-16*e-f)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 179, normalized size = 0.80

$$\frac{1}{144}\sqrt{3}(13d+2f-32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}(13d+2f+32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)-\frac{7(d-f)x^7-8x^6e+5(d-2f)x^5-12x^4e+7(d-2f)x^3-16x^2e-(4d+5f)x-6e}{24(x^8+2x^6+3x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqr
t(3)*(13*d + 2*f + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)*l
og(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*(d - f)*x^7 -
 8*x^6*e + 5*(d - 2*f)*x^5 - 12*x^4*e + 7*(d - 2*f)*x^3 - 16*x^2*e - (4*d +
 5*f)*x - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [A]
time = 0.50, size = 384, normalized size = 1.72

$$-\frac{84(d-f)x^7-96ex^6+60(d-2f)x^5-144ex^4+84(d-2f)x^3-192ex^2-2\sqrt{3}((13d-32e+2f)x^8+2(13d-32e+2f)x^6+3(13d-32e+2f)x^4+2(13d-32e+2f)x^2+13d-32e+2f)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}((13d+32e+2f)x^8+2(13d+32e+2f)x^6+3(13d+32e+2f)x^4+2(13d+32e+2f)x^2+13d+32e+2f)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(4d+5f)x-9((9d-4f)x^8+2(9d-4f)x^6+3(9d-4f)x^4+2(9d-4f)x^2+9d-4f)\log(x^2+x+1)+9((9d-4f)x^8+2(9d-4f)x^6+3(9d-4f)x^4+2(9d-4f)x^2+9d-4f)\log(x^2-x+1)-72e}{288(x^8+2x^6+3x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*(d - f)*x^7 - 96*e*x^6 + 60*(d - 2*f)*x^5 - 144*e*x^4 + 84*(d -
2*f)*x^3 - 192*e*x^2 - 2*sqrt(3)*((13*d - 32*e + 2*f)*x^8 + 2*(13*d - 32*e
+ 2*f)*x^6 + 3*(13*d - 32*e + 2*f)*x^4 + 2*(13*d - 32*e + 2*f)*x^2 + 13*d -
 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f)
*x^8 + 2*(13*d + 32*e + 2*f)*x^6 + 3*(13*d + 32*e + 2*f)*x^4 + 2*(13*d + 32
```

```
*e + 2*f)*x^2 + 13*d + 32*e + 2*f)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(4*d
+ 5*f)*x - 9*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(
9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1) + 9*((9*d - 4*f)*x^8 + 2*(9*d
- 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 - x
 + 1) - 72*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 142.25, size = 4496, normalized size = 20.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] (-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-1025428432*d*
*5*e - 334752912*d**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) +
 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d +
 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(-9*d/32 + f
/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 99170058
24*d**3*e*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 94430016
0*d**3*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 118782443
52*d**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 233164800*
d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*
e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(-9*d/32 + f
/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 754974720*d*e
**4*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1843200*d*e**3*f*
*2 + 3850371072*d*e**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)*
*2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/
288) + 20384317440*d*e**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/28
8)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 - sqrt(3)*I
*(13*d + 32*e + 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 - sqrt(3)*I*(
13*d + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 - sqrt(3)*I*(13*
d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(-9*d/32 + f/8
 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e**3*f**3 - 2151677952*e**3
*f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 287096832*e**2*
f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 5096079360*e**2*
f*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e*f**5
- 859521024*e*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 -
 7648128*f**5*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 4538695
68*f**3*(-9*d/32 + f/8 - sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3)/(217696167*
d**6 - 301346487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2 + 2181
281792*d**3*e**2*f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 1450149888*d
**2*e**2*f**2 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*e**2*f
```

```
**3 + 783648*d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 188352*f**
6)) + (-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)*log(x + (-1025428
432*d**5*e - 334752912*d**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/
288) + 2008961360*d**4*e*f + 1151575920*d**4*f*(-9*d/32 + f/8 + sqrt(3)*I*(
13*d + 32*e + 2*f)/288) - 431308800*d**3*e**3 - 3143688192*d**3*e**2*(-9*d/
32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1598857120*d**3*e*f**2 + 99
17005824*d**3*e*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 - 94
4300160*d**3*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 118
78244352*d**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 2331
64800*d**2*e**3*f + 4409634816*d**2*e**2*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d
 + 32*e + 2*f)/288) + 662937520*d**2*e*f**3 - 13004623872*d**2*e*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 231796080*d**2*f**3*(-9*d/
32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 10089639936*d**2*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 142606336*d*e**5 + 7549747
20*d*e**4*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 1843200*d*e
**3*f**2 + 3850371072*d*e**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)
/288)**2 - 1926291456*d*e**2*f**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e +
 2*f)/288) + 20384317440*d*e**2*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2
*f)/288)**3 - 146756960*d*e*f**4 + 5813379072*d*e*f**2*(-9*d/32 + f/8 + sqr
t(3)*I*(13*d + 32*e + 2*f)/288)**2 + 12679200*d*f**4*(-9*d/32 + f/8 + sqrt(
3)*I*(13*d + 32*e + 2*f)/288) + 1116758016*d*f**2*(-9*d/32 + f/8 + sqrt(3)*
I*(13*d + 32*e + 2*f)/288)**3 - 79691776*e**5*f - 188743680*e**4*f*(-9*d/32
 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 7372800*e**3*f**3 - 215167795
2*e**3*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**2 + 287096832
*e**2*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) - 5096079360
*e**2*f*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3 + 14093632*e
*f**5 - 859521024*e*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288
)**2 - 7648128*f**5*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288) + 4
53869568*f**3*(-9*d/32 + f/8 + sqrt(3)*I*(13*d + 32*e + 2*f)/288)**3)/(2176
96167*d**6 - 301346487*d**5*f - 1217128448*d**4*e**2 + 130506255*d**4*f**2
+ 2181281792*d**3*e**2*f - 5619240*d**3*f**3 - 617611264*d**2*e**4 - 145014
9888*d**2*e**2*f**2 - 8036820*d**2*f**4 + 495976448*d*e**4*f + 430088192*d*
e**2*f**3 + 783648*d*f**5 - 114294784*e**4*f**2 - 47771648*e**2*f**4 + 1883
52*f**6)) + (9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2*f)/288)*log(x + (-10
25428432*d**5*e - 334752912*d**5*(9*d/32 - f/8 - sqrt(3)*I*(13*d - 32*e + 2
*f)/288) + 2008961360*d**4*e*f + 1151575920*d**...
```

<u>Giac</u> [A]
time = 4.01, size = 171, normalized size = 0.77

$$\frac{1}{144}\,\sqrt{3}\,(13\,d+2\,f-32\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\,\sqrt{3}\,(13\,d+2\,f+32\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{32}\,(9\,d-4\,f)\log\left(x^2+x+1\right)-\frac{1}{32}\,(9\,d-4\,f)\log\left(x^2-x+1\right)-\frac{7\,dx^7-7\,fx^7-8\,x^6e+5\,dx^5-10\,fx^5-12\,x^4e+7\,dx^3-14\,fx^3-16\,x^2e-4\,dx-5\,fx-6\,e}{24\,(x^4+x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/144*sqr
t(3)*(13*d + 2*f + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32*(9*d - 4*f)*l
og(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x
^7 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 - 16*x^2*
e - 4*d*x - 5*f*x - 6*e)/(x^4 + x^2 + 1)^2
```

Mupad [B]
time = 1.01, size = 249, normalized size = 1.12

$$\frac{\left(\frac{7f}{24}-\frac{7d}{24}\right)x^7+\frac{ex^6}{3}+\left(\frac{5f}{12}-\frac{5d}{24}\right)x^5+\frac{ex^4}{2}+\left(\frac{7f}{12}-\frac{7d}{24}\right)x^3+\frac{2ex^2}{3}+\left(\frac{d}{6}+\frac{5f}{24}\right)x+\frac{e}{4}}{x^8+2x^6+3x^4+2x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{9d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{f}{8}-\frac{9d}{32}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{9d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1\mathrm{i}}{144}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(x^2 + x^4 + 1)^3,x)
```

```
[Out] (e/4 - x^5*((5*d)/24 - (5*f)/12) - x^3*((7*d)/24 - (7*f)/12) - x^7*((7*d)/2
4 - (7*f)/24) + (2*e*x^2)/3 + (e*x^4)/2 + (e*x^6)/3 + x*(d/6 + (5*f)/24))/(
2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((9*d)/32
- f/8 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144) - log(
x - (3^(1/2)*1i)/2 + 1/2)*(f/8 - (9*d)/32 + (3^(1/2)*d*13i)/288 - (3^(1/2)*
e*1i)/9 + (3^(1/2)*f*1i)/144) + log(x + (3^(1/2)*1i)/2 - 1/2)*(f/8 - (9*d)/
32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144) + log(x +
 (3^(1/2)*1i)/2 + 1/2)*((9*d)/32 - f/8 + (3^(1/2)*d*13i)/288 - (3^(1/2)*e*1
i)/9 + (3^(1/2)*f*1i)/144)
```

3.49 $\int \frac{d+ex+fx^2+gx^3}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=243

$$\frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2}+\frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2}+\frac{(2e-g)\,(1+2x^2)}{12\,(1+x^2+x^4)}+\frac{x(2d+3f-7(d-f)x^2)}{24\,(1+x^2+x^4)}-\frac{(13d+2f)\tan}{48\sqrt{}}$$

```
[Out] 1/12*x*(d+f-(d-2*f)*x^2)/(x^4+x^2+1)^2+1/12*(e-2*g+(2*e-g)*x^2)/(x^4+x^2+1)
^2+1/12*(2*e-g)*(2*x^2+1)/(x^4+x^2+1)+1/24*x*(2*d+3*f-7*(d-f)*x^2)/(x^4+x^2
+1)-1/32*(9*d-4*f)*ln(x^2-x+1)+1/32*(9*d-4*f)*ln(x^2+x+1)-1/144*(13*d+2*f)*
arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/144*(13*d+2*f)*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+1/9*(2*e-g)*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1687, 1192, 1183, 648, 632, 210, 642, 1261, 652, 628}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{3\sqrt{3}}-\frac{1}{32}(9d-4f)\log\left(x^2-x+1\right)+\frac{1}{32}(9d-4f)\log\left(x^2+x+1\right)+\frac{x(-7x^2(d-f)+2d+3f)}{24\left(x^4+x^2+1\right)}+\frac{x(-(x^2(d-2f))+d+f)}{12\left(x^4+x^2+1\right)^2}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (x*(d + f - (d - 2*f)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + (2*e - g)*x
^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2 + x^4)) +
 (x*(2*d + 3*f - 7*(d - f)*x^2))/(24*(1 + x^2 + x^4)) - ((13*d + 2*f)*ArcTa
n[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f)*ArcTan[(1 + 2*x)/Sqrt[3]
])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9
*d - 4*f)*Log[1 - x + x^2])/32 + ((9*d - 4*f)*Log[1 + x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{(1+x^2+x^4)^3}\,dx = \int \frac{d+fx^2}{(1+x^2+x^4)^3}\,dx + \int \frac{x(e+gx^2)}{(1+x^2+x^4)^3}\,dx$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{1}{12}\int \frac{11d-f-5(d-2f)x^2}{(1+x^2+x^4)^2}\,dx + \frac{1}{2}\text{Subst}\bigg(\int \frac{e+}{(1+x}$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(2d+3f-7(d-f)x^2)}{24\,(1+x^2+x^4)} + \frac{1}{72}$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2} + \frac{(2e-g)\,(1+2x^2)}{12\,(1+x^2+x^4)} + \frac{x(2d+3}{24\,(1}$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2} + \frac{(2e-g)\,(1+2x^2)}{12\,(1+x^2+x^4)} + \frac{x(2d+3}{24\,(1}$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2} + \frac{(2e-g)\,(1+2x^2)}{12\,(1+x^2+x^4)} + \frac{x(2d+3}{24\,(1}$$

$$= \frac{x(d+f-(d-2f)x^2)}{12\,(1+x^2+x^4)^2} + \frac{e-2g+(2e-g)x^2}{12\,(1+x^2+x^4)^2} + \frac{(2e-g)\,(1+2x^2)}{12\,(1+x^2+x^4)} + \frac{x(2d+3}{24\,(1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 259, normalized size = 1.07

$$\frac{1}{144}\left(\frac{6(2dx+3fx-7dx^3+7fx^3-2g(1+2x^2)+e(4+8x^2))}{1+x^2+x^4}+\frac{12(e+2ex^2-g(2+x^2)+x(d+f-dx^2+2fx^2))}{(1+x^2+x^4)^2}-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}\,(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((6*(2*d*x + 3*f*x - 7*d*x^3 + 7*f*x^3 - 2*g*(1 + 2*x^2) + e*(4 + 8*x^2)))/
(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g*(2 + x^2) + x*(d + f - d*x^2 + 2*f*x
^2)))/(1 + x^2 + x^4)^2 - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f)*A
rcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])
*d - (17*I + 7*Sqrt[3])*f)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3]
)/6] - 16*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

<u>Maple</u> [A]
time = 0.15, size = 232, normalized size = 0.95

method	result
default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^3+(-6d+4f-2g)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4d+\frac{4f}{3}+2e-2g}{16(x^2+x+1)^2}+\frac{(27d-12f)\ln(x^2+x+1)}{96}+\frac{\left(\frac{13d}{2}-16e+f\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*g)*x^2+(-20/3*d+13/3*f+1/3
*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1)^2+1/96*(27*d-12*f)*ln(x^2+x+1)+1/7
2*(13/2*d-16*e+f+8*g)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/16*((7/3*d-7/3*
f-4/3*e-1/3*g)*x^3+(-6*d+4*f+2*g)*x^2+(20/3*d-13/3*f+1/3*e-8/3*g)*x-4*d+4/3
*f-2*e+2*g)/(x^2-x+1)^2-1/96*(27*d-12*f)*ln(x^2-x+1)-1/72*(-13/2*d-16*e-f+8
*g)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

<u>Maxima</u> [A]
time = 0.49, size = 200, normalized size = 0.82

$$\frac{1}{144}\sqrt{3}(13d+2f+16g-32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{144}\sqrt{3}(13d+2f-16g+32e)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{32}(9d-4f)\log(x^2+x+1)-\frac{1}{32}(9d-4f)\log(x^2-x+1)-\frac{7(d-f)x^7+4(g-2e)x^6+5(d-2f)x^5+6(g-2e)x^4+7(d-2f)x^3+8(g-2e)x^2-(4d+5f)x+6g-6e}{24(x^8+2x^6+3x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/
144*sqrt(3)*(13*d + 2*f - 16*g + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32
*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7
*(d - f)*x^7 + 4*(g - 2*e)*x^6 + 5*(d - 2*f)*x^5 + 6*(g - 2*e)*x^4 + 7*(d -
 2*f)*x^3 + 8*(g - 2*e)*x^2 - (4*d + 5*f)*x + 6*g - 6*e)/(x^8 + 2*x^6 + 3*x
^4 + 2*x^2 + 1)
```

<u>Fricas</u> [A]
time = 0.61, size = 435, normalized size = 1.79

$$-\frac{84(d-f)x^7-48(2e-g)x^6+60(d-2f)x^5-72(2e-g)x^4+84(d-2f)x^3-96(2e-g)x^2-2\sqrt{3}\left((13d-32e+2f+16g)x^8+2(13d-32e+2f+16g)x^6+3(13d-32e+2f+16g)x^4+2(13d-32e+2f+16g)x^2+13d-32e+2f+16g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((13d+32e+2f-16g)x^8+2(13d+32e+2f-16g)x^6+3(13d+32e+2f-16g)x^4+2(13d+32e+2f-16g)x^2+13d+32e+2f-16g\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(4d+5f)x-9\left((9d-4f)x^8+2(9d-4f)x^6+3(9d-4f)x^4+2(9d-4f)x^2+9d-4f\right)\log(x^2+x+1)+9\left((9d-4f)x^8+2(9d-4f)x^6+3(9d-4f)x^4+2(9d-4f)x^2+9d-4f\right)\log(x^2-x+1)-72e+72g}{288(x^8+2x^6+3x^4+2x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(84*(d - f)*x^7 - 48*(2*e - g)*x^6 + 60*(d - 2*f)*x^5 - 72*(2*e - g)
*x^4 + 84*(d - 2*f)*x^3 - 96*(2*e - g)*x^2 - 2*sqrt(3)*((13*d - 32*e + 2*f
+ 16*g)*x^8 + 2*(13*d - 32*e + 2*f + 16*g)*x^6 + 3*(13*d - 32*e + 2*f + 16*
g)*x^4 + 2*(13*d - 32*e + 2*f + 16*g)*x^2 + 13*d - 32*e + 2*f + 16*g)*arcta
n(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f - 16*g)*x^8 + 2*(1
3*d + 32*e + 2*f - 16*g)*x^6 + 3*(13*d + 32*e + 2*f - 16*g)*x^4 + 2*(13*d +
 32*e + 2*f - 16*g)*x^2 + 13*d + 32*e + 2*f - 16*g)*arctan(1/3*sqrt(3)*(2*x
 - 1)) - 12*(4*d + 5*f)*x - 9*((9*d - 4*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d
 - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d - 4*f)*log(x^2 + x + 1) + 9*((9*d - 4
*f)*x^8 + 2*(9*d - 4*f)*x^6 + 3*(9*d - 4*f)*x^4 + 2*(9*d - 4*f)*x^2 + 9*d -
 4*f)*log(x^2 - x + 1) - 72*e + 72*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.62, size = 198, normalized size = 0.81

$$\frac{1}{144}\sqrt{3}\,(13\,d+2\,f+16\,g-32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\,(13\,d+2\,f-16\,g+32\,e)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{32}\,(9\,d-4\,f)\log\left(x^2+x+1\right)-\frac{1}{32}\,(9\,d-4\,f)\log\left(x^2-x+1\right)-\frac{7\,dx^7-7\,fx^7+4\,gx^6-8\,x^6e+5\,dx^5-10\,fx^5+6\,gx^4-12\,x^4e+7\,dx^3-14\,fx^3+8\,gx^2-16\,x^2e-4\,dx-5\,fx+6\,g-6\,e}{24\,\left(x^4+x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/
144*sqrt(3)*(13*d + 2*f - 16*g + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/32
*(9*d - 4*f)*log(x^2 + x + 1) - 1/32*(9*d - 4*f)*log(x^2 - x + 1) - 1/24*(7
*d*x^7 - 7*f*x^7 + 4*g*x^6 - 8*x^6*e + 5*d*x^5 - 10*f*x^5 + 6*g*x^4 - 12*x^
4*e + 7*d*x^3 - 14*f*x^3 + 8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 6*g - 6*e)/
(x^4 + x^2 + 1)^2
```

Mupad [**B**]
time = 1.17, size = 295, normalized size = 1.21

$$\frac{\left(\frac{7\,f}{24}-\frac{7\,d}{24}\right)\,x^7+\left(\frac{e}{3}-\frac{g}{6}\right)\,x^6+\left(\frac{5\,f}{12}-\frac{5\,d}{24}\right)\,x^5+\left(\frac{e}{2}-\frac{g}{4}\right)\,x^4+\left(\frac{7\,f}{12}-\frac{7\,d}{24}\right)\,x^3+\left(\frac{2\,e}{3}-\frac{g}{3}\right)\,x^2+\left(\frac{d}{6}+\frac{5\,f}{24}\right)\,x+\frac{e}{4}-\frac{g}{4}}{x^8+2\,x^6+3\,x^4+2\,x^2+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{9\,d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13{}\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{144}-\frac{\sqrt{3}\,g\,1{}\mathrm{i}}{18}\right)-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{f}{8}-\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13{}\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{144}+\frac{\sqrt{3}\,g\,1{}\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{f}{8}-\frac{9\,d}{32}+\frac{\sqrt{3}\,d\,13{}\mathrm{i}}{288}+\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{144}-\frac{\sqrt{3}\,g\,1{}\mathrm{i}}{18}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{9\,d}{32}-\frac{f}{8}+\frac{\sqrt{3}\,d\,13{}\mathrm{i}}{288}-\frac{\sqrt{3}\,e\,1{}\mathrm{i}}{9}+\frac{\sqrt{3}\,f\,1{}\mathrm{i}}{144}+\frac{\sqrt{3}\,g\,1{}\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2 + x^4 + 1)^3,x)
```

```
[Out] (e/4 - g/4 - x^5*((5*d)/24 - (5*f)/12) - x^3*((7*d)/24 - (7*f)/12) - x^7*((
7*d)/24 - (7*f)/24) + x^2*((2*e)/3 - g/3) + x^4*(e/2 - g/4) + x^6*(e/3 - g/
6) + x*(d/6 + (5*f)/24))/(2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(x - (3^(1/
2)*1i)/2 - 1/2)*((9*d)/32 - f/8 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 +
(3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18) - log(x - (3^(1/2)*1i)/2 + 1/2)*(f/
8 - (9*d)/32 + (3^(1/2)*d*13i)/288 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144
+ (3^(1/2)*g*1i)/18) + log(x + (3^(1/2)*1i)/2 - 1/2)*(f/8 - (9*d)/32 + (3^(
1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18
) + log(x + (3^(1/2)*1i)/2 + 1/2)*((9*d)/32 - f/8 + (3^(1/2)*d*13i)/288 - (
3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 + (3^(1/2)*g*1i)/18)
```

3.50 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(1+x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=263

$$\frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2}+\frac{x\left(d+f-2h-(d-2f+h)x^2\right)}{12\left(1+x^2+x^4\right)^2}+\frac{(2e-g)\left(1+2x^2\right)}{12\left(1+x^2+x^4\right)}+\frac{x(2d+3f-h-(7d-7f+4h}{24\left(1+x^2+x^4\right)}$$

```
[Out] 1/12*(e-2*g+(2*e-g)*x^2)/(x^4+x^2+1)^2+1/12*x*(d+f-2*h-(d-2*f+h)*x^2)/(x^4+
x^2+1)^2+1/12*(2*e-g)*(2*x^2+1)/(x^4+x^2+1)+1/24*x*(2*d+3*f-h-(7*d-7*f+4*h)
*x^2)/(x^4+x^2+1)-1/32*(9*d-4*f+3*h)*ln(x^2-x+1)+1/32*(9*d-4*f+3*h)*ln(x^2+
x+1)-1/144*(13*d+2*f+h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/144*(13*d+2*f
+h)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/9*(2*e-g)*arctan(1/3*(2*x^2+1)*3^
(1/2))*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = $\{1687, 1692, 1192, 1183, 648, 632, 210, 642, 1261, 652, 628\}$

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g)}{3\sqrt{3}}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x\left(-\left(x^2(7d-7f+4h)\right)+2d+3f-h\right)}{24\left(x^4+x^2+1\right)}+\frac{x\left(-\left(x^2(d-2f+h)\right)+d+f-2h\right)}{12\left(x^4+x^2+1\right)^2}+\frac{\left(2x^2+1\right)(2e-g)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g)+e-2g}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (e - 2*g + (2*e - g)*x^2)/(12*(1 + x^2 + x^4)^2) + (x*(d + f - 2*h - (d - 2
*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + ((2*e - g)*(1 + 2*x^2))/(12*(1 + x^2
 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 + x^2 + x^4))
 - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((13*d + 2*f
 + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g)*ArcTan[(1 + 2*x^
2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x^2])/32 + ((9*d
- 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

```
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(1+x^2+x^4)^3}\,dx = \int \frac{x(e+gx^2)}{(1+x^2+x^4)^3}\,dx + \int \frac{d+fx^2+hx^4}{(1+x^2+x^4)^3}\,dx$$

$$= \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{1}{12}\int \frac{11d-f+2h-5(d-2f+h)}{\left(1+x^2+x^4\right)^2}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{x(2d+3f-}{24}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)(1+}{12(1+x^2+}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)(1+}{12(1+x^2+}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)(1+}{12(1+x^2+}$$

$$= \frac{e-2g+(2e-g)x^2}{12\left(1+x^2+x^4\right)^2} + \frac{x(d+f-2h-(d-2f+h)x^2)}{12\left(1+x^2+x^4\right)^2} + \frac{(2e-g)(1+}{12(1+x^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 303, normalized size = 1.15

$$\frac{1}{144}\left(-\frac{6(-4e(1+2x^2)+g(2+4x^2)+x(-2d-3f+h+7dx^2-7fx^2+4hx^2))}{1+x^2+x^4}+\frac{12(e+2ex^2-g(2+x^2)+x(d+f-dx^2+2fx^2-h(2+x^2)))}{(1+x^2+x^4)^2}-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f+2\left(-7i+2\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f+2\left(7i+2\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}(2e-g)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((-6*(-4*e*(1 + 2*x^2) + g*(2 + 4*x^2) + x*(-2*d - 3*f + h + 7*d*x^2 - 7*f*
x^2 + 4*h*x^2)))/(1 + x^2 + x^4) + (12*(e + 2*e*x^2 - g*(2 + x^2) + x*(d +
f - d*x^2 + 2*f*x^2 - h*(2 + x^2))))/(1 + x^2 + x^4)^2 - (((-47*I + 7*Sqrt[
3])*d + (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2*Sqrt[3])*h)*ArcTan[((-I + Sqrt[3
])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I + 7*Sqrt[3])*d - (17*I + 7*Sqrt
[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[(1 - I*Sq
rt[3])/6] - 16*Sqrt[3]*(2*e - g)*ArcTan[Sqrt[3]/(1 + 2*x^2)])/144
```

Maple [A]
time = 0.19, size = 262, normalized size = 1.00

method	result

default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}\right)x^3+(-6d+4f-2h-2g)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}-\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}\right)x-4d+\frac{4f}{3}+2e-2g}{16(x^2+x+1)^2}+\frac{(27d-12f+9h)\ln(x^2+x+1)}{96}+$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*h-2*g)*x^2+(-20/3*d+
13/3*f-5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f+2*e-2*g)/(x^2+x+1)^2+1/96*(27*d-12*f+
9*h)*ln(x^2+x+1)+1/72*(13/2*d-16*e+f+8*g+1/2*h)*arctan(1/3*(2*x+1)*3^(1/2))
*3^(1/2)-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g)*x^3+(-6*d+4*f-2*h+2*g)*x^2+(
20/3*d-13/3*f+5/3*h+1/3*e-8/3*g)*x-4*d+4/3*f-2*e+2*g)/(x^2-x+1)^2-1/96*(27*
d-12*f+9*h)*ln(x^2-x+1)-1/72*(-13/2*d-16*e-f+8*g-1/2*h)*3^(1/2)*arctan(1/3*
(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 217, normalized size = 0.83

$$\frac{1}{144}\sqrt{3}\left(13d+2f+16g+h-32e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+1\right)\right)+\frac{1}{144}\sqrt{3}\left(13d+2f-16g+h+32e\right)\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-1\right)\right)+\frac{1}{32}\left(9d-4f+3h\right)\log\left(x^2+x+1\right)-\frac{1}{32}\left(9d-4f+3h\right)\log\left(x^2-x+1\right)-\frac{(7d-7f+4h)x^7+4(g-2e)x^6+5(d-2f+h)x^5+6(g-2e)x^4+7(d-2f+h)x^3+8(g-2e)x^2-(4d+5f-5h)x+6g-6e}{24\left(x^8+2x^6+3x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*log(x^
2 - x + 1) - 1/24*((7*d - 7*f + 4*h)*x^7 + 4*(g - 2*e)*x^6 + 5*(d - 2*f + h
)*x^5 + 6*(g - 2*e)*x^4 + 7*(d - 2*f + h)*x^3 + 8*(g - 2*e)*x^2 - (4*d + 5*
f - 5*h)*x + 6*g - 6*e)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(236) = 472$.
time = 1.36, size = 485, normalized size = 1.84

$$-\frac{12(7d-7f+4h)x^7-48(2e-g)x^6+60(d-2f+h)x^5-72(2e-g)x^4+84(d-2f+h)x^3-96(2e-g)x^2-2\sqrt{3}\left((13d-32e+2f+16g+h)x^8+2(13d-32e+2f+16g+h)x^6+3(13d-32e+2f+16g+h)x^4+2(13d-32e+2f+16g+h)x^2+13d-32e+2f+16g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}\left((13d+32e+2f-16g+h)x^8+2(13d+32e+2f-16g+h)x^6+3(13d+32e+2f-16g+h)x^4+2(13d+32e+2f-16g+h)x^2+13d+32e+2f-16g+h\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-12(4d+5f-5h)x-9\left((9d-4f+3h)x^8+2(9d-4f+3h)x^6+3(9d-4f+3h)x^4+2(9d-4f+3h)x^2+9d-4f+3h\right)\log(x^2+x+1)+9\left((9d-4f+3h)x^8+2(9d-4f+3h)x^6+3(9d-4f+3h)x^4+2(9d-4f+3h)x^2+9d-4f+3h\right)\log(x^2-x+1)-72e+72g}{288\left(x^8+2x^6+3x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/288*(12*(7*d - 7*f + 4*h)*x^7 - 48*(2*e - g)*x^6 + 60*(d - 2*f + h)*x^5
- 72*(2*e - g)*x^4 + 84*(d - 2*f + h)*x^3 - 96*(2*e - g)*x^2 - 2*sqrt(3)*((
13*d - 32*e + 2*f + 16*g + h)*x^8 + 2*(13*d - 32*e + 2*f + 16*g + h)*x^6 +
3*(13*d - 32*e + 2*f + 16*g + h)*x^4 + 2*(13*d - 32*e + 2*f + 16*g + h)*x^2
```

```
 + 13*d - 32*e + 2*f + 16*g + h)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*
((13*d + 32*e + 2*f - 16*g + h)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h)*x^6
+ 3*(13*d + 32*e + 2*f - 16*g + h)*x^4 + 2*(13*d + 32*e + 2*f - 16*g + h)*x
^2 + 13*d + 32*e + 2*f - 16*g + h)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(4*d
+ 5*f - 5*h)*x - 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*
d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 + x
 + 1) + 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*d - 4*f +
 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x + 1) - 7
2*e + 72*g)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.69, size = 228, normalized size = 0.87

$$\frac{1}{144}\,\sqrt{3}\,(13\,d+2\,f+16\,g+h-32\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\,\sqrt{3}\,(13\,d+2\,f-16\,g+h+32\,e)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{32}\,(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}\,(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)-\frac{7\,dx^7-7\,fx^7+4\,hx^7+4\,gx^6-8\,x^6e+5\,dx^5-10\,fx^5+5\,hx^5+6\,gx^4-12\,x^4e+7\,dx^3-14\,fx^3+7\,hx^3+8\,gx^2-16\,x^2e-4\,dx-5\,fx+5\,hx+6\,g-6\,e}{24\,(x^4+x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e)*arctan(1/3*sqrt(3)*(2*x + 1))
+ 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e)*arctan(1/3*sqrt(3)*(2*x - 1)
) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f + 3*h)*log(x^
2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*h*x^7 + 4*g*x^6 - 8*x^6*e + 5*d*x^
5 - 10*f*x^5 + 5*h*x^5 + 6*g*x^4 - 12*x^4*e + 7*d*x^3 - 14*f*x^3 + 7*h*x^3
+ 8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 5*h*x + 6*g - 6*e)/(x^4 + x^2 + 1)^2
```

Mupad [**B**]
time = 5.45, size = 1611, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(x^2 + x^4 + 1)^3,x)
```

```
[Out] (e/4 - g/4 + x^2*((2*e)/3 - g/3) + x^4*(e/2 - g/4) + x^6*(e/3 - g/6) + x*(d
/6 + (5*f)/24 - (5*h)/24) - x^7*((7*d)/24 - (7*f)/24 + h/6) - x^5*((5*d)/24
 - (5*f)/12 + (5*h)/24) - x^3*((7*d)/24 - (7*f)/12 + (7*h)/24))/(2*x^2 + 3*
```

x^4 + 2*x^6 + x^8 + 1) - log(960*d*g - 2763*d*f - 1920*d*e + 480*e*f + 1971
*d*h - 480*e*h - 240*f*g - 981*f*h + 240*g*h + 3^(1/2)*d^2*1620i + 3^(1/2)*
f^2*180i + 3^(1/2)*h^2*135i - 3807*d^2*x - 612*f^2*x - 378*h^2*x + 2754*d^2
 + 684*f^2 + 351*h^2 + 3^(1/2)*d*e*1088i - 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*
544i - 3^(1/2)*e*f*608i + 3^(1/2)*d*h*945i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g
*304i - 3^(1/2)*f*h*315i - 3^(1/2)*g*h*208i - 672*d*e*x + 3069*d*f*x + 336*
d*g*x + 672*e*f*x - 2403*d*h*x - 384*e*h*x - 336*f*g*x + 963*f*h*x + 192*g*
h*x + 3^(1/2)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2
)*d*f*x*819i + 3^(1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i
 - 3^(1/2)*e*h*x*448i - 3^(1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i + 3^(1/2)*g
*h*x*224i - 3^(1/2)*d*e*x*1504i)*((9*d)/32 - f/8 + (3*h)/32 + (3^(1/2)*d*13
i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18 + (3^(1/
2)*h*1i)/288) - log(1920*d*e - 2763*d*f - 960*d*g - 480*e*f + 1971*d*h + 48
0*e*h + 240*f*g - 981*f*h - 240*g*h - 3^(1/2)*d^2*1620i - 3^(1/2)*f^2*180i
 - 3^(1/2)*h^2*135i + 3807*d^2*x + 612*f^2*x + 378*h^2*x + 2754*d^2 + 684*f^
2 + 351*h^2 + 3^(1/2)*d*e*1088i + 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^
(1/2)*e*f*608i - 3^(1/2)*d*h*945i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g*304i + 3
^(1/2)*f*h*315i - 3^(1/2)*g*h*208i - 672*d*e*x - 3069*d*f*x + 336*d*g*x + 6
72*e*f*x + 2403*d*h*x - 384*e*h*x - 336*f*g*x - 963*f*h*x + 192*g*h*x + 3^(
1/2)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2)*d*f*x*8
19i - 3^(1/2)*d*g*x*752i - 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i + 3^(1/2
)*e*h*x*448i + 3^(1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i - 3^(1/2)*g*h*x*224i
 + 3^(1/2)*d*e*x*1504i)*(f/8 - (9*d)/32 - (3*h)/32 + (3^(1/2)*d*13i)/288 -
(3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 + (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/
288) + log(1920*d*e - 2763*d*f - 960*d*g - 480*e*f + 1971*d*h + 480*e*h + 2
40*f*g - 981*f*h - 240*g*h + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1/2)
*h^2*135i + 3807*d^2*x + 612*f^2*x + 378*h^2*x + 2754*d^2 + 684*f^2 + 351*h
^2 - 3^(1/2)*d*e*1088i - 3^(1/2)*d*f*1125i + 3^(1/2)*d*g*544i + 3^(1/2)*e*f
*608i + 3^(1/2)*d*h*945i - 3^(1/2)*e*h*416i - 3^(1/2)*f*g*304i - 3^(1/2)*f*
h*315i + 3^(1/2)*g*h*208i - 672*d*e*x - 3069*d*f*x + 336*d*g*x + 672*e*f*x
 + 2403*d*h*x - 384*e*h*x - 336*f*g*x - 963*f*h*x + 192*g*h*x - 3^(1/2)*d^2*
x*567i - 3^(1/2)*f^2*x*252i - 3^(1/2)*h^2*x*108i + 3^(1/2)*d*f*x*819i + 3^(
1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i - 3^(1/2)*d*h*x*513i - 3^(1/2)*e*h*x*4
48i - 3^(1/2)*f*g*x*272i + 3^(1/2)*f*h*x*333i + 3^(1/2)*g*h*x*224i - 3^(1/2
)*d*e*x*1504i)*((9*d)/32 - f/8 + (3*h)/32 + (3^(1/2)*d*13i)/288 - (3^(1/2)*
e*1i)/9 + (3^(1/2)*f*1i)/144 + (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/288) + lo
g(1920*d*e + 2763*d*f - 960*d*g - 480*e*f - 1971*d*h + 480*e*h + 240*f*g +
981*f*h - 240*g*h + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1/2)*h^2*135i
 + 3807*d^2*x + 612*f^2*x + 378*h^2*x - 2754*d^2 - 684*f^2 - 351*h^2 + 3^(1
/2)*d*e*1088i - 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^(1/2)*e*f*608i + 3
^(1/2)*d*h*945i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g*304i - 3^(1/2)*f*h*315i -
3^(1/2)*g*h*208i + 672*d*e*x - 3069*d*f*x - 336*d*g*x - 672*e*f*x + 2403*d*
h*x + 384*e*h*x + 336*f*g*x - 963*f*h*x - 192*g*h*x + 3^(1/2)*d^2*x*567i +
3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2)*d*f*x*819i + 3^(1/2)*d*g*
x*752i + 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i - 3^(1/2)*e*h*x*448i - 3^(

```
1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i + 3^(1/2)*g*h*x*224i - 3^(1/2)*d*e*x*1
504i)*(f/8 - (9*d)/32 - (3*h)/32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 +
 (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/288)
```

3.51 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(1+x^2+x^4)^3}\,dx$

Optimal. Leaf size=269

$$\frac{x(d+f-2h-(d-2f+h)x^2)}{12\,(1+x^2+x^4)^2}+\frac{e-2g+i+(2e-g-i)x^2}{12\,(1+x^2+x^4)^2}+\frac{(2e-g+i)\,(1+2x^2)}{12\,(1+x^2+x^4)}+\frac{x(2d+3f-h-(7d}{24\,(1+x^2}$$

```
[Out] 1/12*x*(d+f-2*h-(d-2*f+h)*x^2)/(x^4+x^2+1)^2+1/12*(e-2*g+i+(2*e-g-i)*x^2)/(
x^4+x^2+1)^2+1/12*(2*e-g+i)*(2*x^2+1)/(x^4+x^2+1)+1/24*x*(2*d+3*f-h-(7*d-7*
f+4*h)*x^2)/(x^4+x^2+1)-1/32*(9*d-4*f+3*h)*ln(x^2-x+1)+1/32*(9*d-4*f+3*h)*l
n(x^2+x+1)-1/144*(13*d+2*f+h)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/144*(13
*d+2*f+h)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/9*(2*e-g+i)*arctan(1/3*(2*x
^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1687, 1692, 1192, 1183, 648, 632, 210, 642, 1677, 1674, 12, 628}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)(13d+2f+h)}{48\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)(2e-g+i)}{3\sqrt{3}}-\frac{1}{32}\log\left(x^2-x+1\right)(9d-4f+3h)+\frac{1}{32}\log\left(x^2+x+1\right)(9d-4f+3h)+\frac{x(-(x^2(7d-7f+4h))+2d+3f-h)}{24\left(x^4+x^2+1\right)}+\frac{x(-(x^2(d-2f+h))+d+f-2h)}{12\left(x^4+x^2+1\right)^2}+\frac{\left(2x^2+1\right)(2e-g+i)}{12\left(x^4+x^2+1\right)}+\frac{x^2(2e-g-i)+e-2g+i}{12\left(x^4+x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

```
[Out] (x*(d + f - 2*h - (d - 2*f + h)*x^2))/(12*(1 + x^2 + x^4)^2) + (e - 2*g + i
 + (2*e - g - i)*x^2)/(12*(1 + x^2 + x^4)^2) + ((2*e - g + i)*(1 + 2*x^2))/
(12*(1 + x^2 + x^4)) + (x*(2*d + 3*f - h - (7*d - 7*f + 4*h)*x^2))/(24*(1 +
 x^2 + x^4)) - ((13*d + 2*f + h)*ArcTan[(1 - 2*x)/Sqrt[3]])/(48*Sqrt[3]) +
((13*d + 2*f + h)*ArcTan[(1 + 2*x)/Sqrt[3]])/(48*Sqrt[3]) + ((2*e - g + i)*
ArcTan[(1 + 2*x^2)/Sqrt[3]])/(3*Sqrt[3]) - ((9*d - 4*f + 3*h)*Log[1 - x + x
^2])/32 + ((9*d - 4*f + 3*h)*Log[1 + x + x^2])/32
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+51x^5}{(1+x^2+x^4)^3}\,dx = \int \frac{x(e+gx^2+51x^4)}{(1+x^2+x^4)^3}\,dx + \int \frac{d+fx^2+hx^4}{(1+x^2+x^4)^3}\,dx$$
$$= \frac{x(d+f-2h-(d-2f+h)x^2)}{12\,(1+x^2+x^4)^2} + \frac{1}{12}\int \frac{11d-f+2h-5(d-}{(1+x^2+x^4)}$$
$$= \frac{51+e-2g-(51-2e+g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(d+f-2h-(d-2f+h)x}{12\,(1+x^2+x^4)^2}$$
$$= \frac{51+e-2g-(51-2e+g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(d+f-2h-(d-2f+h)x}{12\,(1+x^2+x^4)^2}$$
$$= \frac{51+e-2g-(51-2e+g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(d+f-2h-(d-2f+h)x}{12\,(1+x^2+x^4)^2}$$
$$= \frac{51+e-2g-(51-2e+g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(d+f-2h-(d-2f+h)x}{12\,(1+x^2+x^4)^2}$$
$$= \frac{51+e-2g-(51-2e+g)x^2}{12\,(1+x^2+x^4)^2} + \frac{x(d+f-2h-(d-2f+h)x}{12\,(1+x^2+x^4)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.51, size = 325, normalized size = 1.21

$$\frac{1}{144}\left(\frac{12(e+i+dx+fx-2hx+2ex^2-ix^2-dx^3+2fx^3-hx^3-g(2+x^2))}{(1+x^2+x^4)^2}+\frac{6(2i+2dx+3fx-hx+4ix^2-7dx^3+7fx^3-4hx^3-2g(1+2x^2)+e(4+8x^2))}{1+x^2+x^4}-\frac{\left(\left(-47i+7\sqrt{3}\right)d+\left(17i-7\sqrt{3}\right)f+2\left(-7i+2\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1+i\sqrt{3}\right)}}-\frac{\left(\left(47i+7\sqrt{3}\right)d-\left(17i+7\sqrt{3}\right)f+2\left(7i+2\sqrt{3}\right)h\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}\left(1-i\sqrt{3}\right)}}-16\sqrt{3}(2e-g+i)\tan^{-1}\left(\frac{\sqrt{3}}{1+2x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(1 + x^2 + x^4)^3,x]
```

```
[Out] ((12*(e + i + d*x + f*x - 2*h*x + 2*e*x^2 - i*x^2 - d*x^3 + 2*f*x^3 - h*x^3
 - g*(2 + x^2)))/(1 + x^2 + x^4)^2 + (6*(2*i + 2*d*x + 3*f*x - h*x + 4*i*x^
2 - 7*d*x^3 + 7*f*x^3 - 4*h*x^3 - 2*g*(1 + 2*x^2) + e*(4 + 8*x^2)))/(1 + x^
2 + x^4) - (((-47*I + 7*Sqrt[3])*d + (17*I - 7*Sqrt[3])*f + 2*(-7*I + 2*Sqr
t[3])*h)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[(1 + I*Sqrt[3])/6] - (((47*I +
7*Sqrt[3])*d - (17*I + 7*Sqrt[3])*f + 2*(7*I + 2*Sqrt[3])*h)*ArcTan[((I + S
qrt[3])*x)/2])/Sqrt[(1 - I*Sqrt[3])/6] - 16*Sqrt[3]*(2*e - g + i)*ArcTan[Sq
rt[3]/(1 + 2*x^2)])/144
```

Maple [A]
time = 0.23, size = 292, normalized size = 1.09

method	result
default	$\frac{\left(-\frac{7d}{3}+\frac{7f}{3}-\frac{4h}{3}-\frac{4e}{3}-\frac{g}{3}+\frac{i}{3}\right)x^3+(-6d+4f-2h-2g+2i)x^2+\left(-\frac{20d}{3}+\frac{13f}{3}-\frac{5h}{3}+\frac{e}{3}-\frac{8g}{3}+\frac{7i}{3}\right)x-4d+\frac{4f}{3}+2e-2g+\frac{4i}{3}}{16(x^2+x+1)^2}+\frac{(27d-12f+9}{}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*((-7/3*d+7/3*f-4/3*h-4/3*e-1/3*g+1/3*i)*x^3+(-6*d+4*f-2*h-2*g+2*i)*x^2
+(-20/3*d+13/3*f-5/3*h+1/3*e-8/3*g+7/3*i)*x-4*d+4/3*f+2*e-2*g+4/3*i)/(x^2+x
+1)^2+1/96*(27*d-12*f+9*h)*ln(x^2+x+1)+1/72*(13/2*d-16*e+f+8*g+1/2*h-8*i)*a
rctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/16*((7/3*d-7/3*f+4/3*h-4/3*e-1/3*g+1/3
*i)*x^3+(-6*d+4*f-2*h+2*g-2*i)*x^2+(20/3*d-13/3*f+5/3*h+1/3*e-8/3*g+7/3*i)*
x-4*d+4/3*f-2*e+2*g-4/3*i)/(x^2-x+1)^2-1/96*(27*d-12*f+9*h)*ln(x^2-x+1)-1/7
2*(-13/2*d-16*e-f+8*g-1/2*h-8*i)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 225, normalized size = 0.84

$$\frac{1}{144}\sqrt{3}\,(13\,d+2\,f+16\,g+h-32\,e-16i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\,(13\,d+2\,f-16\,g+h+32\,e+16i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)-\frac{(7\,d-7\,f+4\,h)x^7+4\,(g-2\,e-i)x^6+5\,(d-2\,f+h)x^5+6\,(g-2\,e-i)x^4+7\,(d-2\,f+h)x^3+4\,(2\,g-4\,e-i)x^2-(4\,d+5\,f-5\,h)x+6\,g-6\,e-4i}{24\,(x^8+2\,x^6+3\,x^4+2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="maxim
a")
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e - 16*I)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e + 16*I)*arctan(1/3*sqr
t(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f
 + 3*h)*log(x^2 - x + 1) - 1/24*((7*d - 7*f + 4*h)*x^7 + 4*(g - 2*e - I)*x^
6 + 5*(d - 2*f + h)*x^5 + 6*(g - 2*e - I)*x^4 + 7*(d - 2*f + h)*x^3 + 4*(2*
g - 4*e - I)*x^2 - (4*d + 5*f - 5*h)*x + 6*g - 6*e - 4*I)/(x^8 + 2*x^6 + 3*
x^4 + 2*x^2 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(242) = 484.
time = 5.43, size = 521, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="frica
s")
```

```
[Out] -1/288*(12*(7*d - 7*f + 4*h)*x^7 - 48*(2*e - g + i)*x^6 + 60*(d - 2*f + h)*
x^5 - 72*(2*e - g + i)*x^4 + 84*(d - 2*f + h)*x^3 - 48*(4*e - 2*g + i)*x^2
- 2*sqrt(3)*((13*d - 32*e + 2*f + 16*g + h - 16*i)*x^8 + 2*(13*d - 32*e + 2
*f + 16*g + h - 16*i)*x^6 + 3*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^4 + 2
*(13*d - 32*e + 2*f + 16*g + h - 16*i)*x^2 + 13*d - 32*e + 2*f + 16*g + h -
```

```
16*i)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((13*d + 32*e + 2*f - 16*g
 + h + 16*i)*x^8 + 2*(13*d + 32*e + 2*f - 16*g + h + 16*i)*x^6 + 3*(13*d +
32*e + 2*f - 16*g + h + 16*i)*x^4 + 2*(13*d + 32*e + 2*f - 16*g + h + 16*i)
*x^2 + 13*d + 32*e + 2*f - 16*g + h + 16*i)*arctan(1/3*sqrt(3)*(2*x - 1)) -
 12*(4*d + 5*f - 5*h)*x - 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^
6 + 3*(9*d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*lo
g(x^2 + x + 1) + 9*((9*d - 4*f + 3*h)*x^8 + 2*(9*d - 4*f + 3*h)*x^6 + 3*(9*
d - 4*f + 3*h)*x^4 + 2*(9*d - 4*f + 3*h)*x^2 + 9*d - 4*f + 3*h)*log(x^2 - x
 + 1) - 72*e + 72*g - 48*i)/(x^8 + 2*x^6 + 3*x^4 + 2*x^2 + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4+x**2+1)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.96, size = 246, normalized size = 0.91

$$\frac{1}{144}\sqrt{3}\,(13\,d+2\,f+16\,g+h-32\,e-16i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{144}\sqrt{3}\,(13\,d+2\,f-16\,g+h+32\,e+16i)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2+x+1\right)-\frac{1}{32}(9\,d-4\,f+3\,h)\log\left(x^2-x+1\right)-\frac{7\,dx^7-7\,fx^7+4\,hx^7+4\,gx^6-8\,x^6e+5\,dx^5-10\,fx^5+5\,hx^5-4i\,x^6+6\,gx^4-12\,x^4e+7\,dx^3-14\,fx^3+7\,hx^3-6i\,x^4+8\,gx^2-16\,x^2e-4\,dx-5\,fx+5\,hx-4i\,x^2+6\,g-6\,e-4i}{24\,(x^4+x^2+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4+x^2+1)^3,x, algorithm="giac"
)
```

```
[Out] 1/144*sqrt(3)*(13*d + 2*f + 16*g + h - 32*e - 16*I)*arctan(1/3*sqrt(3)*(2*x
 + 1)) + 1/144*sqrt(3)*(13*d + 2*f - 16*g + h + 32*e + 16*I)*arctan(1/3*sqr
t(3)*(2*x - 1)) + 1/32*(9*d - 4*f + 3*h)*log(x^2 + x + 1) - 1/32*(9*d - 4*f
 + 3*h)*log(x^2 - x + 1) - 1/24*(7*d*x^7 - 7*f*x^7 + 4*h*x^7 + 4*g*x^6 - 8*
x^6*e + 5*d*x^5 - 10*f*x^5 + 5*h*x^5 - 4*I*x^6 + 6*g*x^4 - 12*x^4*e + 7*d*x
^3 - 14*f*x^3 + 7*h*x^3 - 6*I*x^4 + 8*g*x^2 - 16*x^2*e - 4*d*x - 5*f*x + 5*
h*x - 4*I*x^2 + 6*g - 6*e - 4*I)/(x^4 + x^2 + 1)^2
```

Mupad [**B**]
time = 8.22, size = 1963, normalized size = 7.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(x^2 + x^4 + 1)^3,x)
```

```
[Out] (e/4 - g/4 + i/6 + x*(d/6 + (5*f)/24 - (5*h)/24) - x^7*((7*d)/24 - (7*f)/24
 + h/6) - x^5*((5*d)/24 - (5*f)/12 + (5*h)/24) - x^3*((7*d)/24 - (7*f)/12 +
```

(7*h)/24) + x^4*(e/2 - g/4 + i/4) + x^2*((2*e)/3 - g/3 + i/6) + x^6*(e/3 - g/6 + i/6))/(2*x^2 + 3*x^4 + 2*x^6 + x^8 + 1) - log(960*d*g - 2763*d*f - 1920*d*e + 480*e*f + 1971*d*h - 960*d*i - 480*e*h - 240*f*g - 981*f*h + 240*f*i + 240*g*h - 240*h*i + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1/2)*h^2*135i - 3807*d^2*x - 612*f^2*x - 378*h^2*x + 2754*d^2 + 684*f^2 + 351*h^2 + 3^(1/2)*d*e*1088i - 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^(1/2)*e*f*608i + 3^(1/2)*d*h*945i + 3^(1/2)*d*i*544i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g*304i - 3^(1/2)*f*h*315i - 3^(1/2)*f*i*304i - 3^(1/2)*g*h*208i + 3^(1/2)*h*i*208i - 672*d*e*x + 3069*d*f*x + 336*d*g*x + 672*e*f*x - 2403*d*h*x - 336*d*i*x - 384*e*h*x - 336*f*g*x + 963*f*h*x + 336*f*i*x + 192*g*h*x - 192*h*i*x + 3^(1/2)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2)*d*f*x*819i + 3^(1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i - 3^(1/2)*d*i*x*752i - 3^(1/2)*e*h*x*448i - 3^(1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i + 3^(1/2)*f*i*x*272i + 3^(1/2)*g*h*x*224i - 3^(1/2)*h*i*x*224i - 3^(1/2)*d*e*x*1504i)*((9*d)/32 - f/8 + (3*h)/32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/288 + (3^(1/2)*i*1i)/18) - log(1920*d*e - 2763*d*f - 960*d*g - 480*e*f + 1971*d*h + 960*d*i + 480*e*h + 240*f*g - 981*f*h - 240*f*i - 240*g*h + 240*h*i - 3^(1/2)*d^2*1620i - 3^(1/2)*f^2*180i - 3^(1/2)*h^2*135i + 3807*d^2*x + 612*f^2*x + 378*h^2*x + 2754*d^2 + 684*f^2 + 351*h^2 + 3^(1/2)*d*e*1088i + 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^(1/2)*e*f*608i - 3^(1/2)*d*h*945i + 3^(1/2)*d*i*544i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g*304i + 3^(1/2)*f*h*315i - 3^(1/2)*f*i*304i - 3^(1/2)*g*h*208i + 3^(1/2)*h*i*208i - 672*d*e*x - 3069*d*f*x + 336*d*g*x + 672*e*f*x + 2403*d*h*x - 336*d*i*x - 384*e*h*x - 336*f*g*x - 963*f*h*x + 336*f*i*x + 192*g*h*x - 192*h*i*x + 3^(1/2)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2)*d*f*x*819i - 3^(1/2)*d*g*x*752i - 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i + 3^(1/2)*d*i*x*752i + 3^(1/2)*e*h*x*448i + 3^(1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i - 3^(1/2)*f*i*x*272i - 3^(1/2)*g*h*x*224i + 3^(1/2)*h*i*x*224i + 3^(1/2)*d*e*x*1504i)*(f/8 - (9*d)/32 - (3*h)/32 + (3^(1/2)*d*13i)/288 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 + (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/288 - (3^(1/2)*i*1i)/18) + log(1920*d*e - 2763*d*f - 960*d*g - 480*e*f + 1971*d*h + 960*d*i + 480*e*h + 240*f*g - 981*f*h - 240*f*i - 240*g*h + 240*h*i + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1/2)*h^2*135i + 3807*d^2*x + 612*f^2*x + 378*h^2*x + 2754*d^2 + 684*f^2 + 351*h^2 - 3^(1/2)*d*e*1088i - 3^(1/2)*d*f*1125i + 3^(1/2)*d*g*544i + 3^(1/2)*e*f*608i + 3^(1/2)*d*h*945i - 3^(1/2)*d*i*544i - 3^(1/2)*e*h*416i - 3^(1/2)*f*g*304i - 3^(1/2)*f*h*315i + 3^(1/2)*f*i*304i + 3^(1/2)*g*h*208i - 3^(1/2)*h*i*208i - 672*d*e*x - 3069*d*f*x + 336*d*g*x + 672*e*f*x + 2403*d*h*x - 336*d*i*x - 384*e*h*x - 336*f*g*x - 963*f*h*x + 336*f*i*x + 192*g*h*x - 192*h*i*x - 3^(1/2)*d^2*x*567i - 3^(1/2)*f^2*x*252i - 3^(1/2)*h^2*x*108i + 3^(1/2)*d*f*x*819i + 3^(1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i - 3^(1/2)*d*h*x*513i - 3^(1/2)*d*i*x*752i - 3^(1/2)*e*h*x*448i - 3^(1/2)*f*g*x*272i + 3^(1/2)*f*h*x*333i + 3^(1/2)*f*i*x*272i + 3^(1/2)*g*h*x*224i - 3^(1/2)*h*i*x*224i - 3^(1/2)*d*e*x*1504i)*((9*d)/32 - f/8 + (3*h)/32 + (3^(1/2)*d*13i)/288 - (3^(1/2)*e*1i)/9 + (3^(1/2)*f*1i)/144 + (3^(1/2)*g*1i)/18 + (3

```
^(1/2)*h*1i)/288 - (3^(1/2)*i*1i)/18) + log(1920*d*e + 2763*d*f - 960*d*g -
 480*e*f - 1971*d*h + 960*d*i + 480*e*h + 240*f*g + 981*f*h - 240*f*i - 240
*g*h + 240*h*i + 3^(1/2)*d^2*1620i + 3^(1/2)*f^2*180i + 3^(1/2)*h^2*135i +
3807*d^2*x + 612*f^2*x + 378*h^2*x - 2754*d^2 - 684*f^2 - 351*h^2 + 3^(1/2)
*d*e*1088i - 3^(1/2)*d*f*1125i - 3^(1/2)*d*g*544i - 3^(1/2)*e*f*608i + 3^(1
/2)*d*h*945i + 3^(1/2)*d*i*544i + 3^(1/2)*e*h*416i + 3^(1/2)*f*g*304i - 3^(
1/2)*f*h*315i - 3^(1/2)*f*i*304i - 3^(1/2)*g*h*208i + 3^(1/2)*h*i*208i + 67
2*d*e*x - 3069*d*f*x - 336*d*g*x - 672*e*f*x + 2403*d*h*x + 336*d*i*x + 384
*e*h*x + 336*f*g*x - 963*f*h*x - 336*f*i*x - 192*g*h*x + 192*h*i*x + 3^(1/2
)*d^2*x*567i + 3^(1/2)*f^2*x*252i + 3^(1/2)*h^2*x*108i - 3^(1/2)*d*f*x*819i
 + 3^(1/2)*d*g*x*752i + 3^(1/2)*e*f*x*544i + 3^(1/2)*d*h*x*513i - 3^(1/2)*d
*i*x*752i - 3^(1/2)*e*h*x*448i - 3^(1/2)*f*g*x*272i - 3^(1/2)*f*h*x*333i +
3^(1/2)*f*i*x*272i + 3^(1/2)*g*h*x*224i - 3^(1/2)*h*i*x*224i - 3^(1/2)*d*e*
x*1504i)*(f/8 - (9*d)/32 - (3*h)/32 + (3^(1/2)*d*13i)/288 + (3^(1/2)*e*1i)/
9 + (3^(1/2)*f*1i)/144 - (3^(1/2)*g*1i)/18 + (3^(1/2)*h*1i)/288 + (3^(1/2)*
i*1i)/18)
```

3.52 $\int \frac{d+ex}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=474

$$-\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{dx(b^2-2ac+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{dx((b^2-7ac}{8a^2(b}$$

```
[Out] -1/4*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*d*x*(b*c*x^2-2*a*c+b^
2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*e*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x
^4+b*x^2+a)+1/8*d*x*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+b^2)*x^2)/a^
2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-6*c^2*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(5/2)+3/16*d*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^
(1/2))/a^2/(-4*a*c+b^2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*d*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^3-8*a*b*c+
(-56*a^2*c^2+10*a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.47, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1687, 12, 1106, 1192, 1180, 211, 1121, 628, 632, 212}

$$\frac{3\sqrt{c}\,d\left(56a^2c^2-10ab^2c+b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\,d\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{dx(3bcx^2(b^2-8ac)+(b^2-7ac)(3b^2-4ac))}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{dx(-2ac+b^2+bcx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (d*x*(b^2 -
2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*e*(b + 2
*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (d*x*((b^2 - 7*a*c)*(3*b
^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt
[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[c]
*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*a*c])*d*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 -
4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex}{(a+bx^2+cx^4)^3}\,dx = \int \frac{d}{(a+bx^2+cx^4)^3}\,dx + \int \frac{ex}{(a+bx^2+cx^4)^3}\,dx$$

$$= d\int \frac{1}{(a+bx^2+cx^4)^3}\,dx + e\int \frac{x}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{dx(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{d\int \frac{b^2-2ac-4(b^2-4ac)-5bcx^2}{(a+bx^2+cx^4)^2}\,dx}{4a\,(b^2-4ac)} + \frac{1}{2}e\text{Subst}\Bigg(\int \frac{}{(a+}$$

$$= -\frac{e(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{dx(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{dx((b^2-7ac)}{8a^2\,(b^2}$$

$$= -\frac{e(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{dx(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{3ce(b+}{2\,(b^2-4ac)^2\,(}$$

$$= -\frac{e(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{dx(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{3ce(b+}{2\,(b^2-4ac)^2\,(}$$

$$= -\frac{e(b+2cx^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{dx(b^2-2ac+bcx^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{3ce(b+}{2\,(b^2-4ac)^2\,(}$$

<u>Mathematica</u> [A]
time = 1.19, size = 488, normalized size = 1.03

$$\frac{1}{16}\left(\frac{4abe+8acx(d+ex)-4bdx(b+cx^2)}{a\,(-b^2+4ac)\,(a+bx^2+cx^4)^2}+\frac{6b^3dx(b+cx^2)-2abcdx(25b+24cx^2)+8a^2c(3be+cx(7d+6ex))}{a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{3\sqrt{2}\,\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2\,(b^2-4ac)^{5/2}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\,\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b^3\sqrt{b^2-4ac}+8abc\sqrt{b^2-4ac}\right)d\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^2\,(b^2-4ac)^{5/2}\,\sqrt{b+\sqrt{b^2-4ac}}}+\frac{48c^2e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}}-\frac{48c^2e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*a*b*e + 8*a*c*x*(d + e*x) - 4*b*d*x*(b + c*x^2))/(a*(-b^2 + 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) - 2*a*b*c*d*x*(25*b + 24*c*x^2)
 + 8*a^2*c*(3*b*e + c*x*(7*d + 6*e*x)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 -
 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^3*Sqrt[b^2 - 4*a*c]
 + 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (48*c
```

^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48*c^2*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(421) = 842$.

time = 0.29, size = 889, normalized size = 1.88

method	result
risch	$\frac{-\frac{3bc^2d(8ac-b^2)x^7}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{3c^3ex^6}{16a^2c^2-8ab^2c+b^4}+\frac{cd(28a^2c^2-49ab^2c+6b^4)x^5}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{9bc^2ex^4}{2(16a^2c^2-8ab^2c+b^4)}-\frac{bd(4a^2c^2+20ab^2c-3b^4)x^3}{8a^2(16a^2c^2-8ab^2c+b^4)}+\frac{(5ac+b\cdots}{16a^2c^2-8\cdots}}{(cx^4+bx^2+a)^2}$
default	$64c^3\left(\frac{-\frac{3\left(-24a^2c^2\sqrt{-4ac+b^2}+10ab^2c\sqrt{-4ac+b^2}-\sqrt{-4ac+b^2}\,b^4+32a^2bc^2-12ab^3c+b^5\right)dx^3}{64a^2c^3}+\frac{3e(4ac-b^2)x^2}{8c^2}-\frac{d(\cdots}{\cdots}}{\left(x^2+\frac{\sqrt{-\cdots}}{\cdots}\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 64*c^3*(1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-3/64/a^2/c^3*(-24*a^
2*c^2*(-4*a*c+b^2)^(1/2)+10*a*b^2*c*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*b
^4+32*a^2*b*c^2-12*a*b^3*c+b^5)*d*x^3+3/8*e*(4*a*c-b^2)/c^2*x^2-1/64*d*(20*
(-4*a*c+b^2)^(1/2)*a*b*c-5*(-4*a*c+b^2)^(1/2)*b^3+176*a^2*c^2-64*a*b^2*c+5*
b^4)/a/c^3*x+1/16*e*(16*a*c*(-4*a*c+b^2)^(1/2)-4*b^2*(-4*a*c+b^2)^(1/2)+12*
a*b*c-3*b^3)/c^3)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2+3/32/a^2/c^2*(8*
(-4*a*c+b^2)^(1/2)*a^2*c*e*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(56*(-4*a*c
+b^2)^(1/2)*a^2*c^2*d-10*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+(-4*a*c+b^2)^(1/2)*b^
4*d-32*a^2*b*c^2*d+12*a*b^3*c*d-b^5*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+1/16/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-3/64/a^2/c^3*(24*a^2*c^2*(-4*a*c+b^2)^(1/2)
-10*a*b^2*c*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*b^4+32*a^2*b*c^2-12*a*b^3
*c+b^5)*d*x^3+3/8*e*(4*a*c-b^2)/c^2*x^2-1/64*d*(-20*(-4*a*c+b^2)^(1/2)*a*b*
c+5*(-4*a*c+b^2)^(1/2)*b^3+176*a^2*c^2-64*a*b^2*c+5*b^4)/a/c^3*x+1/16*e*(-1

```
6*a*c*(-4*a*c+b^2)^(1/2)+4*b^2*(-4*a*c+b^2)^(1/2)+12*a*b*c-3*b^3)/c^3)/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2+3/32/a^2/c^2*(-8*(-4*a*c+b^2)^(1/2)*a^
2*c*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(56*(-4*a*c+b^2)^(1/2)*a^2*c^2*
d-10*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+(-4*a*c+b^2)^(1/2)*b^4*d+32*a^2*b*c^2*d-1
2*a*b^3*c*d+b^5*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*x^6*e + 36*a^2*b*c^2*x^4*e + 3*(b^3*c^2 - 8*a*b*c^3)*d*x^7
+ (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d*x^5 - 2*a^2*b^3*e + 20*a^3*b*c*e
+ (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d*x^3 + (5*a*b^4 - 37*a^2*b^2*c + 44*a
^3*c^2)*d*x + 8*(a^2*b^2*c*e + 5*a^3*c^2*e)*x^2)/((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) - 3/8*integrate(-(16*a^2
*c^2*x*e + (b^3*c - 8*a*b*c^2)*d*x^2 + (b^4 - 9*a*b^2*c + 28*a^2*c^2)*d)/(c
*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3399 vs. $2(423) = 846$.
time = 8.69, size = 3399, normalized size = 7.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7
*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 -
 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^
2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c
^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 8
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)
*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 -
4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4
 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*d*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3
)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2
*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^
3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*
b^2*c^4 - 64*a^6*c^5)*abs(c)) + 3/32*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2 +
2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^4 - 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^6*c + 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^
2 - 4*a*c)*b^5*c^2 - 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3
*c^3 + 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*d*arctan(2*s
qrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^
4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3
)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^
2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a
^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) - 3*(b^2*c^4
- 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4
*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c
^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))/((b^8 - 16*a*b^6*c - 2*b^7*
c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*
c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*
c^5)*c^2) + 3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x
^2 + 1/2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 ...
```

Mupad [B]
time = 2.34, size = 2500, normalized size = 5.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] symsum(log(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 +
 47185920*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a
^13*b^4*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5
*z^4 + 3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a
^15*c^10*z^4 + 65536*a^5*b^20*z^4 + 6936330240*a^8*b^3*c^8*d^2*z^2 + 246487
```

4496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2 - 1509949440*a^9*b^
2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a*b^17*c*d^2*z^2 + 7
54974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2*z^2 - 188743680*a^
7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 23592960*a^6*b^8*c^5*e
^2*z^2 - 19860480*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 177
1776*a^2*b^15*c^2*d^2*z^2 + 1207959552*a^10*c^9*e^2*z^2 + 2304*b^19*d^2*z^2
 - 428544*a*b^12*c^3*d^2*e*z + 1022754816*a^6*b^2*c^8*d^2*e*z - 642318336*a
^5*b^4*c^7*d^2*e*z + 223395840*a^4*b^6*c^6*d^2*e*z - 46725120*a^3*b^8*c^5*d
^2*e*z + 5930496*a^2*b^10*c^4*d^2*e*z - 693633024*a^7*c^9*d^2*e*z + 13824*b
^14*c^2*d^2*e*z + 34836480*a^4*b*c^8*d^2*e^2 - 435456*a*b^7*c^5*d^2*e^2 - 1
7418240*a^3*b^3*c^7*d^2*e^2 + 3919104*a^2*b^5*c^6*d^2*e^2 + 20736*b^9*c^4*d
^2*e^2 - 27433728*a^3*b^2*c^8*d^4 + 6446304*a^2*b^4*c^7*d^4 - 734832*a*b^6*
c^6*d^4 + 49787136*a^4*c^9*d^4 + 5308416*a^5*c^8*e^4 + 35721*b^8*c^5*d^4, z
, k)*(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 4718
5920*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b
^4*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4
+ 3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c
^10*z^4 + 65536*a^5*b^20*z^4 + 6936330240*a^8*b^3*c^8*d^2*z^2 + 2464874496*
a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2 - 1509949440*a^9*b^2*c^8
*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a*b^17*c*d^2*z^2 + 754974
720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2*z^2 - 188743680*a^7*b^6
*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 23592960*a^6*b^8*c^5*e^2*z^
2 - 19860480*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 1771776*
a^2*b^15*c^2*d^2*z^2 + 1207959552*a^10*c^9*e^2*z^2 + 2304*b^19*d^2*z^2 - 42
8544*a*b^12*c^3*d^2*e*z + 1022754816*a^6*b^2*c^8*d^2*e*z - 642318336*a^5*b^
4*c^7*d^2*e*z + 223395840*a^4*b^6*c^6*d^2*e*z - 46725120*a^3*b^8*c^5*d^2*e*
z + 5930496*a^2*b^10*c^4*d^2*e*z - 693633024*a^7*c^9*d^2*e*z + 13824*b^14*c
^2*d^2*e*z + 34836480*a^4*b*c^8*d^2*e^2 - 435456*a*b^7*c^5*d^2*e^2 - 174182
40*a^3*b^3*c^7*d^2*e^2 + 3919104*a^2*b^5*c^6*d^2*e^2 + 20736*b^9*c^4*d^2*e^
2 - 27433728*a^3*b^2*c^8*d^4 + 6446304*a^2*b^4*c^7*d^4 - 734832*a*b^6*c^6*d
^4 + 49787136*a^4*c^9*d^4 + 5308416*a^5*c^8*e^4 + 35721*b^8*c^5*d^4, z, k)*
((x*(786432*a^9*c^9*e - 768*a^4*b^10*c^4*e + 15360*a^5*b^8*c^5*e - 122880*a
^6*b^6*c^6*e + 491520*a^7*b^4*c^7*e - 983040*a^8*b^2*c^8*e))/(32*(a^4*b^12
+ 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840
*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (3*(7340032*a^9*c^9*d - 256*a^2*b^14*c^
2*d + 7424*a^3*b^12*c^3*d - 94208*a^4*b^10*c^4*d + 675840*a^5*b^8*c^5*d - 2
949120*a^6*b^6*c^6*d + 7798784*a^7*b^4*c^7*d - 11534336*a^8*b^2*c^8*d))/(51
2*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^
6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + (root(56371445760*a^11*b^8*
c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 47185920*a^7*b^16*c^2*z^4 - 17179869
1840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4*c^8*z^4 - 128849018880*a^12*b
^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 + 3523215360*a^9*b^12*c^4*z^4 -
2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^10*z^4 + 65536*a^5*b^20*z^4 + 6
936330240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280
*a^9*b*c^9*d^2*z^2 - 1509949440*a^9*b^2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^

7*d^2*z^2 - 94464*a*b^17*c*d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054
656*a^5*b^9*c^5*d^2*z^2 - 188743680*a^7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^1
1*c^4*d^2*z^2 + 23592960*a^6*b^8*c^5*e^2*z^2 - 19860480*a^3*b^13*c^3*d^2*z^
2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 1771776*a^2*b^15*c^2*d^2*z^2 + 120795955
2*a^10*c^9*e^2*z^2 + 2304*b^19*d^2*z^2 - 428544*a*b^12*c^3*d^2*e*z + 102275
4816*a^6*b^2*c^8*d^2*e*z - 642318336*a^5*b^4*c^7*d^2*e*z + 223395840*a^4*b^
6*c^6*d^2*e*z - 46725120*a^3*b^8*c^5*d^2*e*z + 5930496*a^2*b^10*c^4*d^2*e*z
 - 693633024*a^7*c^9*d^2*e*z + 13824*b^14*c^2*d^2*e*z + 34836480*a^4*b*c^8*
d^2*e^2 - 435456*a*b^7*c^5*d^2*e^2 - 17418240*a^3*b^3*c^7*d^2*e^2 + 3919104
*a^2*b^5*c^6*d^2*e^2 + 20736*b^9*c^4*d^2*e^2 - 27433728*a^3*b^2*c^8*d^4 + 6
446304*a^2*b^4*c^7*d^4 - 734832*a*b^6*c^6*d^4 + 49787136*a^4*c^9*d^4 + 5308
416*a^5*c^8*e^4 + 35721*b^8*c^5*d^4, z, k)*x*(4194304*a^11*b*c^9 - 256*a^4*
b^15*c^2 + 7168*a^5*b^13*c^3 - 86016*a^6*b^11*c^4 + 573440*a^7*b^9*c^5 - 22
93760*a^8*b^7*c^6 + 5505024*a^9*b^5*c^7 - 7340032*a^10*b^3*c^8))/(32*(a^4*b
^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 +
3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5))) + (3*(1081344*a^6*b*c^8*d*e + 1536*a
^2*b^9*c^4*d*e - 29184*a^3*b^7*c^5*d*e + 227328...

3.53 $\int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{e(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3ce(b+2cx^2)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x(3b^4$$

```
[Out] -1/4*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*x*(b^2*d-2*a*c*d-a*b*
f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*e*(2*c*x^2+b)/
(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d-25*a*b^2*c*d+28*a^2*c^2*d+a*b
^3*f+8*a^2*b*c*f+c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)*x^2)/a^2/(-4*a*c
+b^2)^2/(c*x^4+b*x^2+a)-6*c^2*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(5/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*c^(1/2)*(3*b^4*d+b^3*(a*f+3*d*(-4*a*c+b^2)^(1/2))-4*a*b*c*(13*a*f+6*d*(-4*
a*c+b^2)^(1/2))-a*b^2*(30*c*d-f*(-4*a*c+b^2)^(1/2))+4*a^2*c*(42*c*d+5*f*(-4
*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b
^3*d-24*a*b*c*d+a*b^2*f+20*a^2*c*f+(52*a^2*b*c*f-168*a^2*c^2*d-a*b^3*f+30*a
*b^2*c*d-3*b^4*d)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.26, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3b^4d\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{x(cx^2(20a^2cf+ab^2f-24abcd+3b^3d)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3ce(b+2cx^2)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{e(b+2cx^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d -
2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x
*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*
d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*
x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a
*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f
) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f -
```

```
(3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx = \int \frac{ex}{(a+bx^2+cx^4)^3}\,dx + \int \frac{d+fx^2}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{-3b^2d+14acd-abf-5c(bd-2af)x^2}{(a+bx^2+cx^4)^2}\,dx}{4a(b^2-4ac)} + e\int \frac{}{(}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(3b^4d-25ab^2cd+28a^2c^2d+ab^3f+8a}{8a^2(b^2-4ac}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(3b^4d}{}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{}{2(b^2-}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{}{2(b^2-}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{}{2(b^2-}$$

Mathematica [A]
time = 2.18, size = 625, normalized size = 1.01

$$\frac{1}{16}\left(\frac{4ab(e+fx)-4bdx(b+cx^2)+8acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)^2} + \frac{6b^3dx(b+cx^2)+2abx(-25bcd+b^2f-24c^2dx^2+bcfx^2)+8a^2c(b(3e+2fx)+cx(7d+6ex+5fx^2))}{a^2(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d+af\right)-4abc\left(6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(-30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d-af\right)+4abc\left(-6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(-42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{48c^2e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}} - \frac{48c^2e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-25*
b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c*x*
(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[
b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*
```

```
(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*S
qrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2
*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48
*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(561) = 1122$.
time = 0.35, size = 1311, normalized size = 2.11

method	result
risch	$\frac{\frac{c^{2}\left(20a^{2}cf+a\,b^{2}f-24abcd+3b^{3}d\right)x^{7}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3c^{3}e\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{c\left(28a^{2}bcf+28a^{2}c^{2}d+2a\,b^{3}f-49a\,b^{2}cd+6b^{4}d\right)x^{5}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{9b\,c^{2}e\,x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(36a^{3}c^{2}f\cdots\right.}{}}{\left(c\,x^{4}+b\,x^{2}\cdots\right.}$
default	$64c^{3}\left(-\frac{-\frac{\left(720\sqrt{-4ac+b^{2}}\,a^{2}c^{2}d-312\sqrt{-4ac+b^{2}}\,a\,b^{2}cd+33\sqrt{-4ac+b^{2}}\,b^{4}d+800a^{3}c^{2}f-112a^{2}b^{2}cf-1104a^{2}b\,c^{2}d-22a\cdots\right.}{64a^{2}c^{2}\left(100ac+11b^{2}\right)}\cdots}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-1/64*(720*(-4*a*c
+b^2)^(1/2)*a^2*c^2*d-312*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+33*(-4*a*c+b^2)^(1/2
)*b^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b*c^2*d-22*a*b^4*f+408*a*b^3
*c*d-33*b^5*d)*(-b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)/a^2/c^2/(100*a*c+11*b^2)*
x^3-3/4*e*(4*a*c-b^2)/c*x^2-1/32*(1232*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-568*(-4
*a*c+b^2)^(1/2)*a*b^2*c*d+65*(-4*a*c+b^2)^(1/2)*b^4*d+1568*a^3*c^2*f-496*a^
2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*c*d-65*b^5*d)*(7*(-4*a*c+b^
2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/8*e*(16*a*c*(-4*a*c+b^2)^(1/2)-4*b
```

```
^2*(-4*a*c+b^2)^(1/2)+12*a*b*c-3*b^3)/c^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/
2*b/c)^2+1/16/a^2/c*(-24*(-4*a*c+b^2)^(1/2)*a^2*c*e*ln(b+2*c*x^2+(-4*a*c+b^
2)^(1/2))+1/2*(52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f-168*(-4*a*c+b^2)^(1/2)*a^2*c
^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*f+30*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-3*(-4*a*c+b
^2)^(1/2)*b^4*d-80*a^3*c^2*f+16*a^2*b^2*c*f+96*a^2*b*c^2*d+a*b^4*f-36*a*b^3
*c*d+3*b^5*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)*((-1/64*(-720*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+312*(-4*a*c+b^2)^(1/2)*a*b
^2*c*d-33*(-4*a*c+b^2)^(1/2)*b^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b
*c^2*d-22*a*b^4*f+408*a*b^3*c*d-33*b^5*d)*(b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)
/a^2/c^2/(100*a*c+11*b^2)*x^3-3/4*e*(4*a*c-b^2)/c*x^2-1/32*(-1232*(-4*a*c+b
^2)^(1/2)*a^2*c^2*d+568*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-65*(-4*a*c+b^2)^(1/2)*
b^4*d+1568*a^3*c^2*f-496*a^2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*
c*d-65*b^5*d)*(-7*(-4*a*c+b^2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/8*e*(-
16*a*c*(-4*a*c+b^2)^(1/2)+4*b^2*(-4*a*c+b^2)^(1/2)+12*a*b*c-3*b^3)/c^2)/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2+1/16/a^2/c*(24*(-4*a*c+b^2)^(1/2)*a^2
*c*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f
-168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*f+30*(-4*a*c+b^2
)^(1/2)*a*b^2*c*d-3*(-4*a*c+b^2)^(1/2)*b^4*d+80*a^3*c^2*f-16*a^2*b^2*c*f-96
*a^2*b*c^2*d-a*b^4*f+36*a*b^3*c*d-3*b^5*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*x^6*e + 36*a^2*b*c^2*x^4*e + (3*(b^3*c^2 - 8*a*b*c^3)*d + (
a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d +
 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 - 2*a^2*b^3*e + 20*a^3*b*c*e + ((3*b^5 -
 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 +
8*(a^2*b^2*c*e + 5*a^3*c^2*e)*x^2 + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*
d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((48*a^2*c^2*x*e + (3*(b^
3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 2
8*a^2*c^2)*d + (a*b^3 - 16*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5292 vs. $2(564) = 1128$.
time = 8.25, size = 5292, normalized size = 8.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))/((b^8 - 16*a*b^
6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 -
 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*c^2) + 3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*
a*c)*e*log(x^2 + 1/2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3))/((b^8 - 16*a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*
c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a
^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*c^2) + 1/32*(3*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6
*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 2*b^8*c + 116*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2
 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 232*a^2*b^4*c^3 - 30*a*b
^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*
```

sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^6*c - 26*(b^2 - 4*a*c)
*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(
b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b
*c^4)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48*a^2*b^5*c^2 + 2*a*b^6*c^2 -
256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*c^4 + 64*a^3*b^2*c^4 + 320*a
^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 +
22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 32*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 36*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 160*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 18*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 40*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^5
c - 40(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 128*(b^2 -
4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3 + 80*(b^2 - 4*a*c)*a^3*c^4)
*f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(
(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 +
24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b
^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c))
+ 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^

```
2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*...
```

Mupad [B]
time = 3.26, size = 2500, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^2*(5*a*c^2*e + b^2*c*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3*e - 10*a
*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*a^2*c^3*d + 6*b^4*c*d
 + 2*a*b^3*c*f - 49*a*b^2*c^2*d + 28*a^2*b*c^2*f))/(8*a^2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x*(5*b^4*d + 44*a^2*c^2*d - a*b^3*f - 37*a*b^2*c*d + 16*a
^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e*x^6)/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^3*(3*b^5*d + 36*a^3*c^2*f + a*b^4*f - 20*a*b^3*c*d
 - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (9*b*c^2*e*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a^2*c^2*f
+ 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + symsum(
log(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 471859
20*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4
*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 +
3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^1
0*z^4 + 65536*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321205760*a^9*b^2*
c^8*d*f*z^2 + 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2
 - 330301440*a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 15175680
*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 440401920*a^10*b*c^8
*f^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*c*f^2*z^2 + 6936330
240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b
*c^9*d^2*z^2 - 1509949440*a^9*b^2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*
z^2 - 94464*a*b^17*c*d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^
5*b^9*c^5*d^2*z^2 + 477102080*a^9*b^3*c^7*f^2*z^2 - 174325760*a^8*b^5*c^6*f
^2*z^2 - 188743680*a^7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 1
1206656*a^7*b^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^2*z^2 + 23592960*a^6*b^
8*c^5*e^2*z^2 - 2600960*a^5*b^11*c^3*f^2*z^2 + 291840*a^4*b^13*c^2*f^2*z^2
- 19860480*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 1771776*a^
2*b^15*c^2*d^2*z^2 + 1536*a*b^18*d*f*z^2 + 1207959552*a^10*c^9*e^2*z^2 + 25
6*a^2*b^17*f^2*z^2 + 2304*b^19*d^2*z^2 + 169869312*a^7*b*c^8*d*e*f*z + 9216
*a*b^13*c^2*d*e*f*z - 221773824*a^6*b^3*c^7*d*e*f*z + 117964800*a^5*b^5*c^6
*d*e*f*z - 32440320*a^4*b^7*c^5*d*e*f*z + 4792320*a^3*b^9*c^4*d*e*f*z - 350
208*a^2*b^11*c^3*d*e*f*z - 428544*a*b^12*c^3*d^2*e*z + 1022754816*a^6*b^2*c
^8*d^2*e*z - 642318336*a^5*b^4*c^7*d^2*e*z + 223395840*a^4*b^6*c^6*d^2*e*z
- 50724864*a^7*b^2*c^7*e*f^2*z + 26542080*a^6*b^4*c^6*e*f^2*z - 46725120*a^
3*b^8*c^5*d^2*e*z - 7127040*a^5*b^6*c^5*e*f^2*z + 1013760*a^4*b^8*c^4*e*f^2
```

```
*z - 69120*a^3*b^10*c^3*e*f^2*z + 1536*a^2*b^12*c^2*e*f^2*z + 5930496*a^2*b
^10*c^4*d^2*e*z - 693633024*a^7*c^9*d^2*e*z + 39321600*a^8*c^8*e*f^2*z + 13
824*b^14*c^2*d^2*e*z + 13824*a*b^8*c^4*d*e^2*f - 7741440*a^4*b^2*c^7*d*e^2*
f + 2903040*a^3*b^4*c^6*d*e^2*f - 387072*a^2*b^6*c^5*d*e^2*f + 37310976*a^3
*b^3*c^7*d^3*f + 3870720*a^5*b*c^7*e^2*f^2 + 34836480*a^4*b*c^8*d^2*e^2 - 8
068032*a^2*b^5*c^6*d^3*f - 5623296*a^4*b^3*c^6*d*f^3 + 1737792*a^3*b^5*c^5*
d*f^3 - 260190*a*b^8*c^4*d^2*f^2 - 211680*a^2*b^7*c^4*d*f^3 - 435456*a*b^7*
c^5*d^2*e^2 - 75188736*a^4*b*c^8*d^3*f - 15482880*a^5*c^8*d*e^2*f - 4262400
*a^5*b*c^7*d*f^3 + 852768*a*b^7*c^5*d^3*f + 7350*a*b^9*c^3*d*f^3 + 35525376
*a^4*b^2*c^7*d^2*f^2 + 645120*a^4*b^3*c^6*e^2*f^2 - 80640*a^3*b^5*c^5*e^2*f
^2 + 2304*a^2*b^7*c^4*e^2*f^2 - 15269184*a^3*b^4*c^6*d^2*f^2 + 2870784*a^2*
b^6*c^5*d^2*f^2 - 17418240*a^3*b^3*c^7*d^2*e^2 + 3919104*a^2*b^5*c^6*d^2*e^
2 + 11025*b^10*c^3*d^2*f^2 + 5644800*a^5*c^8*d^2*f^2 + 20736*b^9*c^4*d^2*e^
2 + 492800*a^5*b^2*c^6*f^4 + 351456*a^4*b^4*c^5*f^4 - 43120*a^3*b^6*c^4*f^4
 + 1225*a^2*b^8*c^3*f^4 - 27433728*a^3*b^2*c^8*d^4 + 6446304*a^2*b^4*c^7*d^
4 - 39690*b^9*c^4*d^3*f - 734832*a*b^6*c^6*d^4 + 49787136*a^4*c^9*d^4 + 160
000*a^6*c^7*f^4 + 5308416*a^5*c^8*e^4 + 35721*b^8*c^5*d^4, z, k)*(root(5637
1445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 47185920*a^7*b^16*c
^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4*c^8*z^4 - 12
8849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 + 3523215360*a^
9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^10*z^4 + 65536
*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321205760*a^9*b^2*c^8*d*f*z^2 +
 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2 - 330301440*
a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 15175680*a^4*b^12*c^3
*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 440401920*a^10*b*c^8*f^2*z^2 + 17
61607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*c*f^2*z^2 + 6936330240*a^8*b^3*c
^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2
- 1509949440*a^9*b^2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a
*b^17*c*d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2
*z^2 + 477102080*a^9*b^3*c^7*f^2*z^2 - 174325760*a^8*b^5*c^6*f^2*z^2 - 1887
43680*a^7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 11206656*a^7*b
^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^2*z^2 + ...
```

3.54 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=646

$$\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{be-2ag+(2ce-bg)x^2}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{3(2ce-bg)\,(b+2cx^2)}{4\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{x(3b^4d$$

```
[Out] 1/4*x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)^2+1/4*(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*(
-b*g+2*c*e)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d-25*a*
b^2*c*d+28*a^2*c^2*d+a*b^3*f+8*a^2*b*c*f+c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3
*b^3*d)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-3*c*(-b*g+2*c*e)*arctanh((2
*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)+1/16*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^4*d+b^3*(a*f+3*d*(-4*a*c+b^2
)^(1/2))-4*a*b*c*(13*a*f+6*d*(-4*a*c+b^2)^(1/2))-a*b^2*(30*c*d-f*(-4*a*c+b^
2)^(1/2))+4*a^2*c*(42*c*d+5*f*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(5/2)*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3*d-24*a*b*c*d+a*b^2*f+20*a^2*c*f+(52*a^2
*b*c*f-168*a^2*c^2*d-a*b^3*f+30*a*b^2*c*d-3*b^4*d)/(-4*a*c+b^2)^(1/2))/a^2/
(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.48, antiderivative size = 646, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1687, 1192, 1180, 211, 1261, 652, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)}{8\sqrt{2}\,a^2\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3b^4d\right)}{8\sqrt{2}\,a^2\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{x(cx^2(20a^2cf+ab^2f-24abcd+3b^3d)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d)}{8a^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{4a\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3(b+2cx^2)\,(2ce-bg)}{4\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{-2ag+x^2(2ce-bg)+be}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(4*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + (x*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f +
8*a^2*b*c*f + c*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2
- 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*
d - Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*
a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d +
```

```
a*b^2*f + 20*a^2*c*f - (3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f -
52*a^2*b*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
- (3*c*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c
)^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^3}\,dx = \int \frac{d+fx^2}{(a+bx^2+cx^4)^3}\,dx + \int \frac{x(e+gx^2)}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{(a+bx+cx^2)^3}\,dx, x, x^2\right.$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(3b^4}{}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(}{4(b^2}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(}{4(b^2}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(}{4(b^2}$$

Mathematica [A]
time = 2.57, size = 661, normalized size = 1.02

$$\frac{1}{16}\left(\frac{-8a^2g-4bdx(b+cx^2)+8acx(d+x(e+fx))+4ab(e+x(f-gx))}{a(-b^2+4ac)(a+bx^2+cx^4)^2}+\frac{2(3b^3dx(b+cx^2)+abx(-25bcd+b^2f-24c^2dx^2+bcfx^2)+a^2(-6b^2g+4c^2x(7d+6ex+5fx^2)+4bc(3e+2fx-3gx^2)))}{a^2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d+af\right)-4abc\left(6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(-30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d-af\right)+4abc\left(-6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(-42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{24c(-2ce+bg)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}}+\frac{24c(-2ce+bg)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-8*a^2*g - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)) + 4*a*b*(e + x
*(f - g*x)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*(3*b^3*d*x*(b +
c*x^2) + a*b*x*(-25*b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + a^2*(-6*b^2
*g + 4*c^2*x*(7*d + 6*e*x + 5*f*x^2) + 4*b*c*(3*e + 2*f*x - 3*g*x^2))))/(a^
2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3
*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) + a*
b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4
*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-3*b^4*d + b^3
*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*Sqrt[b^2 - 4*a*c]*d + 13*a*f)
+ a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*(-42*c*d + 5*Sqrt[b^2 - 4*
```

```
a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2
 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (24*c*(-2*c*e + b*g)*Log[-b
+ Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (24*c*(-2*c*e + b*g)*
Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1637 vs. $2(588) = 1176$.
time = 0.32, size = 1638, normalized size = 2.54

method	result
risch	$\frac{\frac{c^2\left(20a^2cf+a\,b^2f-24abcd+3b^3d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3(bg-2ce)c^2x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(28a^2bcf+28a^2c^2d+2a\,b^3f-49a\,b^2cd+6b^4d\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9b(bg-2ce)c\,x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{(36a^3 \cdots}{}}{(c\,x^4 \cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-1/64*(720*(-4*a*c
+b^2)^(1/2)*a^2*c^2*d-312*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+33*(-4*a*c+b^2)^(1/2
)*b^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b*c^2*d-22*a*b^4*f+408*a*b^3
*c*d-33*b^5*d)*(-b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)/a^2/c^2/(100*a*c+11*b^2)*
x^3-1/8/c^2*(-4*(-4*a*c+b^2)^(1/2)*a*c*g+(-4*a*c+b^2)^(1/2)*b^2*g-12*a*b*g*
c+24*a*c^2*e+3*b^3*g-6*b^2*c*e)*x^2-1/32*(1232*(-4*a*c+b^2)^(1/2)*a^2*c^2*d
-568*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+65*(-4*a*c+b^2)^(1/2)*b^4*d+1568*a^3*c^2*
f-496*a^2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*c*d-65*b^5*d)*(7*(-
4*a*c+b^2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/16*(-128*(-4*a*c+b^2)^(1/2
)*a^2*c^2*g+52*(-4*a*c+b^2)^(1/2)*a*b^2*c*g-5*(-4*a*c+b^2)^(1/2)*b^4*g-224*
a^2*c^2*b*g+512*a^2*c^3*e+76*a*b^3*c*g-184*a*b^2*c^2*e-5*b^5*g+14*b^4*c*e)*
(4*(-4*a*c+b^2)^(1/2)+3*b)/(64*a*c-7*b^2)/c^3)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2
)+1/2*b/c)^2+1/16/a^2/c*(1/4*(48*(-4*a*c+b^2)^(1/2)*a^2*b*c*g-96*(-4*a*c+b^
2)^(1/2)*a^2*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(52*(-4*a*c+b^2)
^(1/2)*a^2*b*c*f-168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*
f+30*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-3*(-4*a*c+b^2)^(1/2)*b^4*d-80*a^3*c^2*f+1
6*a^2*b^2*c*f+96*a^2*b*c^2*d+a*b^4*f-36*a*b^3*c*d+3*b^5*d)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))))-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-1/64*(-720*(-4*a*c+b^
2)^(1/2)*a^2*c^2*d+312*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-33*(-4*a*c+b^2)^(1/2)*b
^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b*c^2*d-22*a*b^4*f+408*a*b^3*c*
d-33*b^5*d)*(b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)/a^2/c^2/(100*a*c+11*b^2)*x^3-
1/8/c^2*(4*(-4*a*c+b^2)^(1/2)*a*c*g-(-4*a*c+b^2)^(1/2)*b^2*g-12*a*b*g*c+24*
a*c^2*e+3*b^3*g-6*b^2*c*e)*x^2-1/32*(-1232*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+568
*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-65*(-4*a*c+b^2)^(1/2)*b^4*d+1568*a^3*c^2*f-49
```

```
6*a^2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*c*d-65*b^5*d)*(-7*(-4*a
*c+b^2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/16*(128*(-4*a*c+b^2)^(1/2)*a^
2*c^2*g-52*(-4*a*c+b^2)^(1/2)*a*b^2*c*g+5*(-4*a*c+b^2)^(1/2)*b^4*g-224*a^2*
c^2*b*g+512*a^2*c^3*e+76*a*b^3*c*g-184*a*b^2*c^2*e-5*b^5*g+14*b^4*c*e)*(-4*
(-4*a*c+b^2)^(1/2)+3*b)/(64*a*c-7*b^2)/c^3)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)
^(1/2))^2+1/16/a^2/c*(-1/4*(48*(-4*a*c+b^2)^(1/2)*a^2*b*c*g-96*(-4*a*c+b^2)
^(1/2)*a^2*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(52*(-4*a*c+b^2)^
(1/2)*a^2*b*c*f-168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*f
+30*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-3*(-4*a*c+b^2)^(1/2)*b^4*d+80*a^3*c^2*f-16
*a^2*b^2*c*f-96*a^2*b*c^2*d-a*b^4*f+36*a*b^3*c*d-3*b^5*d)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*(b^3*c^2 - 8*a*b*c^3)*d + (a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 - 12*(a^2
*b*c^2*g - 2*a^2*c^3*e)*x^6 + ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d + 2*
(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 - 2*a^2*b^3*e + 20*a^3*b*c*e - 18*(a^2*b^2*
c*g - 2*a^2*b*c^2*e)*x^4 + ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 +
 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 + 4*(2*a^2*b^2*c*e + 10*a^3*c^2*e - (a^2*
b^3 + 5*a^3*b*c)*g)*x^2 - 2*(a^3*b^2 + 8*a^4*c)*g + ((5*a*b^4 - 37*a^2*b^2*
c + 44*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*
c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x
^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate(((3*(b^3
*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 28
*a^2*c^2)*d + (a*b^3 - 16*a^2*b*c)*f - 24*(a^2*b*c*g - 2*a^2*c^2*e)*x)/(c*x
^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5439 vs. 2(588) = 1176.
time = 9.38, size = 5439, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176
*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^
4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c
- 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2
- 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2
 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*
c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*
```

```
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48
*a^2*b^5*c^2 + 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*
b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*
c^4 + 64*a^3*b^2*c^4 + 320*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^2 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*
a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3
 + 80*(b^2 - 4*a*c)*a^3*c^4)*f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*
b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*
a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3
 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2
- 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 - 736*a^3*b^2*c^4 - 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b^2...
```

Mupad [B]
time = 4.56, size = 2500, normalized size = 3.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] symsum(log((x*(13824*a^4*c^8*e^3 - 54*b^7*c^5*d^2*e + 27*b^8*c^4*d^2*g - 17
28*a^4*b^3*c^5*g^3 - 20160*a^4*c^8*d*e*f + 972*a*b^5*c^6*d^2*e + 24192*a^3*
b*c^8*d^2*e - 486*a*b^6*c^5*d^2*g + 6240*a^4*b*c^7*e*f^2 - 20736*a^4*b*c^7*
e^2*g - 7344*a^2*b^3*c^7*d^2*e + 3672*a^2*b^4*c^6*d^2*g - 6*a^2*b^5*c^5*e*f
^2 - 12096*a^3*b^2*c^7*d^2*g + 192*a^3*b^3*c^6*e*f^2 + 10368*a^4*b^2*c^6*e*
g^2 + 3*a^2*b^6*c^4*f^2*g - 96*a^3*b^4*c^5*f^2*g - 3120*a^4*b^2*c^6*f^2*g -
 36*a*b^6*c^5*d*e*f + 18*a*b^7*c^4*d*f*g + 10080*a^4*b*c^7*d*f*g + 900*a^2*
b^4*c^6*d*e*f - 4896*a^3*b^2*c^7*d*e*f - 450*a^2*b^5*c^5*d*f*g + 2448*a^3*b
^3*c^6*d*f*g))/(64*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*
c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) - root(56371
445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 47185920*a^7*b^16*c^
2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4*c^8*z^4 - 128
849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 + 3523215360*a^9
*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^10*z^4 + 65536*
a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 + 1509949440*a^9*b^3*c^7*e*g*z^2 -
1321205760*a^9*b^2*c^8*d*f*z^2 - 754974720*a^8*b^5*c^6*e*g*z^2 + 732168192*
a^7*b^6*c^6*d*f*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2 - 330301440*a^8*b^4*c^7
*d*f*z^2 + 188743680*a^7*b^7*c^5*e*g*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 -
23592960*a^6*b^9*c^4*e*g*z^2 + 1179648*a^5*b^11*c^3*e*g*z^2 - 15175680*a^4*
b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 1207959552*a^10*b*c^8*e*g
*z^2 - 440401920*a^10*b*c^8*f^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 - 14080*a
^3*b^15*c*f^2*z^2 + 6936330240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6
*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2 - 1509949440*a^9*b^2*c^8*e^2*z^2 -
5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a*b^17*c*d^2*z^2 + 754974720*a^8*b^4
*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2*z^2 + 477102080*a^9*b^3*c^7*f^2*z^
2 - 377487360*a^9*b^4*c^6*g^2*z^2 + 301989888*a^10*b^2*c^7*g^2*z^2 + 188743
680*a^8*b^6*c^5*g^2*z^2 - 174325760*a^8*b^5*c^6*f^2*z^2 - 188743680*a^7*b^6
*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 - 47185920*a^7*b^8*c^4*g^2*z^
2 + 5898240*a^6*b^10*c^3*g^2*z^2 - 294912*a^5*b^12*c^2*g^2*z^2 + 11206656*a
^7*b^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^2*z^2 + 23592960*a^6*b^8*c^5*e^2
*z^2 - 2600960*a^5*b^11*c^3*f^2*z^2 + 291840*a^4*b^13*c^2*f^2*z^2 - 1986048
0*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 1771776*a^2*b^15*c^
2*d^2*z^2 + 1536*a*b^18*d*f*z^2 + 1207959552*a^10*c^9*e^2*z^2 + 256*a^2*b^1
7*f^2*z^2 + 2304*b^19*d^2*z^2 + 169869312*a^7*b*c^8*d*e*f*z + 9216*a*b^13*c
^2*d*e*f*z - 4608*a*b^14*c*d*f*g*z - 221773824*a^6*b^3*c^7*d*e*f*z + 110886
912*a^6*b^4*c^6*d*f*g*z - 84934656*a^7*b^2*c^7*d*f*g*z + 117964800*a^5*b^5*
c^6*d*e*f*z - 58982400*a^5*b^6*c^5*d*f*g*z + 16220160*a^4*b^8*c^4*d*f*g*z -
 2396160*a^3*b^10*c^3*d*f*g*z + 175104*a^2*b^12*c^2*d*f*g*z - 32440320*a^4*
b^7*c^5*d*e*f*z + 4792320*a^3*b^9*c^4*d*e*f*z - 350208*a^2*b^11*c^3*d*e*f*z
 + 346816512*a^7*b*c^8*d^2*g*z - 19660800*a^8*b*c^7*f^2*g*z - 768*a^2*b^13*
c*f^2*g*z + 214272*a*b^13*c^2*d^2*g*z - 428544*a*b^12*c^3*d^2*e*z + 1022754
```

816*a^6*b^2*c^8*d^2*e*z - 642318336*a^5*b^4*c^7*d^2*e*z - 511377408*a^6*b^3
*c^7*d^2*g*z + 321159168*a^5*b^5*c^6*d^2*g*z + 223395840*a^4*b^6*c^6*d^2*e*
z - 111697920*a^4*b^7*c^5*d^2*g*z + 25362432*a^7*b^3*c^6*f^2*g*z - 50724864
*a^7*b^2*c^7*e*f^2*z - 13271040*a^6*b^5*c^5*f^2*g*z + 3563520*a^5*b^7*c^4*f
^2*g*z - 506880*a^4*b^9*c^3*f^2*g*z + 34560*a^3*b^11*c^2*f^2*g*z + 26542080
*a^6*b^4*c^6*e*f^2*z + 23362560*a^3*b^9*c^4*d^2*g*z - 46725120*a^3*b^8*c^5*
d^2*e*z - 7127040*a^5*b^6*c^5*e*f^2*z - 2965248*a^2*b^11*c^3*d^2*g*z + 1013
760*a^4*b^8*c^4*e*f^2*z - 69120*a^3*b^10*c^3*e*f^2*z + 1536*a^2*b^12*c^2*e*
f^2*z + 5930496*a^2*b^10*c^4*d^2*e*z - 693633024*a^7*c^9*d^2*e*z + 39321600
*a^8*c^8*e*f^2*z + 13824*b^14*c^2*d^2*e*z - 6912*b^15*c*d^2*g*z + 15482880*
a^5*b*c^7*d*e*f*g - 13824*a*b^9*c^3*d*e*f*g + 7741440*a^4*b^3*c^6*d*e*f*g -
 2903040*a^3*b^5*c^5*d*e*f*g + 387072*a^2*b^7*c^4*d*e*f*g + 3456*a*b^10*c^2
*d*f*g^2 + 435456*a*b^8*c^4*d^2*e*g + 13824*a*b^8*c^4*d*e^2*f - 3870720*a^5
*b^2*c^6*e*f^2*g - 34836480*a^4*b^2*c^7*d^2*e*g - 645120*a^4*b^4*c^5*e*f^2*
g + 80640*a^3*b^6*c^4*e*f^2*g - 2304*a^2*b^8*c^3*e*f^2*g - 3870720*a^5*b^2*
c^6*d*f*g^2 - 1935360*a^4*b^4*c^5*d*f*g^2 + 725760*a^3*b^6*c^4*d*f*g^2 + 17
418240*a^3*b^4*c^6*d^2*e*g - 96768*a^2*b^8*c^3*d*f*g^2 - 3919104*a^2*b^6*c^
5*d^2*e*g - 7741440*a^4*b^2*c^7*d*e^2*f + 2903040*a^3*b^4*c^6*d*e^2*f - 387
072*a^2*b^6*c^5*d*e^2*f + 37310976*a^3*b^3*c^7*d^3*f - 2654208*a^5*b^3*c^5*
e*g^3 + 3870720*a^5*b*c^7*e^2*f^2 + 34836480*a^4*b*c^8*d^2*e^2 - 108864*a*b
^9*c^3*d^2*g^2 - 8068032*a^2*b^5*c^6*d^3*f - 5623296*a^4*b^3*c^6*d*f^3 + 17
37792*a^3*b^5*c^5*d*f^3 - 260190*a*b^8*c^4*d^2*f^2 - 211680*a^2*b^7*c^4*d*f
^3 - 435456*a*b^7*c^5*d^2*e^2 - 20736*b^10*c^3*d^2*e*g - 75188736*a^4*b*c^8
*d^3*f - 15482880*a^5*c^8*d*e^2*f - 10616832*a^5*b*c^7*e^3*g - 4262400*a^5*
b*c^7*d*f^3 + 852768*a*b^7*c^5*d^3*f + 7350*a*b^9*c^3*d*f^3 + 967680*a^5*b^
3*c^5*f^2*g^2 + 161280*a^4*b^5*c^4*f^2*g^2 - 20...

3.55 $\int \frac{d+ex+fx^2+gx^3+hx^4}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=679

$$-\frac{be-2ag+(2ce-bg)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3(2ce-bg)\,(b+2}{4\left(b^2-4ac\right)^2\left(a+bx^2\right.}$$

```
[Out] 1/4*(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*x*(b^2
*d-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*
x^2+a)^2+3/4*(-b*g+2*c*e)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*
(3*b^4*d+a*b^3*f+8*a^2*b*c*f+4*a^2*c*(a*h+7*c*d)-a*b^2*(7*a*h+25*c*d)+c*(3*
b^3*d+a*b^2*f+20*a^2*c*f-12*a*b*(a*h+2*c*d))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4
+b*x^2+a)-3*c*(-b*g+2*c*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(5/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1
/2)*(3*b^3*d+a*b^2*f+20*a^2*c*f-12*a*b*(a*h+2*c*d)+(3*b^4*d+a*b^3*f-52*a^2*
b*c*f-6*a*b^2*(-3*a*h+5*c*d)+24*a^2*c*(a*h+7*c*d))/(-4*a*c+b^2)^(1/2))/a^2/
(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3*d+a*b^2*f+20*a^2*c*f-12
*a*b*(a*h+2*c*d)+(-3*b^4*d-a*b^3*f+52*a^2*b*c*f+6*a*b^2*(-3*a*h+5*c*d)-24*a
^2*c*(a*h+7*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.86, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1687, 1692, 1192, 1180, 211, 1261, 652, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3c(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3\left(b+2cx^2\right)(2ce-bg)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2ag+x^2(2ce-bg)+be}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2
) + (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(
4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*(2*c*e - b*g)*(b + 2*c*x^2))/
(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c
*f + 4*a^2*c*(7*c*d + a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f
+ 20*a^2*c*f - 12*a*b*(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*
h) + (3*b^4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c
*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
```

```
[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[c]*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^
4*d + a*b^3*f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d +
a*h))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (3*c*(
2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x(e+gx^2)}{(a+bx^2+cx^4)^3}\,dx + \int \frac{d+fx^2+hx^4}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{1}{2}\text{Subst}\Bigg(\int \frac{}{(a+}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2}{4a(b^2-4ac)(a+bx^2+cx}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2}{4a(b^2-4ac)(a+bx^2+cx}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2}{4a(b^2-4ac)(a+bx^2+cx}$$

$$= -\frac{be-2ag+(2ce-bg)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2}{4a(b^2-4ac)(a+bx^2+cx}$$

Mathematica [A]
time = 4.13, size = 739, normalized size = 1.09

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-8*a^2*(g + h*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)) + 4*a*
b*(e + x*(f - x*(g + h*x)))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*
(4*a^3*c*h*x + 3*b^3*d*x*(b + c*x^2) + a*b*x*(b^2*f - 24*c^2*d*x^2 + b*c*(-
25*d + f*x^2)) + a^2*(-(b^2*(6*g + 7*h*x)) + 4*c^2*x*(7*d + x*(6*e + 5*f*x)
) + 4*b*c*(3*e + x*(2*f - 3*x*(g + h*x))))))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^
2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f)
 + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f + 6*a*h) + a*b^2*(-30*c*d + Sqrt
[b^2 - 4*a*c]*f + 18*a*h) - 4*a*b*(6*c*Sqrt[b^2 - 4*a*c]*d + 13*a*c*f + 3*a
*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[
c]*(3*b^4*d + b^3*(-3*Sqrt[b^2 - 4*a*c]*d + a*f) + 4*a^2*c*(42*c*d - 5*Sqrt
```

```
[b^2 - 4*a*c]*f + 6*a*h) + a*b^2*(-30*c*d - Sqrt[b^2 - 4*a*c]*f + 18*a*h) +
 4*a*b*(6*c*Sqrt[b^2 - 4*a*c]*d - 13*a*c*f + 3*a*Sqrt[b^2 - 4*a*c]*h))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (24*c*(-2*c*e + b*g)*Log[-b + Sqrt[b^2 -
4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (24*c*(-2*c*e + b*g)*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [A]
time = 0.14, size = 1165, normalized size = 1.72

method	result
risch	$-\frac{c^2\left(12a^2bh-20a^2cf-a\,b^2f+24abcd-3b^3d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3(bg-2ce)c^2x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(4a^3ch-19a^2b^2h+28a^2bcf+28a^2c^2d+2a\,b^3f-49a\,b^2cd+6b^4d\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{}{4(1}$
default	$-\frac{c^2\left(12a^2bh-20a^2cf-a\,b^2f+24abcd-3b^3d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3(bg-2ce)c^2x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{c\left(4a^3ch-19a^2b^2h+28a^2bcf+28a^2c^2d+2a\,b^3f-49a\,b^2cd+6b^4d\right)x^5}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{}{4(1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*c^2*(12*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^7-3/2*(b*g-2*c*e)*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8
/a^2*c*(4*a^3*c*h-19*a^2*b^2*h+28*a^2*b*c*f+28*a^2*c^2*d+2*a*b^3*f-49*a*b^2
*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-9/4*b*(b*g-2*c*e)*c/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^4-1/8*(16*a^3*b*c*h-36*a^3*c^2*f+5*a^2*b^3*h-5*a^2*b^2*c
*f+4*a^2*b*c^2*d-a*b^4*f+20*a*b^3*c*d-3*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/2*(5*a*c+b^2)*(b*g-2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(12*a
^3*c*h+3*a^2*b^2*h-16*a^2*b*c*f-44*a^2*c^2*d+a*b^3*f+37*a*b^2*c*d-5*b^4*d)/
(16*a^2*c^2-8*a*b^2*c+b^4)/a*x-1/4*(8*a^2*c*g+a*b^2*g-10*a*b*c*e+b^3*e)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/2/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*c*(1/(16*a*c-4*b^2)*(-1/4*(-48*(-4*a*c+b^2)^(1/2)*a^2*b*c*g+96*(-4*a*c+b
^2)^(1/2)*a^2*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(24*(-4*a*c+b^
2)^(1/2)*a^3*c*h+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*h-52*(-4*a*c+b^2)^(1/2)*a^2*
b*c*f+168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+(-4*a*c+b^2)^(1/2)*a*b^3*f-30*(-4*a*
```

```
c+b^2)^(1/2)*a*b^2*c*d+3*(-4*a*c+b^2)^(1/2)*b^4*d+48*a^3*b*c*h-80*a^3*c^2*f
-12*a^2*b^3*h+16*a^2*b^2*c*f+96*a^2*b*c^2*d+a*b^4*f-36*a*b^3*c*d+3*b^5*d)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(-48*(-4*a*c+b^2)^(1/2)*a^2*b*c
*g+96*(-4*a*c+b^2)^(1/2)*a^2*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*
(24*(-4*a*c+b^2)^(1/2)*a^3*c*h+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*h-52*(-4*a*c+b
^2)^(1/2)*a^2*b*c*f+168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+(-4*a*c+b^2)^(1/2)*a*b
^3*f-30*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+3*(-4*a*c+b^2)^(1/2)*b^4*d-48*a^3*b*c*
h+80*a^3*c^2*f+12*a^2*b^3*h-16*a^2*b^2*c*f-96*a^2*b*c^2*d-a*b^4*f+36*a*b^3*
c*d-3*b^5*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima"
)
```

```
[Out] -1/8*((12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^2 + 20*a^2*c^3
)*f)*x^7 + 12*(a^2*b*c^2*g - 2*a^2*c^3*e)*x^6 - ((6*b^4*c - 49*a*b^2*c^2 +
28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f - (19*a^2*b^2*c - 4*a^3*c^2)*h
)*x^5 + 2*a^2*b^3*e - 20*a^3*b*c*e + 18*(a^2*b^2*c*g - 2*a^2*b*c^2*e)*x^4 -
 ((3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)
*f - (5*a^2*b^3 + 16*a^3*b*c)*h)*x^3 - 4*(2*a^2*b^2*c*e + 10*a^3*c^2*e - (a
^2*b^3 + 5*a^3*b*c)*g)*x^2 + 2*(a^3*b^2 + 8*a^4*c)*g - ((5*a*b^4 - 37*a^2*b
^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f - 3*(a^3*b^2 + 4*a^4*c)*h)*
x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6
- 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*
x^2) - 1/8*integrate(((12*a^2*b*c*h - 3*(b^3*c - 8*a*b*c^2)*d - (a*b^2*c +
20*a^2*c^2)*f)*x^2 - 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d - (a*b^3 - 16*a^2*b
*c)*f - 3*(a^2*b^2 + 4*a^3*c)*h + 24*(a^2*b*c*g - 2*a^2*c^2*e)*x)/(c*x^4 +
b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7044 vs. 2(629) = 1258.
time = 8.50, size = 7044, normalized size = 10.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 +
 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 - 2*b^7*c^2 - 368*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
3 - 232*a^2*b^4*c^3 + 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 - 176
*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^
4*c^5 + 352*a^3*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^7 - 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^5*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c
+ 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2
+ 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 +
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 176*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 88*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 11*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 44*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a
*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 + 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2
 - 4*a*c)*a^2*b^2*c^3 - 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*
c^4 + 88*(b^2 - 4*a*c)*a^2*b*c^4)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2
```

```
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48
*a^2*b^5*c^2 - 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 + 44*a^2*
b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*
c^4 - 64*a^3*b^2*c^4 - 320*a^4*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^6 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c^2 + 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^2 + 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 + 2*(b^2 - 4*
a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 - 36*(b^2 - 4*a*c)*a^2*b^2*c^3
 - 80*(b^2 - 4*a*c)*a^3*c^4)*f + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*a^2*b^6*c - 16*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^4*c^2 + 8*a^3*b^4*c^2 - 2*a^2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^5*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^3 + 32*a^4*b^2*c^3 - 16*a^3*b^3*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^4 - 128*a^5*c^4 + 96*a^4*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b^4*c + 2*(b^2 - 4*a*c)*a
^2*b^3*c^2 - 32*(b^2 - 4*a*c)*a^4*c^3 + 24*(b^2 - 4*a*c)*a^3*b*c^3)*h)*arct
an(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5
 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(
a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a
^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*
b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 +
 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*
(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8...
```

Mupad [B]

time = 5.35, size = 2500, normalized size = 3.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((9*x^4*(2*b*c^2*e - b^2*c*g))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(b
^3*g - 10*a*c^2*e - 2*b^2*c*e + 5*a*b*c*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) - (b^3*e + a*b^2*g + 8*a^2*c*g - 10*a*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x^5*(28*a^2*c^3*d + 4*a^3*c^2*h + 6*b^4*c*d + 2*a*b^3*c*f - 49*
a*b^2*c^2*d + 28*a^2*b*c^2*f - 19*a^2*b^2*c*h))/(8*a^2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^3*(3*b^5*d + 36*a^3*c^2*f - 5*a^2*b^3*h + a*b^4*f - 20*a*b
^3*c*d - 16*a^3*b*c*h - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x*(3*a^2*b^2*h - 44*a^2*c^2*d - 5*b^4*d + a*b^3*f +
12*a^3*c*h + 37*a*b^2*c*d - 16*a^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (3*c^2*x^6*(2*c*e - b*g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^
7*(20*a^2*c^2*f + 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f - 12*a^2*b*c*h))/(8*
a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2
*a*b*x^2 + 2*b*c*x^6) + symsum(log((10368*a*b^5*c^6*d^3 - 8000*a^5*c^7*f^3
- 567*b^7*c^5*d^3 + 169344*a^3*b*c^8*d^3 + 193536*a^4*c^8*d*e^2 - 141120*a^
4*c^8*d^2*f + 1728*a^6*b*c^5*h^3 + 315*b^8*c^4*d^2*f + 27648*a^5*c^7*e^2*h
- 135*b^9*c^3*d^2*h - 2880*a^6*c^6*f*h^2 - 67824*a^2*b^3*c^7*d^3 + 35*a^2*b
^6*c^4*f^3 + 84*a^3*b^4*c^5*f^3 - 12720*a^4*b^2*c^6*f^3 + 540*a^4*b^5*c^3*h
^3 + 4320*a^5*b^3*c^4*h^3 - 40320*a^5*c^7*d*f*h - 6237*a*b^6*c^5*d^2*f + 21
0*a*b^7*c^4*d*f^2 + 116160*a^4*b*c^7*d*f^2 - 36864*a^4*b*c^7*e^2*f + 2430*a
*b^7*c^4*d^2*h + 133056*a^4*b*c^7*d^2*h + 27648*a^5*b*c^6*d*h^2 + 26880*a^5
*b*c^6*f^2*h + 6912*a^2*b^4*c^6*d*e^2 - 62208*a^3*b^2*c^7*d*e^2 + 42372*a^2
*b^4*c^6*d^2*f - 1764*a^2*b^5*c^5*d*f^2 - 96048*a^3*b^2*c^7*d^2*f - 4608*a^
3*b^3*c^6*d*f^2 + 1728*a^2*b^6*c^4*d*g^2 + 2304*a^3*b^3*c^6*e^2*f - 15552*a
^3*b^4*c^5*d*g^2 + 48384*a^4*b^2*c^6*d*g^2 - 13716*a^2*b^5*c^5*d^2*h + 405*
a^2*b^7*c^3*d*h^2 + 12096*a^3*b^3*c^6*d^2*h - 5400*a^3*b^5*c^4*d*h^2 + 2894
4*a^4*b^3*c^5*d*h^2 + 576*a^3*b^5*c^4*f*g^2 + 6912*a^4*b^2*c^6*e^2*h - 9216
*a^4*b^3*c^5*f*g^2 - 15*a^2*b^7*c^3*f^2*h - 360*a^3*b^5*c^4*f^2*h + 135*a^3
*b^6*c^3*f*h^2 + 15696*a^4*b^3*c^5*f^2*h - 5580*a^4*b^4*c^4*f*h^2 - 20592*a
^5*b^2*c^5*f*h^2 + 1728*a^4*b^4*c^4*g^2*h + 6912*a^5*b^2*c^5*g^2*h - 193536
*a^4*b*c^7*d*e*g - 90*a*b^8*c^3*d*f*h - 27648*a^5*b*c^6*e*g*h - 6912*a^2*b^
5*c^5*d*e*g + 62208*a^3*b^3*c^6*d*e*g - 270*a^2*b^6*c^4*d*f*h + 16056*a^3*b
^4*c^5*d*f*h - 2304*a^3*b^4*c^5*e*f*g - 127008*a^4*b^2*c^6*d*f*h + 36864*a^
4*b^2*c^6*e*f*g - 6912*a^4*b^3*c^5*e*g*h)/(512*(a^4*b^12 + 4096*a^10*c^6 -
24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 614
4*a^9*b^2*c^5)) - root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^
3*z^4 + 47185920*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 1932735
28320*a^13*b^4*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b
^10*c^5*z^4 + 3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 687194
```

76736*a^15*c^10*z^4 + 65536*a^5*b^20*z^4 - 46080*a^4*b^14*c*f*h*z^2 - 10598
4*a^3*b^15*c*d*h*z^2 - 73728*a^2*b^16*c*d*f*z^2 + 2548039680*a^9*b^3*c^7*d*
h*z^2 + 1509949440*a^9*b^3*c^7*e*g*z^2 - 1401421824*a^8*b^5*c^6*d*h*z^2 - 1
321205760*a^9*b^2*c^8*d*f*z^2 - 754974720*a^8*b^5*c^6*e*g*z^2 + 732168192*a
^7*b^6*c^6*d*f*z^2 - 456130560*a^9*b^4*c^6*f*h*z^2 + 390463488*a^7*b^7*c^5*
d*h*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2 - 330301440*a^8*b^4*c^7*d*f*z^2 + 2
54017536*a^8*b^6*c^5*f*h*z^2 - 1887436800*a^10*b*c^8*d*h*z^2 + 188743680*a^
10*b^2*c^7*f*h*z^2 + 188743680*a^7*b^7*c^5*e*g*z^2 - 61931520*a^7*b^8*c^4*f
*h*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 51609600*a^6*b^9*c^4*d*h*z^2 + 614
4000*a^6*b^10*c^3*f*h*z^2 + 61440*a^5*b^12*c^2*f*h*z^2 - 23592960*a^6*b^9*c
^4*e*g*z^2 + 1179648*a^5*b^11*c^3*e*g*z^2 + 829440*a^4*b^13*c^2*d*h*z^2 + 3
68640*a^5*b^11*c^3*d*h*z^2 - 15175680*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^
14*c^2*d*f*z^2 - 1207959552*a^10*b*c^8*e*g*z^2 - 440401920*a^10*b*c^8*f^2*z
^2 - 188743680*a^11*b*c^7*h^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 + 46080*a^5
*b^13*c*h^2*z^2 - 14080*a^3*b^15*c*f^2*z^2 + 6936330240*a^8*b^3*c^8*d^2*z^2
 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2 - 15099494
40*a^9*b^2*c^8*e^2*z^2 + 251658240*a^11*c^8*f*h*z^2 + 1536*a^3*b^16*f*h*z^2
 + 4608*a^2*b^17*d*h*z^2 - 5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a*b^17*c*
d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2*z^2 + 4
77102080*a^9*b^3*c^7*f^2*z^2 - 377487360*a^9*b^4*c^6*g^2*z^2 + 301989888*a^
10*b^2*c^7*g^2*z^2 + 188743680*a^8*b^6*c^5*g^2*z^2 + 141557760*a^10*b^3*c^6
*h^2*z^2 - 174325760*a^8*b^5*c^6*f^2*z^2 - 188743680*a^7*b^6*c^6*e^2*z^2 +
146165760*a^4*b^11*c^4*d^2*z^2 - 47185920*a^7*b^8*c^4*g^2*z^2 - 26542080*a^
8*b^7*c^4*h^2*z^2 + 9584640*a^7*b^9*c^3*h^2*z^2 - 2359296*a^9*b^5*c^5*h^2*z
^2 - 1290240*a^6*b^11*c^2*h^2*z^2 + 5898240*a^6*b^10*c^3*g^2*z^2 - 294912*a
^5*b^12*c^2*g^2*z^2 + 11206656*a^7*b^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^
2*z^2 + 23592960*a^6*b^8*c^5*e^2*z^2 - 2600960*a^5*b^11*c^3*f^2*z^2 + 29184
0*a^4*b^13*c^2*f^2*z^2 - 19860480*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c
^4*e^2*z^2 + 1771776*a^2*b^15*c^2*d^2*z^2 + 153...

3.56 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=728

$$\frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2acg-b(ce+ai)-\left(2c^2e-bcg+b^2i-2aci\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(6c}{}$$

```
[Out] 1/4*x*(b^2*d-a*b*f-2*a*(-a*h+c*d)+(a*b*h-2*a*c*f+b*c*d)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)^2+1/4*(2*a*c*g-b*(a*i+c*e)-(-2*a*c*i+b^2*i-b*c*g+2*c^2*e)*
x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(6*c*e-3*b*g+2*a*i+b^2*i/c)*(2*c*
x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d+a*b^3*f+8*a^2*b*c*f+4*
a^2*c*(a*h+7*c*d)-a*b^2*(7*a*h+25*c*d)+c*(3*b^3*d+a*b^2*f+20*a^2*c*f-12*a*b
*(a*h+2*c*d))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-(2*a*c*i+b^2*i-3*b*c*
g+6*c^2*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)+1/16*
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3*d+a*b
^2*f+20*a^2*c*f-12*a*b*(a*h+2*c*d)+(3*b^4*d+a*b^3*f-52*a^2*b*c*f-6*a*b^2*(-
3*a*h+5*c*d)+24*a^2*c*(a*h+7*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3*d+a*b^2*f+20*a^2*c*f-12*a*b*(a*h+2*c*d)
+(-3*b^4*d-a*b^3*f+52*a^2*b*c*f+6*a*b^2*(-3*a*h+5*c*d)-24*a^2*c*(a*h+7*c*d)
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]
time = 1.86, antiderivative size = 728, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$,
Rules used = {1687, 1692, 1192, 1180, 211, 1677, 1674, 12, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+24a^2c(ah+7cd)+ab^3f-6ab^2(5cd-3ah)+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(cx^2\left(20a^2cf+ab^2f-12ab(ah+2cd)+3b^3d\right)+8a^2bcf+4a^2c(ah+7cd)+ab^3f-ab^2(7ah+25cd)+3b^4d\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2aci+b^2i-3bcg+6c^2e\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{-\left(x^2\left(-2aci+b^2i-bcg+2c^2e\right)\right)-b(ai+ce)+2acg}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(x^2(abh-2acf+bcd)-abf-2a(cd-ah)+b^2d\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\left(b+2cx^2\right)\left(2ai+\frac{b^2i}{c}-3bg+6ce\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(b^2*d - a*b*f - 2*a*(c*d - a*h) + (b*c*d - 2*a*c*f + a*b*h)*x^2))/(4*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*a*c*g - b*(c*e + a*i) - (2*c^2*e
- b*c*g + b^2*i - 2*a*c*i)*x^2)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 ((6*c*e - 3*b*g + 2*a*i + (b^2*i)/c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)) + (x*(3*b^4*d + a*b^3*f + 8*a^2*b*c*f + 4*a^2*c*(7*c*d +
a*h) - a*b^2*(25*c*d + 7*a*h) + c*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*
(2*c*d + a*h))*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]
*(3*b^3*d + a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) + (3*b^4*d + a*b^3*
```

```
f - 52*a^2*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d
+ a*b^2*f + 20*a^2*c*f - 12*a*b*(2*c*d + a*h) - (3*b^4*d + a*b^3*f - 52*a^2
*b*c*f - 6*a*b^2*(5*c*d - 3*a*h) + 24*a^2*c*(7*c*d + a*h))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((6*c^2*e - 3*b*c*g + b^2*i
+ 2*a*c*i)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+56x^5}{(a+bx^2+cx^4)^3}\,dx = \int \frac{x(e+gx^2+56x^4)}{(a+bx^2+cx^4)^3}\,dx + \int \frac{d+fx^2+hx^4}{(a+bx^2+cx^4)^3}\,dx$$

$$= \frac{x(b^2d-abf-2a(cd-ah)+(bcd-2acf+abh)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{1}{2}\text{Subst}\Big($$

$$= -\frac{56ab+bce-2acg+(56b^2-2c(56a-ce)-bcg)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-}{}$$

$$= -\frac{56ab+bce-2acg+(56b^2-2c(56a-ce)-bcg)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-}{}$$

$$= -\frac{56ab+bce-2acg+(56b^2-2c(56a-ce)-bcg)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-}{}$$

$$= -\frac{56ab+bce-2acg+(56b^2-2c(56a-ce)-bcg)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-}{}$$

$$= -\frac{56ab+bce-2acg+(56b^2-2c(56a-ce)-bcg)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b^2d-}{}$$

Mathematica [A]
time = 4.96, size = 821, normalized size = 1.13

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x
]
```

```
[Out] ((2*(3*b^3*c*d*x*(b + c*x^2) + 4*a^3*c*(b*i + c*x*(h + 2*i*x)) + a*b*c*x*(b
^2*f - 24*c^2*d*x^2 + b*c*(-25*d + f*x^2)) + a^2*(2*b^3*i + 4*c^3*x*(7*d +
```

```
x*(6*e + 5*f*x)) + b^2*c*(-6*g + x*(-7*h + 4*i*x)) + 4*b*c^2*(3*e + x*(2*f
- 3*x*(g + h*x))))))/(a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-(b*
c*d*x*(b + c*x^2)) + a^2*(b*i - 2*c*(g + x*(h + i*x))) + a*(b^2*i*x^2 + 2*c
^2*x*(d + x*(e + f*x)) + b*c*(e + x*(f - x*(g + h*x))))))/(a*c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4
*a*c]*d + a*f) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f + 6*a*h) + a*b^2*(
-30*c*d + Sqrt[b^2 - 4*a*c]*f + 18*a*h) - 4*a*b*(6*c*Sqrt[b^2 - 4*a*c]*d +
13*a*c*f + 3*a*Sqrt[b^2 - 4*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
(Sqrt[2]*Sqrt[c]*(3*b^4*d + b^3*(-3*Sqrt[b^2 - 4*a*c]*d + a*f) + 4*a^2*c*(4
2*c*d - 5*Sqrt[b^2 - 4*a*c]*f + 6*a*h) + a*b^2*(-30*c*d - Sqrt[b^2 - 4*a*c]
*f + 18*a*h) + 4*a*b*(6*c*Sqrt[b^2 - 4*a*c]*d - 13*a*c*f + 3*a*Sqrt[b^2 - 4
*a*c]*h))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a^2*(b^
2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (8*(6*c^2*e - 3*b*c*g + b^2
*i + 2*a*c*i)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) -
(8*(6*c^2*e - 3*b*c*g + b^2*i + 2*a*c*i)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [A]
time = 0.17, size = 1295, normalized size = 1.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBO
SE)
```

```
[Out] (-1/8*c^2*(12*a^2*b*h-20*a^2*c*f-a*b^2*f+24*a*b*c*d-3*b^3*d)/a^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^7+1/2*c*(2*a*c*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^6+1/8/a^2*c*(4*a^3*c*h-19*a^2*b^2*h+28*a^2*b*c*f+28*a^2*c^2*d+
2*a*b^3*f-49*a*b^2*c*d+6*b^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+3/4*b*(2*a*c
*i+b^2*i-3*b*c*g+6*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-1/8*(16*a^3*b*c*h-
36*a^3*c^2*f+5*a^2*b^3*h-5*a^2*b^2*c*f+4*a^2*b*c^2*d-a*b^4*f+20*a*b^3*c*d-3
*b^5*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(2*a^2*c*i-5*a*b^2*i+5*a*b*c
*g-10*a*c^2*e+b^3*g-2*b^2*c*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(12*a^3*c
*h+3*a^2*b^2*h-16*a^2*b*c*f-44*a^2*c^2*d+a*b^3*f+37*a*b^2*c*d-5*b^4*d)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/4*(6*a^2*b*i-8*a^2*c*g-a*b^2*g+10*a*b*c*e-b^3*
e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/2/a^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*c*(1/(16*a*c-4*b^2)*(-1/4*(32*(-4*a*c+b^2)^(1/2)*a^3*c*i+16*(-4*a*
c+b^2)^(1/2)*a^2*b^2*i-48*(-4*a*c+b^2)^(1/2)*a^2*b*c*g+96*(-4*a*c+b^2)^(1/2
)*a^2*c^2*e)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(24*(-4*a*c+b^2)^(1/2)
*a^3*c*h+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*h-52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f+16
8*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+(-4*a*c+b^2)^(1/2)*a*b^3*f-30*(-4*a*c+b^2)^(
1/2)*a*b^2*c*d+3*(-4*a*c+b^2)^(1/2)*b^4*d+48*a^3*b*c*h-80*a^3*c^2*f-12*a^2*
b^3*h+16*a^2*b^2*c*f+96*a^2*b*c^2*d+a*b^4*f-36*a*b^3*c*d+3*b^5*d)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))+1/(16*a*c-4*b^2)*(1/4*(32*(-4*a*c+b^2)^(1/2)*a^3*c*i+16*(-4*a
*c+b^2)^(1/2)*a^2*b^2*i-48*(-4*a*c+b^2)^(1/2)*a^2*b*c*g+96*(-4*a*c+b^2)^(1/
```

```
2)*a^2*c^2*e)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(24*(-4*a*c+b^2)^(1/2)
*a^3*c*h+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*h-52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f+16
8*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+(-4*a*c+b^2)^(1/2)*a*b^3*f-30*(-4*a*c+b^2)^(
1/2)*a*b^2*c*d+3*(-4*a*c+b^2)^(1/2)*b^4*d-48*a^3*b*c*h+80*a^3*c^2*f+12*a^2*
b^3*h-16*a^2*b^2*c*f-96*a^2*b*c^2*d-a*b^4*f+36*a*b^3*c*d-3*b^5*d)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="m
axima")
```

```
[Out] -1/8*((12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*b^2*c^2 + 20*a^2*c^3
)*f)*x^7 + 4*(3*a^2*b*c^2*g - 6*a^2*c^3*e - I*a^2*b^2*c - 2*I*a^3*c^2)*x^6
- ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d + 2*(a*b^3*c + 14*a^2*b*c^2)*f -
 (19*a^2*b^2*c - 4*a^3*c^2)*h)*x^5 + 2*a^2*b^3*e - 20*a^3*b*c*e - 12*I*a^4*
b + 6*(3*a^2*b^2*c*g - 6*a^2*b*c^2*e - I*a^2*b^3 - 2*I*a^3*b*c)*x^4 - ((3*b
^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f - (
5*a^2*b^3 + 16*a^3*b*c)*h)*x^3 - 4*(10*a^3*c^2*e + 5*I*a^3*b^2 + 2*(a^2*b^2
*e - I*a^4)*c - (a^2*b^3 + 5*a^3*b*c)*g)*x^2 + 2*(a^3*b^2 + 8*a^4*c)*g - ((
5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d - (a^2*b^3 - 16*a^3*b*c)*f - 3*(a^3*
b^2 + 4*a^4*c)*h)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*
b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^
3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*x^2) - 1/8*integrate(((12*a^2*b*c*h - 3*(b^3*c - 8*a*b*c^
2)*d - (a*b^2*c + 20*a^2*c^2)*f)*x^2 - 3*(b^4 - 9*a*b^2*c + 28*a^2*c^2)*d -
 (a*b^3 - 16*a^2*b*c)*f - 3*(a^2*b^2 + 4*a^3*c)*h + 8*(3*a^2*b*c*g - 6*a^2*
c^2*e - I*a^2*b^2 - 2*I*a^3*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27123 vs. 2(674) = 1348.
time = 9.94, size = 27123, normalized size = 37.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x, algorithm="g
iac")
```

```
[Out] -1/512*(-64*I*a^2*b^7 + 384*I*a^3*b^5*c - 128*I*a^2*b^6*c + 256*I*a^3*b^4*c
^2 - 2048*I*a^5*b*c^3 + 1024*I*a^4*b^2*c^3 - 32*I*a^2*b^6 + 192*I*a^3*b^4*c
 - 128*I*a^2*b^5*c + 256*I*a^3*b^3*c^2 - 1024*I*a^5*c^3 + 1024*I*a^4*b*c^3
- 32*I*a^2*b^4*c + 64*I*a^3*b^2*c^2 + 256*I*a^4*c^3 - 3*(-4*I*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^8 + 72*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^6*c - 8*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 608*I*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 112*I*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 2432*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^3 - 768*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
3 - 3584*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 1792*I*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 4*sqrt(2)*sqrt(-b^2 + 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 56*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 4*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^6*c - 8*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^6*c - 4*sqrt(b^2 - 4*a*c)*b^7*c - 384*sqrt(2)*sqrt(-b^2 +
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 56*I*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 80*sqrt(2)*sqrt(-b^2 +
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 56*sqrt(b^2 - 4*a*c)*a*
b^5*c^2 - 8*sqrt(b^2 - 4*a*c)*b^6*c^2 + 896*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 384*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 448*sqrt(2)*sqrt(-b^2 + 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 384*sqrt(b^2 - 4*a*c)*a^2*b^3*
c^3 + 80*sqrt(b^2 - 4*a*c)*a*b^4*c^3 - 896*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 896*sqrt(b^2 - 4*a*c)*a^3*b*c^4 - 448
*sqrt(b^2 - 4*a*c)*a^2*b^2*c^4 + 2*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
```

)*b^7 - 32*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 12*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*b^6*c - 2*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 160*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 168*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a*b^4*c^2 + 36*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a*b^5*c^2 + 4*b^6*c^2 - 256*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 1152*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a^2*b^2*c^3 - 448*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 192*a^2*b^3*c^3 - 40*a*b^4*c^3 - 2688*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a^3*c^4 + 1344*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 448*a^3*b*c^4 + 224*a^2*b^2*c^4 + 2*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 24*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*sqrt(b^2 - 4*a*c)*b^6*c - 6*I*sqrt(-b^2 + 4*a*c)*b^6*c + 64*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 24*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 8*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 24*sqrt(b^2 - 4*a*c)*a*b^4*c^2 + 84*I*sqrt(-b^2 + 4*a*c)*a*b^4*c^2 - 64*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 224*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 64*sqrt(b^2 - 4*a*c)*a^2*b^2*c^3 - 576*I*sqrt(-b^2 + 4*a*c)*a^2*b^2*c^3 + 8*sqrt(b^2 - 4*a*c)*a*b^3*c^3 + 1344*I*sqrt(-b^2 + 4*a*c)*a^3*c^4 - 224*sqrt(b^2 - 4*a*c)*a^2*b*c^4 - 6*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*b^5*c + 3*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - b^6*c + 72*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a*b^3*c^2 - 36*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 12*a*b^4*c^2 - 192*I*sqrt(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)*a^2*b*c^3 + 96*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^2*b^2*c^3 - 4*a*b^3*c^3 + 112*a^2*b*c^4 + 2*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 3*I*sqrt(-b^2 + 4*a*c)*b^5*c - 16*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 36*I*sqrt(-b^2 + 4*a*c)*a*b^3*c^2 + 2*sqrt(b^2 - 4*a*c)*b^4*c^2 + 96*I*sqrt(-b^2 + 4*a*c)*a^2*b*c^3 - 16*sqrt(b^2 - 4*a*c)*a*b^2*c^3 - b^4*c^2 + 8*a*b^2*c^3)*d + (4*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 240*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c + 8*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c + 1728*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 448*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 3328*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 1664*I*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 4*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 224*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 4*I*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 8*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 4*sqrt(b^2 - 4*a*c)*a*b^6*c + 832*sqrt(2)*sqrt(-b^2 + 4*a*c)*sqrt(b*c + sqrt(b^2 - 4...

Mupad [B]

time = 7.16, size = 2500, normalized size = 3.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^5*(28*a^2*c^3*d + 4*a^3*c^2*h + 6*b^4*c*d + 2*a*b^3*c*f - 49*a*b^2*c^2*
d + 28*a^2*b*c^2*f - 19*a^2*b^2*c*h))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) - (x^2*(b^3*g - 10*a*c^2*e - 2*b^2*c*e - 5*a*b^2*i + 2*a^2*c*i + 5*a*b*c*
g))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (b^3*e + a*b^2*g + 8*a^2*c*g - 6*a
^2*b*i - 10*a*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*x^4*(6*c^2*e
 + b^2*i - 3*b*c*g + 2*a*c*i))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^6*
(6*c^2*e + b^2*i - 3*b*c*g + 2*a*c*i))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (x^3*(3*b^5*d + 36*a^3*c^2*f - 5*a^2*b^3*h + a*b^4*f - 20*a*b^3*c*d - 16*a
^3*b*c*h - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)) - (x*(3*a^2*b^2*h - 44*a^2*c^2*d - 5*b^4*d + a*b^3*f + 12*a^3*c*h +
37*a*b^2*c*d - 16*a^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7
*(20*a^2*c^2*f + 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f - 12*a^2*b*c*h))/(8*a
^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*
a*b*x^2 + 2*b*c*x^6) + symsum(log((10368*a*b^5*c^6*d^3 - 8000*a^5*c^7*f^3 -
 567*b^7*c^5*d^3 + 169344*a^3*b*c^8*d^3 + 193536*a^4*c^8*d*e^2 - 141120*a^4
*c^8*d^2*f + 1728*a^6*b*c^5*h^3 + 315*b^8*c^4*d^2*f + 27648*a^5*c^7*e^2*h +
 21504*a^6*c^6*d*i^2 - 135*b^9*c^3*d^2*h - 2880*a^6*c^6*f*h^2 + 3072*a^7*c^
5*h*i^2 - 67824*a^2*b^3*c^7*d^3 + 35*a^2*b^6*c^4*f^3 + 84*a^3*b^4*c^5*f^3 -
 12720*a^4*b^2*c^6*f^3 + 540*a^4*b^5*c^3*h^3 + 4320*a^5*b^3*c^4*h^3 + 12902
4*a^5*c^7*d*e*i - 40320*a^5*c^7*d*f*h + 18432*a^6*c^6*e*h*i - 6237*a*b^6*c^
5*d^2*f + 210*a*b^7*c^4*d*f^2 + 116160*a^4*b*c^7*d*f^2 - 36864*a^4*b*c^7*e^
2*f + 2430*a*b^7*c^4*d^2*h + 133056*a^4*b*c^7*d^2*h + 27648*a^5*b*c^6*d*h^2
 + 26880*a^5*b*c^6*f^2*h - 4096*a^6*b*c^5*f*i^2 + 6912*a^2*b^4*c^6*d*e^2 -
62208*a^3*b^2*c^7*d*e^2 + 42372*a^2*b^4*c^6*d^2*f - 1764*a^2*b^5*c^5*d*f^2
- 96048*a^3*b^2*c^7*d^2*f - 4608*a^3*b^3*c^6*d*f^2 + 1728*a^2*b^6*c^4*d*g^2
 + 2304*a^3*b^3*c^6*e^2*f - 15552*a^3*b^4*c^5*d*g^2 + 48384*a^4*b^2*c^6*d*g
^2 - 13716*a^2*b^5*c^5*d^2*h + 405*a^2*b^7*c^3*d*h^2 + 12096*a^3*b^3*c^6*d^
2*h - 5400*a^3*b^5*c^4*d*h^2 + 28944*a^4*b^3*c^5*d*h^2 + 192*a^2*b^8*c^2*d*
i^2 + 576*a^3*b^5*c^4*f*g^2 - 960*a^3*b^6*c^3*d*i^2 + 6912*a^4*b^2*c^6*e^2*
h - 9216*a^4*b^3*c^5*f*g^2 - 768*a^4*b^4*c^4*d*i^2 + 14592*a^5*b^2*c^5*d*i^
2 - 15*a^2*b^7*c^3*f^2*h - 360*a^3*b^5*c^4*f^2*h + 135*a^3*b^6*c^3*f*h^2 +
15696*a^4*b^3*c^5*f^2*h - 5580*a^4*b^4*c^4*f*h^2 - 20592*a^5*b^2*c^5*f*h^2
+ 64*a^3*b^7*c^2*f*i^2 + 1728*a^4*b^4*c^4*g^2*h - 768*a^4*b^5*c^3*f*i^2 + 6
912*a^5*b^2*c^5*g^2*h - 3840*a^5*b^3*c^4*f*i^2 + 192*a^4*b^6*c^2*h*i^2 + 15
36*a^5*b^4*c^3*h*i^2 + 3840*a^6*b^2*c^4*h*i^2 - 193536*a^4*b*c^7*d*e*g - 90
*a*b^8*c^3*d*f*h - 64512*a^5*b*c^6*d*g*i - 24576*a^5*b*c^6*e*f*i - 27648*a^
5*b*c^6*e*g*h - 9216*a^6*b*c^5*g*h*i - 6912*a^2*b^5*c^5*d*e*g + 62208*a^3*b
^3*c^6*d*e*g + 2304*a^2*b^6*c^4*d*e*i - 270*a^2*b^6*c^4*d*f*h - 16128*a^3*b
```

^4*c^5*d*e*i + 16056*a^3*b^4*c^5*d*f*h - 2304*a^3*b^4*c^5*e*f*g + 23040*a^4
*b^2*c^6*d*e*i - 127008*a^4*b^2*c^6*d*f*h + 36864*a^4*b^2*c^6*e*f*g - 1152*
a^2*b^7*c^3*d*g*i + 8064*a^3*b^5*c^4*d*g*i + 768*a^3*b^5*c^4*e*f*i - 11520*
a^4*b^3*c^5*d*g*i - 10752*a^4*b^3*c^5*e*f*i - 6912*a^4*b^3*c^5*e*g*h - 384*
a^3*b^6*c^3*f*g*i + 2304*a^4*b^4*c^4*e*h*i + 5376*a^4*b^4*c^4*f*g*i + 13824
*a^5*b^2*c^5*e*h*i + 12288*a^5*b^2*c^5*f*g*i - 1152*a^4*b^5*c^3*g*h*i - 691
2*a^5*b^3*c^4*g*h*i)/(512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a
^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)) + roo
t(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 47185920*a^7*
b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4*c^8*z^
4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 + 3523215
360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^10*z^4 +
65536*a^5*b^20*z^4 + 196608*a^5*b^13*c*g*i*z^2 - 46080*a^4*b^14*c*f*h*z^2
- 105984*a^3*b^15*c*d*h*z^2 - 73728*a^2*b^16*c*d*f*z^2 + 2548039680*a^9*b^3
*c^7*d*h*z^2 + 1509949440*a^9*b^3*c^7*e*g*z^2 - 1401421824*a^8*b^5*c^6*d*h*
z^2 - 1321205760*a^9*b^2*c^8*d*f*z^2 - 754974720*a^8*b^5*c^6*e*g*z^2 + 7321
68192*a^7*b^6*c^6*d*f*z^2 - 603979776*a^10*b^2*c^7*e*i*z^2 - 456130560*a^9*
b^4*c^6*f*h*z^2 + 390463488*a^7*b^7*c^5*d*h*z^2 + 301989888*a^10*b^3*c^6*g*
i*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2 - 330301440*a^8*b^4*c^7*d*f*z^2 + 254
017536*a^8*b^6*c^5*f*h*z^2 - 1887436800*a^10*b*c^8*d*h*z^2 + 188743680*a^10
*b^2*c^7*f*h*z^2 + 188743680*a^7*b^7*c^5*e*g*z^2 + 125829120*a^8*b^6*c^5*e*
i*z^2 - 62914560*a^8*b^7*c^4*g*i*z^2 - 61931520*a^7*b^8*c^4*f*h*z^2 + 23592
960*a^7*b^9*c^3*g*i*z^2 - 47185920*a^7*b^8*c^4*e*i*z^2 - 3538944*a^6*b^11*c
^2*g*i*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 51609600*a^6*b^9*c^4*d*h*z^2 +
7077888*a^6*b^10*c^3*e*i*z^2 + 6144000*a^6*b^10*c^3*f*h*z^2 - 393216*a^5*b
^12*c^2*e*i*z^2 + 61440*a^5*b^12*c^2*f*h*z^2 - 23592960*a^6*b^9*c^4*e*g*z^2
+ 1179648*a^5*b^11*c^3*e*g*z^2 + 829440*a^4*b^13*c^2*d*h*z^2 + 368640*a^5*
b^11*c^3*d*h*z^2 - 15175680*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f
*z^2 - 1207959552*a^10*b*c^8*e*g*z^2 - 40265318...

3.57 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=1150

$$-\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-c^2(bg+2aj)-b^3l+bc(bj+3al)\right)x^2}{4c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x(abc(cf+ak)-b^2(c^2\cdots}{\cdots}$$

```
[Out] 1/4*(-b*c*(a*j+c*e)+a*b^2*l+2*a*c*(-a*l+c*g)-(2*c^3*e-c^2*(2*a*j+b*g)-b^3*l
+b*c*(3*a*l+b*j))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/4*x*(a*b*c*(a*k
+c*f)-b^2*(a^2*m+c^2*d)+2*a*c*(a^2*m-a*c*h+c^2*d)+(a*b^2*c*k+2*a*c^2*(-a*k+
c*f)-a*b^3*m-b*c*(-3*a^2*m+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^2+1/4*(b^3*j/c+2*b*(a*j+3*c*e)-16*a^2*l-b^4*l/c^2-b^2*(3*g-5*a*l/c)+2
*(-3*a*b*l+2*a*c*j+b^2*j-3*b*c*g+6*c^2*e)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+
a)+1/8*x*(4*a^2*b*c^2*(a*k+2*c*f)+a*b^3*c*(2*a*k+c*f)-a*b^2*c*(-11*a^2*m+7*
a*c*h+25*c^2*d)+4*a^2*c^2*(-9*a^2*m+a*c*h+7*c^2*d)+b^4*(-2*a^2*m+3*c^2*d)+c
*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b^3*(a^2*m+3*c^2*d)-4*a*b*c*(
4*a^2*m+3*a*c*h+6*c^2*d))*x^2)/a^2/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-(-3*a
*b*l+2*a*c*j+b^2*j-3*b*c*g+6*c^2*e)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(5/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b^3*(a^2*m+3*c^2*d)-4*a*b
*c*(4*a^2*m+3*a*c*h+6*c^2*d)+(a*b^3*c*(-3*a*k+c*f)-4*a^2*b*c^2*(9*a*k+13*c*
f)-6*a*b^2*c*(-3*a^2*m-3*a*c*h+5*c^2*d)+b^4*(-a^2*m+3*c^2*d)+8*a^2*c^2*(5*a
^2*m+3*a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))/a^2/c^(3/2)/(-4*a*c+b^2)^2*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(a*b^2*c*(3*a*k+c*f)+4*a^2*c^2*(3*a*k+5*c*f)+b^3*(a^2*m+3
*c^2*d)-4*a*b*c*(4*a^2*m+3*a*c*h+6*c^2*d)+(-a*b^3*c*(-3*a*k+c*f)+4*a^2*b*c^
2*(9*a*k+13*c*f)+6*a*b^2*c*(-3*a^2*m-3*a*c*h+5*c^2*d)-b^4*(-a^2*m+3*c^2*d)-
8*a^2*c^2*(5*a^2*m+3*a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))/a^2/c^(3/2)/(-4*a
*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 5.10, antiderivative size = 1144, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 9, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.164$, Rules used = {1687, 1692, 1180, 211, 1677, 1674, 652, 632, 212}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(b*c*(c*e + a*j) - a*b^2*l - 2*a*c*(c*g - a*l) + (2*c^3*e - c^2*(b*g +
 2*a*j) - b^3*l + b*c*(b*j + 3*a*l))*x^2)/(c^2*(b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)^2) - (x*(a*b*c*(c*f + a*k) - b^2*(c^2*d + a^2*m) + 2*a*c*(c^2*d - a*c
*h + a^2*m) + (a*b^2*c*k + 2*a*c^2*(c*f - a*k) - a*b^3*m - b*c*(c^2*d + a*c
*h - 3*a^2*m))*x^2))/(4*a*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + ((b^3*
j)/c + 2*b*(3*c*e + a*j) - 16*a^2*l - (b^4*l)/c^2 - b^2*(3*g - (5*a*l)/c) +
 2*(6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*x^2)/(4*(b^2 - 4*a*c)^2*
(a + b*x^2 + c*x^4)) + (x*(4*a^2*b*c*(2*c*f + a*k) + a*b^3*(c*f + 2*a*k) -
a*b^2*(25*c^2*d + 7*a*c*h - 11*a^2*m) + 4*a^2*c*(7*c^2*d + a*c*h - 9*a^2*m)
 + b^4*(3*c*d - (2*a^2*m)/c) + (a*b^2*c*(c*f + 3*a*k) + 4*a^2*c^2*(5*c*f +
3*a*k) + b^3*(3*c^2*d + a^2*m) - 4*a*b*c*(6*c^2*d + 3*a*c*h + 4*a^2*m))*x^2
))/(8*a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((a*b^2*(c*f + 3*a*k) +
4*a^2*c*(5*c*f + 3*a*k) - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d
+ (a^2*m)/c) + (a*b^3*c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*
b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21
*c^2*d + 3*a*c*h + 5*a^2*m))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + ((a*b^2*(c*f + 3*a*k) + 4*a^2*c*(5*c*f + 3*a*k
) - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^2*m) + b^3*(3*c*d + (a^2*m)/c) - (a*b^3*
c*(c*f - 3*a*k) - 4*a^2*b*c^2*(13*c*f + 9*a*k) - 6*a*b^2*c*(5*c^2*d - 3*a*c
*h - 3*a^2*m) + b^4*(3*c^2*d - a^2*m) + 8*a^2*c^2*(21*c^2*d + 3*a*c*h + 5*a
^2*m))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) - ((6*c^2*e - 3*b*c*g + b^2*j + 2*a*c*j - 3*a*b*l)*ArcTanh[(b + 2*c*x^
2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
```

```
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
```

```
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{x\left(e+gx^2+jx^4+lx^6\right)}{\left(a+bx^2+cx^4\right)^3}\,dx + \int \frac{d+fx^2+}{(a+}$$

$$= -\frac{x\left(abc(cf+ak)-b^2\left(c^2d+a^2m\right)+2ac\left(c^2d\right.\right.}{}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-\right.}{4c^2\left(b^2-4ac\right)(a+}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-\right.}{4c^2\left(b^2-4ac\right)(a+}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-\right.}{4c^2\left(b^2-4ac\right)(a+}$$

$$= -\frac{bc(ce+aj)-ab^2l-2ac(cg-al)+\left(2c^3e-\right.}{4c^2\left(b^2-4ac\right)(a+}$$

Mathematica [**A**]
time = 6.90, size = 1590, normalized size = 1.38

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*c^2*e - 2*a^2*c^2*g + a^2*b*c*j - a^2*b^2*l + 2*a^3*c*l - b^2*c^2*d*x
+ 2*a*c^3*d*x + a*b*c^2*f*x - 2*a^2*c^2*h*x + a^2*b*c*k*x - a^2*b^2*m*x + 2
*a^3*c*m*x + 2*a*c^3*e*x^2 - a*b*c^2*g*x^2 + a*b^2*c*j*x^2 - 2*a^2*c^2*j*x^
```

```
2 - a*b^3*l*x^2 + 3*a^2*b*c*l*x^2 - b*c^3*d*x^3 + 2*a*c^3*f*x^3 - a*b*c^2*h
*x^3 + a*b^2*c*k*x^3 - 2*a^2*c^2*k*x^3 - a*b^3*m*x^3 + 3*a^2*b*c*m*x^3)/(4*
a*c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^3*e - 6*a^2*b^2*c
^2*g + 2*a^2*b^3*c*j + 4*a^3*b*c^2*j - 2*a^2*b^4*l + 10*a^3*b^2*c*l - 32*a^
4*c^2*l + 3*b^4*c^2*d*x - 25*a*b^2*c^3*d*x + 28*a^2*c^4*d*x + a*b^3*c^2*f*x
 + 8*a^2*b*c^3*f*x - 7*a^2*b^2*c^2*h*x + 4*a^3*c^3*h*x + 2*a^2*b^3*c*k*x +
4*a^3*b*c^2*k*x - 2*a^2*b^4*m*x + 11*a^3*b^2*c*m*x - 36*a^4*c^2*m*x + 24*a^
2*c^4*e*x^2 - 12*a^2*b*c^3*g*x^2 + 4*a^2*b^2*c^2*j*x^2 + 8*a^3*c^3*j*x^2 -
12*a^3*b*c^2*l*x^2 + 3*b^3*c^3*d*x^3 - 24*a*b*c^4*d*x^3 + a*b^2*c^3*f*x^3 +
 20*a^2*c^4*f*x^3 - 12*a^2*b*c^3*h*x^3 + 3*a^2*b^2*c^2*k*x^3 + 12*a^3*c^3*k
*x^3 + a^2*b^3*c*m*x^3 - 16*a^3*b*c^2*m*x^3)/(8*a^2*c^2*(-b^2 + 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 168*a^2*c^4*d + 3*b^3
*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d + a*b^3*c^2*f - 5
2*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c
]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h -
3*a^2*b^3*c*k - 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 - 4*a*c]*k + 12*a^3*c
^2*Sqrt[b^2 - 4*a*c]*k - a^2*b^4*m + 18*a^3*b^2*c*m + 40*a^4*c^2*m + a^2*b^
3*Sqrt[b^2 - 4*a*c]*m - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*m)*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((-3*b^4*c^2*d + 30*a*b^2*c^3*d - 168*
a^2*c^4*d + 3*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d
- a*b^3*c^2*f + 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3
*Sqrt[b^2 - 4*a*c]*f - 18*a^2*b^2*c^2*h - 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[
b^2 - 4*a*c]*h + 3*a^2*b^3*c*k + 36*a^3*b*c^2*k + 3*a^2*b^2*c*Sqrt[b^2 - 4*
a*c]*k + 12*a^3*c^2*Sqrt[b^2 - 4*a*c]*k + a^2*b^4*m - 18*a^3*b^2*c*m - 40*a
^4*c^2*m + a^2*b^3*Sqrt[b^2 - 4*a*c]*m - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*m)*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/
2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((6*c^2*e - 3*b*c*g +
 b^2*j + 2*a*c*j - 3*a*b*l)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(2*(b^2
- 4*a*c)^(5/2)) + ((-6*c^2*e + 3*b*c*g - b^2*j - 2*a*c*j + 3*a*b*l)*Log[b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*(b^2 - 4*a*c)^(5/2))
```

Maple [A]
time = 1.54, size = 1987, normalized size = 1.73

method	result
risch	$-\frac{-\frac{\left(16 a^{3} b c m-12 a^{3} c^{2} k-a^{2} b^{3} m-3 a^{2} b^{2} c k+12 a^{2} b\, c^{2} h-20 a^{2} c^{3} f-a\, b^{2} c^{2} f+24 a b\, c^{3} d-3 b^{3} c^{2} d\right) x^{7}}{8 a^{2} \left(16 a^{2} c^{2}-8 a\, b^{2} c+b^{4}\right)}-\frac{c \left(3 a b l-2 a c j-b^{2} j+3 b c g-6 c^{2} e\right) x^{6}}{2 \left(16 a^{2} c^{2}-8 a\, b^{2} c+b^{4}\right)}-\frac{\left(36 a^{4} c^{2} m+5 a\right.}{}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,m
ethod=_RETURNVERBOSE)
```

```
[Out] (-1/8*(16*a^3*b*c*m-12*a^3*c^2*k-a^2*b^3*m-3*a^2*b^2*c*k+12*a^2*b*c^2*h-20*
a^2*c^3*f-a*b^2*c^2*f+24*a*b*c^3*d-3*b^3*c^2*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^7-1/2*c*(3*a*b*l-2*a*c*j-b^2*j+3*b*c*g-6*c^2*e)/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^6-1/8/a^2*(36*a^4*c^2*m+5*a^3*b^2*c*m-16*a^3*b*c^2*k-4*a^3*c^3*h+a^
2*b^4*m-5*a^2*b^3*c*k+19*a^2*b^2*c^2*h-28*a^2*b*c^3*f-28*a^2*c^4*d-2*a*b^3*
c^2*f+49*a*b^2*c^3*d-6*b^4*c^2*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^5-1/4*(16*
a^2*c^2*l+a*b^2*c*l-6*a*b*c^2*j+b^4*l-3*b^3*c*j+9*b^2*c^2*g-18*b*c^3*e)/(16
*a^2*c^2-8*a*b^2*c+b^4)/c*x^4-1/8/c*(28*a^4*b*c*m+4*a^4*c^2*k+2*a^3*b^3*m-1
9*a^3*b^2*c*k+16*a^3*b*c^2*h-36*a^3*c^3*f+5*a^2*b^3*c*h-5*a^2*b^2*c^2*f+4*a
^2*b*c^3*d-a*b^4*c*f+20*a*b^3*c^2*d-3*b^5*c*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/2/c*(5*a^2*b*c*l+2*a^2*c^2*j+a*b^3*l-5*a*b^2*c*j+5*a*b*c^2*g-10*a*
c^3*e+b^3*c*g-2*b^2*c^2*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(20*a^4*c*m+a
^3*b^2*m-12*a^3*b*c*k+12*a^3*c^2*h+3*a^2*b^2*c*h-16*a^2*b*c^2*f-44*a^2*c^3*
d+a*b^3*c*f+37*a*b^2*c^2*d-5*b^4*c*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c/a*x-1/4/
c*(8*a^3*c*l+a^2*b^2*l-6*a^2*b*c*j+8*a^2*c^2*g+a*b^2*c*g-10*a*b*c^2*e+b^3*c
*e)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/2/a^2/(16*a^2*c^2-8*a*b
^2*c+b^4)*(1/4/c/(4*a*c-b^2)*(-1/4*(-48*(-4*a*c+b^2)^(1/2)*a^3*b*c^2*l+32*(
-4*a*c+b^2)^(1/2)*a^3*c^3*j+16*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2*j-48*(-4*a*c+
b^2)^(1/2)*a^2*b*c^3*g+96*(-4*a*c+b^2)^(1/2)*a^2*c^4*e)/c*ln(-b-2*c*x^2+(-4
*a*c+b^2)^(1/2))+1/2*(-12*a^2*b^3*c^2*h+16*a^2*b^2*c^3*f+96*a^2*b*c^4*d+a*b
^4*c^2*f-36*a*b^3*c^3*d+64*a^4*b*c^2*m-20*a^3*b^3*c*m+48*a^3*b*c^3*h+3*a^2*
b^4*c*k+40*(-4*a*c+b^2)^(1/2)*a^4*c^2*m+24*(-4*a*c+b^2)^(1/2)*a^3*c^3*h-(-4
*a*c+b^2)^(1/2)*a^2*b^4*m+168*(-4*a*c+b^2)^(1/2)*a^2*c^4*d+3*(-4*a*c+b^2)^(
1/2)*b^4*c^2*d+a^2*b^5*m+3*b^5*c^2*d-48*a^4*c^3*k-80*a^3*c^4*f+18*(-4*a*c+b
^2)^(1/2)*a^3*b^2*c*m-36*(-4*a*c+b^2)^(1/2)*a^3*b*c^2*k-3*(-4*a*c+b^2)^(1/2
)*a^2*b^3*c*k+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2*h-52*(-4*a*c+b^2)^(1/2)*a^2
*b*c^3*f+(-4*a*c+b^2)^(1/2)*a*b^3*c^2*f-30*(-4*a*c+b^2)^(1/2)*a*b^2*c^3*d)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(-48*(-4*a*c+b^2)^(1/2)*a^3*b
*c^2*l+32*(-4*a*c+b^2)^(1/2)*a^3*c^3*j+16*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2*j-
48*(-4*a*c+b^2)^(1/2)*a^2*b*c^3*g+96*(-4*a*c+b^2)^(1/2)*a^2*c^4*e)/c*ln(b+2
*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(12*a^2*b^3*c^2*h-16*a^2*b^2*c^3*f-96*a^2*b*
c^4*d-a*b^4*c^2*f+36*a*b^3*c^3*d-64*a^4*b*c^2*m+20*a^3*b^3*c*m-48*a^3*b*c^3
*h-3*a^2*b^4*c*k+40*(-4*a*c+b^2)^(1/2)*a^4*c^2*m+24*(-4*a*c+b^2)^(1/2)*a^3*
c^3*h-(-4*a*c+b^2)^(1/2)*a^2*b^4*m+168*(-4*a*c+b^2)^(1/2)*a^2*c^4*d+3*(-4*a
*c+b^2)^(1/2)*b^4*c^2*d-a^2*b^5*m-3*b^5*c^2*d+48*a^4*c^3*k+80*a^3*c^4*f+18*
(-4*a*c+b^2)^(1/2)*a^3*b^2*c*m-36*(-4*a*c+b^2)^(1/2)*a^3*b*c^2*k-3*(-4*a*c+
b^2)^(1/2)*a^2*b^3*c*k+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2*h-52*(-4*a*c+b^2)^
(1/2)*a^2*b*c^3*f+(-4*a*c+b^2)^(1/2)*a*b^3*c^2*f-30*(-4*a*c+b^2)^(1/2)*a*b^
2*c^3*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="maxima")
```

```
[Out] -1/8*((12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4)*d - (a*b^2*c^3 + 20*a^2*c^4
)*f - 3*(a^2*b^2*c^2 + 4*a^3*c^3)*k - (a^2*b^3*c - 16*a^3*b*c^2)*m)*x^7 - 1
2*a^4*b*c*j + 4*(3*a^2*b*c^3*g + 3*a^3*b*c^2*l - 6*a^2*c^4*e - (a^2*b^2*c^2
 + 2*a^3*c^3)*j)*x^6 + 2*a^2*b^3*c*e - 20*a^3*b*c^2*e - ((6*b^4*c^2 - 49*a*
b^2*c^3 + 28*a^2*c^4)*d + 2*(a*b^3*c^2 + 14*a^2*b*c^3)*f - (19*a^2*b^2*c^2
- 4*a^3*c^3)*h + (5*a^2*b^3*c + 16*a^3*b*c^2)*k - (a^2*b^4 + 5*a^3*b^2*c +
36*a^4*c^2)*m)*x^5 + 2*(9*a^2*b^2*c^2*g - 18*a^2*b*c^3*e - 3*(a^2*b^3*c + 2
*a^3*b*c^2)*j + (a^2*b^4 + a^3*b^2*c + 16*a^4*c^2)*l)*x^4 - ((3*b^5*c - 20*
a*b^3*c^2 - 4*a^2*b*c^3)*d + (a*b^4*c + 5*a^2*b^2*c^2 + 36*a^3*c^3)*f - (5*
a^2*b^3*c + 16*a^3*b*c^2)*h + (19*a^3*b^2*c - 4*a^4*c^2)*k - 2*(a^3*b^3 + 1
4*a^4*b*c)*m)*x^3 - 4*(2*a^2*b^2*c^2*e + 10*a^3*c^3*e - (a^2*b^3*c + 5*a^3*
b*c^2)*g + (5*a^3*b^2*c - 2*a^4*c^2)*j - (a^3*b^3 + 5*a^4*b*c)*l)*x^2 + 2*(
a^3*b^2*c + 8*a^4*c^2)*g + 2*(a^4*b^2 + 8*a^5*c)*l - (12*a^4*b*c*k + (5*a*b
^4*c - 37*a^2*b^2*c^2 + 44*a^3*c^3)*d - (a^2*b^3*c - 16*a^3*b*c^2)*f - 3*(a
^3*b^2*c + 4*a^4*c^2)*h - (a^4*b^2 + 20*a^5*c)*m)*x)/(a^4*b^4*c - 8*a^5*b^2
*c^2 + 16*a^6*c^3 + (a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*x^8 + 2*(a^2
*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*x^6 + (a^2*b^6*c - 6*a^3*b^4*c^2 +
 32*a^5*c^4)*x^4 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^2) - 1/8*
integrate((12*a^3*b*c*k + (12*a^2*b*c^2*h - 3*(b^3*c^2 - 8*a*b*c^3)*d - (a*
b^2*c^2 + 20*a^2*c^3)*f - 3*(a^2*b^2*c + 4*a^3*c^2)*k - (a^2*b^3 - 16*a^3*b
*c)*m)*x^2 - 3*(b^4*c - 9*a*b^2*c^2 + 28*a^2*c^3)*d - (a*b^3*c - 16*a^2*b*c
^2)*f - 3*(a^2*b^2*c + 4*a^3*c^2)*h - (a^3*b^2 + 20*a^4*c)*m + 8*(3*a^2*b*c
^2*g + 3*a^3*b*c*l - 6*a^2*c^3*e - (a^2*b^2*c + 2*a^3*c^2)*j)*x)/(c*x^4 + b
*x^2 + a), x)/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+
b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22437 vs. $2(1102) = 2204$.
time = 9.73, size = 22437, normalized size = 19.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)
^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(2*b^5*c^4 - 24*a*b^3*c^5 + 64*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 16*(b^2 - 4*a*c)*a*b*c^5)*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*d + (2*a*b^4*c^4 + 32*a^2*b^2*c^5
- 160*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^3 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
4 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 20*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2
- 4*a*c)*a*b^2*c^4 - 40*(b^2 - 4*a*c)*a^2*c^5)*(a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)^2*f - 12*(2*a^2*b^3*c^4 - 8*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b*c^4)*(a^2*b^
4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*h + 3*(2*a^2*b^4*c^3 - 32*a^4*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*
```

```
c^3 - 8*(b^2 - 4*a*c)*a^3*c^4)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*k
 + (2*a^2*b^5*c^2 - 40*a^3*b^3*c^3 + 128*a^4*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 32*(b^2 - 4
*a*c)*a^3*b*c^3)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)^2*m + 6*(sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^3 - 21*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^8*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^9*c^4 - 2*a^2*b^10*c^4 + 184*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b^6*c^5 + 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 42*a^3*b^8*c^5 - 832*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^6 - 232*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^5*c^6 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^6*c^6 - 368*a^4*b^6*c^6 + 1920*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^2*c^7 + 736*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 116*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1664*a^5*b^4*c^7 - 17
92*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^8 - 896*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^5*b^2*c^8 - 3840*a^6*b^2*c^8 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^6*c^9 + 3584*a^7*c^9 + 2*(b^2 - 4*a*c)*a^2*b^8*c^4 - 34*(b^2 - 4*a*c
)*a^3*b^6*c^5 + 232*(b^2 - 4*a*c)*a^4*b^4*c^6 - 736*(b^2 - 4*a*c)*a^5*b^2*c
^7 + 896*(b^2 - 4*a*c)*a^6*c^8)*d*abs(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^
3) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^3 - 28*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^8*c^4 - 2*a^3*b^9*c^4 + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^5*c^5 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 56*a^4*b^7*c^5
- 832*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^6 - 288*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^6 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*b^5*c^6 - 480*a^5*b^5*c^6 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^7*b*c^7 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^
2*c^7 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 1664*a^6*
b^3*c^7 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 - 2048*a^7*
b*c^8 + 2*(b^2 - 4*a*c)*a^3*b^7*c^4 - 48*(b^2 -...
```

Mupad [B]
time = 20.57, size = 2500, normalized size = 2.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a + b*x^2 + c*x^4)^3,x)

[Out] symsum(log(root(56371445760*a^11*b^8*c^9*z^4 - 503316480*a^8*b^14*c^6*z^4 + 47185920*a^7*b^16*c^5*z^4 - 2621440*a^6*b^18*c^4*z^4 + 65536*a^5*b^20*c^3*z^4 - 171798691840*a^14*b^2*c^12*z^4 + 193273528320*a^13*b^4*c^11*z^4 - 128849018880*a^12*b^6*c^10*z^4 - 16911433728*a^10*b^10*c^8*z^4 + 3523215360*a^9*b^12*c^7*z^4 + 68719476736*a^15*c^13*z^4 + 1536*a^5*b^16*c*k*m*z^2 + 1536*a*b^18*c^3*d*f*z^2 - 2571632640*a^9*b^5*c^8*d*m*z^2 + 2548039680*a^9*b^3*c^10*d*h*z^2 + 1509949440*a^10*b^3*c^9*e*l*z^2 + 1509949440*a^9*b^3*c^10*e*g*z^2 - 1401421824*a^8*b^5*c^9*d*h*z^2 - 1321205760*a^9*b^2*c^11*d*f*z^2 - 2793406464*a^11*b*c^10*d*m*z^2 + 890634240*a^8*b^7*c^7*d*m*z^2 - 754974720*a^10*b^4*c^8*g*l*z^2 - 754974720*a^9*b^5*c^8*e*l*z^2 + 719585280*a^8*b^6*c^8*d*k*z^2 - 707788800*a^9*b^4*c^9*d*k*z^2 - 754974720*a^8*b^5*c^9*e*g*z^2 + 603979776*a^11*b^2*c^9*g*l*z^2 - 581959680*a^10*b^4*c^8*f*m*z^2 + 732168192*a^7*b^6*c^9*d*f*z^2 + 534773760*a^11*b^3*c^8*h*m*z^2 - 456130560*a^11*b^4*c^7*k*m*z^2 - 603979776*a^10*b^2*c^10*e*j*z^2 + 534773760*a^10*b^3*c^9*f*k*z^2 + 384040960*a^9*b^6*c^7*f*m*z^2 + 377487360*a^9*b^6*c^7*g*l*z^2 - 456130560*a^9*b^4*c^9*f*h*z^2 + 301989888*a^11*b^3*c^8*j*l*z^2 - 415236096*a^10*b^2*c^10*d*k*z^2 + 254017536*a^10*b^6*c^6*k*m*z^2 - 330301440*a^10*b^4*c^8*h*k*z^2 + 390463488*a^7*b^7*c^8*d*h*z^2 + 188743680*a^12*b^2*c^8*k*m*z^2 + 301989888*a^10*b^3*c^9*g*j*z^2 - 297861120*a^7*b^8*c^7*d*k*z^2 - 366280704*a^6*b^8*c^8*d*f*z^2 + 188743680*a^11*b^2*c^9*h*k*z^2 - 330301440*a^8*b^4*c^10*d*f*z^2 + 254017536*a^8*b^6*c^8*f*h*z^2 - 1887436800*a^10*b*c^11*d*h*z^2 + 188743680*a^8*b^7*c^7*e*l*z^2 + 153354240*a^9*b^6*c^7*h*k*z^2 - 185303040*a^7*b^9*c^6*d*m*z^2 - 117964800*a^10*b^5*c^7*h*m*z^2 - 61931520*a^9*b^8*c^5*k*m*z^2 + 121634816*a^11*b^2*c^9*f*m*z^2 - 115671040*a^8*b^8*c^6*f*m*z^2 - 62914560*a^9*b^7*c^6*j*l*z^2 + 188743680*a^10*b^2*c^10*f*h*z^2 - 94371840*a^8*b^8*c^6*g*l*z^2 + 6144000*a^8*b^10*c^4*k*m*z^2 - 117964800*a^9*b^5*c^8*f*k*z^2 + 61440*a^7*b^12*c^3*k*m*z^2 - 46080*a^6*b^14*c^2*k*m*z^2 + 23592960*a^8*b^9*c^5*j*l*z^2 + 188743680*a^7*b^7*c^8*e*g*z^2 - 37355520*a^9*b^7*c^6*h*m*z^2 + 125829120*a^8*b^6*c^8*e*j*z^2 + 23101440*a^8*b^9*c^5*h*m*z^2 - 3538944*a^7*b^11*c^4*j*l*z^2 + 196608*a^6*b^13*c^3*j*l*z^2 - 4349952*a^7*b^11*c^4*h*m*z^2 + 337920*a^6*b^13*c^3*h*m*z^2 - 7680*a^5*b^15*c^2*h*m*z^2 - 62914560*a^8*b^7*c^7*g*j*z^2 - 26542080*a^8*b^8*c^6*h*k*z^2 + 17940480*a^7*b^10*c^5*f*m*z^2 + 11796480*a^7*b^10*c^5*g*l*z^2 - 37355520*a^8*b^7*c^7*f*k*z^2 - 1347584*a^6*b^12*c^4*f*m*z^2 + 68272128*a^6*b^10*c^6*d*k*z^2 - 589824*a^6*b^12*c^4*g*l*z^2 + 552960*a^6*b^12*c^4*h*k*z^2 - 147456*a^7*b^10*c^5*h*k*z^2 - 46080*a^5*b^14*c^3*h*k*z^2 + 35840*a^5*b^14*c^3*f*m*z^2 + 23592960*a^7*b^9*c^6*g*j*z^2 - 23592960*a^7*b^9*c^6*e*l*z^2 + 23371776*a^6*b^11*c^5*d*m*z^2 + 23101440*a^7*b^9*c^6*f*k*z^2 - 47185920*a^7*b^8*c^7*e*j*z^2 - 61931520*a^7*b^8*c^7*f*h*z^2 - 4349952*a^6*b^11*c^5*f*k*z^2 - 3538944*a^6*b^11*c^5*g*j*z^2 - 1677312*a^5*b^13*c^4*d*m*z^2 + 1179648*a^6*b^11*c^5*e*l*z^2 + 337920*a^5*b^13*c^4*f*k*z^2 + 196608*a^5*b^13*c^4*g*j*z^2 + 53760*a^4*b^15*c^3*d*m*z^2 - 7680*a^4*b^15*c^3*f*k*z^2 + 96583680*a^5*b^10*c^7*d*f*z

```
^2 - 9179136*a^5*b^12*c^5*d*k*z^2 + 7077888*a^6*b^10*c^6*e*j*z^2 - 51609600
*a^6*b^9*c^7*d*h*z^2 + 691200*a^4*b^14*c^4*d*k*z^2 - 393216*a^5*b^12*c^5*e*
j*z^2 - 23040*a^3*b^16*c^3*d*k*z^2 + 6144000*a^6*b^10*c^6*f*h*z^2 + 61440*a
^5*b^12*c^5*f*h*z^2 - 46080*a^4*b^14*c^4*f*h*z^2 + 1536*a^3*b^16*c^3*f*h*z^
2 - 23592960*a^6*b^9*c^7*e*g*z^2 + 1179648*a^5*b^11*c^6*e*g*z^2 + 829440*a^
4*b^13*c^5*d*h*z^2 + 368640*a^5*b^11*c^6*d*h*z^2 - 105984*a^3*b^15*c^4*d*h*
z^2 + 4608*a^2*b^17*c^3*d*h*z^2 - 15175680*a^4*b^12*c^6*d*f*z^2 + 1428480*a
^3*b^14*c^5*d*f*z^2 - 73728*a^2*b^16*c^4*d*f*z^2 + 4108320768*a^10*b^3*c^9*
d*m*z^2 - 1207959552*a^11*b*c^10*e*l*z^2 - 1207959552*a^10*b*c^11*e*g*z^2 -
 578813952*a^12*b*c^9*h*m*z^2 - 578813952*a^11*b*c^10*f*k*z^2 - 402653184*a
^12*b*c^9*j*l*z^2 - 402653184*a^11*b*c^10*g*j*z^2 - 440401920*a^10*b*c^11*f
^2*z^2 - 188743680*a^12*b*c^9*k^2*z^2 - 188743680*a^11*b*c^10*h^2*z^2 + 176
1607680*a^10*c^12*d*f*z^2 - 14080*a^6*b^15*c*m^2*z^2 - 94464*a*b^17*c^4*d^2
*z^2 + 6936330240*a^8*b^3*c^11*d^2*z^2 + 2464874496*a^6*b^7*c^9*d^2*z^2 - 3
963617280*a^9*b*c^12*d^2*z^2 + 1056964608*a^11*c^11*d*k*z^2 + 805306368*a^1
1*c^11*e*j*z^2 + 419430400*a^12*c^10*f*m*z^2 + 251658240*a^13*c^9*k*m*z^2 -
 1509949440*a^9*b^2*c^11*e^2*z^2 + 251658240*a^11*c^11*f*h*z^2 + 150994944*
a^12*c^10*h*k*z^2 - 5400428544*a^7*b^5*c^10*d^2*z^2 + 754974720*a^8*b^4*c^1
0*e^2*z^2 - 730054656*a^5*b^9*c^8*d^2*z^2 + 477102080*a^12*b^3*c^7*m^2*z^2
- 377487360*a^11*b^4*c^7*l^2*z^2 + 477102080*a^9*b^3*c^10*f^2*z^2 + 3019898
88*a^12*b^2*c^8*l^2*z^2 - 377487360*a^9*b^4*c^9*g^2*z^2 + 301989888*a^10*b^
2*c^10*g^2*z^2 - 174325760*a^11*b^5*c^6*m^2*z^2 + 188743680*a^10*b^6*c^6*l^
2*z^2 + 141557760*a^11*b^3*c^8*k^2*z^2 + 188743680*a^8*b^6*c^8*g^2*z^2 + 14
1557760*a^10*b^3*c^9*h^2*z^2 - 174325760*a^8*b^5*c^9*f^2*z^2 - 188743680*a^
7*b^6*c^9*e^2*z^2 - 47185920*a^9*b^8*c^5*l^2*z^...
```

3.58 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^6+kx^7}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=645

$$\frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+\left(bc(cd+ah)-ab^2j-2ac(cf-aj)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bc(ce+ai)-ab^2k-2ac(}{}$$

```
[Out] 1/2*x*(c*(b^2*d-2*a*(-a*h+c*d)-a*b*(a*j+c*f)/c)+(b*c*(a*h+c*d)-a*b^2*j-2*a*
c*(-a*j+c*f))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(-b*c*(a*i+c*e)+a*b
^2*k+2*a*c*(-a*k+c*g)-(2*c^3*e-c^2*(2*a*i+b*g)-b^3*k+b*c*(3*a*k+b*i))*x^2)/
c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(4*c^3*e-c^2*(-4*a*i+2*b*g)+b^3*k-6*a*
b*c*k)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*k
*ln(c*x^4+b*x^2+a)/c^2+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(b*(a*h+c*d)+a*b^2*j/c-2*a*(3*a*j+c*f)+(b^2*c*(-a*h+c*d)-4*a*c^2*(a*
h+3*c*d)-a*b^3*j+4*a*b*c*(2*a*j+c*f))/c/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*
2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*h+c*d)+a*b^2*j/c-2*a*(3*a*j+c*f)+(-b^2*c
*(-a*h+c*d)+4*a*c^2*(a*h+3*c*d)+a*b^3*j-4*a*b*c*(2*a*j+c*f))/c/(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.22, antiderivative size = 645, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1687, 1692, 1180, 211, 1677, 1674, 648, 632, 212, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2j}{c}+\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{ab^2j}{c}-\frac{-ab^3j+b^2c(cd-ah)+4abc(2aj+cf)-4ac^2(ah+3cd)}{c\sqrt{b^2-4ac}}+b(ah+cd)-2a(3aj+cf)\right)}{2\sqrt{2}a\sqrt{c}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(x^2\left(-ab^2j+bc(ah+cd)-2ac(cf-aj)\right)+c\left(-\frac{ab(aj+cf)}{c}-2a(cd-ah)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2bg-4ai)-6abck+b^3k+4c^3e\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{x^2\left(-c^2(2ai+bg)+bc(3ak+bi)+b^3(-k)+2c^3e\right)-ab^2k+bc(ai+ce)-2ac(cg-ak)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{k\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 +
c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*h) - (a*b*(c*f + a*j))/c) + (b*c*(c*d + a*h) -
a*b^2*j - 2*a*c*(c*f - a*j))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) - (b*c*(c*e + a*i) - a*b^2*k - 2*a*c*(c*g - a*k) + (2*c^3*e - c^2*(b*g +
2*a*i) - b^3*k + b*c*(b*i + 3*a*k))*x^2)/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + ((b*(c*d + a*h) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) + (b^2*c*(c*d -
 a*h) - 4*a*c^2*(3*c*d + a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^
2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sq
rt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*h
) + (a*b^2*j)/c - 2*a*(c*f + 3*a*j) - (b^2*c*(c*d - a*h) - 4*a*c^2*(3*c*d +
 a*h) - a*b^3*j + 4*a*b*c*(c*f + 2*a*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqr
```

```
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4
*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((4*c^3*e - c^2*(2*b*g - 4*a*i) + b^3*
k - 6*a*b*c*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*
c)^(3/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+58x^5+jx^6+kx^7}{(a+bx^2+cx^4)^2}\,dx = \int \frac{d+fx^2+hx^4+jx^6}{(a+bx^2+cx^4)^2}\,dx + \int \frac{x(e+gx^2+58x^4+}{(a+bx^2+cx^4)^2}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+(bc(cd+ah)\right.}{2ac\,(b^2-4ac)\,(a+bx^2+cx}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+(bc(cd+ah)\right.}{2ac\,(b^2-4ac)\,(a+bx^2+cx}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+(bc(cd+ah)\right.}{2ac\,(b^2-4ac)\,(a+bx^2+cx}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+(bc(cd+ah)\right.}{2ac\,(b^2-4ac)\,(a+bx^2+cx}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-ah)-\frac{ab(cf+aj)}{c}\right)+(bc(cd+ah)\right.}{2ac\,(b^2-4ac)\,(a+bx^2+cx}$$

Mathematica [A]
time = 2.69, size = 775, normalized size = 1.20

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*
x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a^3*c*k - b*c^2*d*x*(b + c*x^2) + a*(-(b^3*k*x^2) + b^2*c*x^2*(i + j
*x) + 2*c^3*x*(d + x*(e + f*x)) + b*c^2*(e + x*(f - x*(g + h*x)))) + a^2*(-
(b^2*k) + b*c*(i + x*(j + 3*k*x)) - 2*c^2*(g + x*(h + x*(i + j*x))))))/(a*(
-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*Sqrt[c]*(a*b^3*j - b*c*(c*Sqr
t[b^2 - 4*a*c]*d + 4*a*c*f + a*Sqrt[b^2 - 4*a*c]*h + 8*a^2*j) - b^2*(c^2*d
- a*c*h + a*Sqrt[b^2 - 4*a*c]*j) + 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*f +
 2*a*c*h + 3*a*Sqrt[b^2 - 4*a*c]*j))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
```

```
rt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*Sqrt[c]*(a*b^3*j + b*c*(c*Sqrt[b^2 - 4*a*c]*d - 4*a*c*f + a*Sqrt[b^2
 - 4*a*c]*h - 8*a^2*j) + 2*a*c*(6*c^2*d - c*Sqrt[b^2 - 4*a*c]*f + 2*a*c*h -
 3*a*Sqrt[b^2 - 4*a*c]*j) + b^2*(-(c^2*d) + a*c*h + a*Sqrt[b^2 - 4*a*c]*j))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-4*c^3*e + 2*c^2*(b*g - 2*a*i) + b^2
*(-b + Sqrt[b^2 - 4*a*c])*k + a*c*(6*b*k - 4*Sqrt[b^2 - 4*a*c]*k))*Log[-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((4*c^3*e + c^2*(-2*b*
g + 4*a*i) + b^2*(b + Sqrt[b^2 - 4*a*c])*k - 2*a*c*(3*b + 2*Sqrt[b^2 - 4*a*
c])*k)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*c^2)
```

Maple [A]
time = 0.59, size = 1072, normalized size = 1.66

method	result
risch	$\frac{-\frac{(2a^2cj-ab^2j+abch-2ac^2f+bc^2d)x^3}{2a(4ac-b^2)c}+\frac{(3abck-2ac^2i-b^3k+b^2ci-bc^2g+2c^3e)x^2}{2(4ac-b^2)c^2}+\frac{(a^2bj-2a^2ch+abcf+2ac^2d-b^2cd)x}{2ac(4ac-b^2)}+\frac{2a^2ck-ab^2k+abc\cdots}{2(4ac-\cdots}}{cx^4+bx^2+a}$
default	$\frac{-\frac{(2a^2cj-ab^2j+abch-2ac^2f+bc^2d)x^3}{2a(4ac-b^2)c}+\frac{(3abck-2ac^2i-b^3k+b^2ci-bc^2g+2c^3e)x^2}{2(4ac-b^2)c^2}+\frac{(a^2bj-2a^2ch+abcf+2ac^2d-b^2cd)x}{2ac(4ac-b^2)}+\frac{2a^2ck-ab^2k+abc\cdots}{2(4ac-\cdots}}{cx^4+bx^2+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,method=
_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(2*a^2*c*j-a*b^2*j+a*b*c*h-2*a*c^2*f+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2
*(3*a*b*c*k-2*a*c^2*i-b^3*k+b^2*c*i-b*c^2*g+2*c^3*e)/(4*a*c-b^2)/c^2*x^2+1/
2*(a^2*b*j-2*a^2*c*h+a*b*c*f+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x+1/2*(2*a^
2*c*k-a*b^2*k+a*b*c*i-2*a*c^2*g+b*c^2*e)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+2
/a/(4*a*c-b^2)*(1/4/c/(4*a*c-b^2)*(-1/4*(-12*(-4*a*c+b^2)^(1/2)*a^2*b*c*k+8
*(-4*a*c+b^2)^(1/2)*a^2*c^2*i+2*(-4*a*c+b^2)^(1/2)*a*b^3*k-4*(-4*a*c+b^2)^(
1/2)*a*b*c^2*g+8*(-4*a*c+b^2)^(1/2)*a*c^3*e-32*a^3*c^2*k+16*a^2*b^2*c*k-2*a
*b^4*k)/c*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-8*(-4*a*c+b^2)^(1/2)*a^2*
b*c*j+4*(-4*a*c+b^2)^(1/2)*a^2*c^2*h+(-4*a*c+b^2)^(1/2)*a*b^3*j+(-4*a*c+b^2
)^(1/2)*a*b^2*c*h-4*(-4*a*c+b^2)^(1/2)*a*b*c^2*f+12*(-4*a*c+b^2)^(1/2)*a*c^
3*d-(-4*a*c+b^2)^(1/2)*b^2*c^2*d-24*a^3*c^2*j+10*a^2*b^2*c*j+4*a^2*b*c^2*h-
8*a^2*c^3*f-a*b^4*j-a*b^3*c*h+2*a*b^2*c^2*f+4*a*b*c^3*d-b^3*c^2*d)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(-12*(-4*a*c+b^2)^(1/2)*a^2*b*c*k+8*(
```

```
-4*a*c+b^2)^(1/2)*a^2*c^2*i+2*(-4*a*c+b^2)^(1/2)*a*b^3*k-4*(-4*a*c+b^2)^(1/
2)*a*b*c^2*g+8*(-4*a*c+b^2)^(1/2)*a*c^3*e+32*a^3*c^2*k-16*a^2*b^2*c*k+2*a*b
^4*k)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-8*(-4*a*c+b^2)^(1/2)*a^2*b*c
*j+4*(-4*a*c+b^2)^(1/2)*a^2*c^2*h+(-4*a*c+b^2)^(1/2)*a*b^3*j+(-4*a*c+b^2)^(
1/2)*a*b^2*c*h-4*(-4*a*c+b^2)^(1/2)*a*b*c^2*f+12*(-4*a*c+b^2)^(1/2)*a*c^3*d
-(-4*a*c+b^2)^(1/2)*b^2*c^2*d+24*a^3*c^2*j-10*a^2*b^2*c*j-4*a^2*b*c^2*h+8*a
^2*c^3*f+a*b^4*j+a*b^3*c*h-2*a*b^2*c^2*f-4*a*b*c^3*d+b^3*c^2*d)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*c^2*g - a*b*c^2*e - I*a^2*b*c + (b*c^3*d - 2*a*c^3*f + a*b*c^2*h
 - (a*b^2*c - 2*a^2*c^2)*j)*x^3 + (a*b*c^2*g - 2*a*c^3*e - I*a*b^2*c + 2*I*
a^2*c^2 + (a*b^3 - 3*a^2*b*c)*k)*x^2 + (a^2*b^2 - 2*a^3*c)*k - (a*b*c^2*f -
 2*a^2*c^2*h + a^2*b*c*j - (b^2*c^2 - 2*a*c^3)*d)*x)/(a^2*b^2*c^2 - 4*a^3*c
^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^4 + (a*b^3*c^2 - 4*a^2*b*c^3)*x^2) - 1/2*int
egrate(-(2*(a*b^2 - 4*a^2*c)*k*x^3 + a*b*c*f - 2*a^2*c*h + a^2*b*j + (b*c^2
*d - 2*a*c^2*f + a*b*c*h + (a*b^2 - 6*a^2*c)*j)*x^2 + (b^2*c - 6*a*c^2)*d +
 2*(a*b*c*g + a^2*b*k - 2*a*c^2*e - 2*I*a^2*c)*x)/(c*x^4 + b*x^2 + a), x)/(
a*b^2*c - 4*a^2*c^2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x,
algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((k*x**7+j*x**6+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16720 vs. $2(593) = 1186$.
time = 8.16, size = 16720, normalized size = 25.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((k*x^7+j*x^6+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] 1/4*k*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/16*((a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)^2*(2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*d - 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)^2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*f + (a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)^2*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*h + (a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)^2*(2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*j + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2*a^2*b^8*c^6 + 120*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 36*a^3*b^6*c^7 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 14*sqrt(2)*sqrt(b*c

```
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 240*a^4*b^4*c^8 + 384*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*c^9 + 192*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b*c^9 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 70
4*a^5*b^2*c^9 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^10 + 768*a
^6*c^10 - 2*(b^2 - 4*a*c)*a^2*b^6*c^6 + 28*(b^2 - 4*a*c)*a^3*b^4*c^7 - 128*
(b^2 - 4*a*c)*a^4*b^2*c^8 + 192*(b^2 - 4*a*c)*a^5*c^9)*d*abs(-a^2*b^4*c^3 +
 8*a^3*b^2*c^4 - 16*a^4*c^5) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^7*c^5 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 2*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 + 2*a^3*b^7*c^6 + 48*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^5*c^7 - 24*a^4*b^5*c^7 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b
*c^8 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 + 96*a^5*b^3*c^8 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 128*a^6*b*c^9 - 2*(b^2 - 4*a*c)*a
^3*b^5*c^6 + 16*(b^2 - 4*a*c)*a^4*b^3*c^7 - 32*(b^2 - 4*a*c)*a^5*b*c^8)*f*a
bs(-a^2*b^4*c^3 + 8*a^3*b^2*c^4 - 16*a^4*c^5) - 4*(sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^6*c^5 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^4*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 + 2*a^4*b
^6*c^6 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^7 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^4*c^7 - 24*a^5*b^4*c^7 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^7*c^8 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 + 96*a^6*b^2*c^8 +
16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^9 - 128*a^7*c^9 - 2*(b^2 -
 4*a*c)*a^4*b^4*c^6 + 16*(b^2 - 4*a*c)*a^5*b^2*c^7 - 32*(b^2 - 4*a*c)*a^6*c
^8)*h*abs(-a^2*b^4*c^3 + 8*a^3*b^2*c^4 - 16*a^4*c^5) + 2*(sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^4 - 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^5*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^5 +
2*a^4*b^7*c^5 + 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^6 + 16
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^6 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 24*a^5*b^5*c^6 - 64*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^7*b*c^7 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^6*b^2*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 96*a^6
*b^3*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 - 128*a^7*b
*c^8 - 2*(b^2 - 4*a*c)*a^4*b^5*c^5 + 16*(b^2 - ...
```

Mupad [B]
time = 8.85, size = 2500, normalized size = 3.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^6 + k*x^7)/(a + b*x^2 +
c*x^4)^2,x)
```

```
[Out] ((b*c^2*e - 2*a*c^2*g - a*b^2*k + 2*a^2*c*k + a*b*c*i)/(2*c^2*(4*a*c - b^2)
) + (x^2*(2*c^3*e - b^3*k - b*c^2*g - 2*a*c^2*i + b^2*c*i + 3*a*b*c*k))/(2*
c^2*(4*a*c - b^2)) + (x*(2*a*c^2*d - b^2*c*d - 2*a^2*c*h + a^2*b*j + a*b*c*
f))/(2*a*c*(4*a*c - b^2)) - (x^3*(b*c^2*d - 2*a*c^2*f - a*b^2*j + 2*a^2*c*j
 + a*b*c*h))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1
572864*a^8*b^2*c^9*z^4 - 983040*a^7*b^4*c^8*z^4 + 327680*a^6*b^6*c^7*z^4 -
61440*a^5*b^8*c^6*z^4 + 6144*a^4*b^10*c^5*z^4 - 256*a^3*b^12*c^4*z^4 - 1048
576*a^9*c^10*z^4 - 1572864*a^8*b^2*c^7*k*z^3 + 983040*a^7*b^4*c^6*k*z^3 - 3
27680*a^6*b^6*c^5*k*z^3 + 61440*a^5*b^8*c^4*k*z^3 - 6144*a^4*b^10*c^3*k*z^3
 + 256*a^3*b^12*c^2*k*z^3 + 1048576*a^9*c^8*k*z^3 + 98304*a^8*b*c^6*i*k*z^2
 + 98304*a^7*b*c^7*e*k*z^2 + 57344*a^7*b*c^7*f*j*z^2 + 32768*a^7*b*c^7*g*i*
z^2 + 57344*a^6*b*c^8*d*h*z^2 + 32768*a^6*b*c^8*e*g*z^2 - 32*a*b^10*c^4*d*f
*z^2 - 90112*a^7*b^3*c^5*i*k*z^2 + 30720*a^6*b^5*c^4*i*k*z^2 - 4608*a^5*b^7
*c^3*i*k*z^2 + 256*a^4*b^9*c^2*i*k*z^2 - 49152*a^7*b^2*c^6*g*k*z^2 + 45056*
a^6*b^4*c^5*g*k*z^2 + 24576*a^7*b^2*c^6*h*j*z^2 - 15360*a^5*b^6*c^4*g*k*z^2
 - 3072*a^5*b^6*c^4*h*j*z^2 + 2304*a^4*b^8*c^3*g*k*z^2 + 2048*a^6*b^4*c^5*h
*j*z^2 + 576*a^4*b^8*c^3*h*j*z^2 - 128*a^3*b^10*c^2*g*k*z^2 - 32*a^3*b^10*c
^2*h*j*z^2 - 90112*a^6*b^3*c^6*e*k*z^2 - 49152*a^6*b^3*c^6*f*j*z^2 + 30720*
a^5*b^5*c^5*e*k*z^2 - 24576*a^6*b^3*c^6*g*i*z^2 + 15360*a^5*b^5*c^5*f*j*z^2
 + 6144*a^5*b^5*c^5*g*i*z^2 - 4608*a^4*b^7*c^4*e*k*z^2 - 2048*a^4*b^7*c^4*f
*j*z^2 - 512*a^4*b^7*c^4*g*i*z^2 + 256*a^3*b^9*c^3*e*k*z^2 + 96*a^3*b^9*c^3
*f*j*z^2 + 131072*a^6*b^2*c^7*d*j*z^2 + 49152*a^6*b^2*c^7*e*i*z^2 - 43008*a
^5*b^4*c^6*d*j*z^2 - 12288*a^5*b^4*c^6*e*i*z^2 + 6144*a^5*b^4*c^6*f*h*z^2 +
 6144*a^4*b^6*c^5*d*j*z^2 - 2048*a^4*b^6*c^5*f*h*z^2 + 1024*a^4*b^6*c^5*e*i
*z^2 - 320*a^3*b^8*c^4*d*j*z^2 + 192*a^3*b^8*c^4*f*h*z^2 - 49152*a^5*b^3*c^
7*d*h*z^2 - 24576*a^5*b^3*c^7*e*g*z^2 + 15360*a^4*b^5*c^6*d*h*z^2 + 6144*a^
4*b^5*c^6*e*g*z^2 - 2048*a^3*b^7*c^5*d*h*z^2 - 512*a^3*b^7*c^5*e*g*z^2 + 96
*a^2*b^9*c^4*d*h*z^2 + 24576*a^5*b^2*c^8*d*f*z^2 - 3072*a^3*b^6*c^6*d*f*z^2
 + 2048*a^4*b^4*c^7*d*f*z^2 + 576*a^2*b^8*c^5*d*f*z^2 + 1536*a^4*b^10*c*k^2
*z^2 + 61440*a^8*b*c^6*j^2*z^2 - 16*a^3*b^11*c*j^2*z^2 + 12288*a^7*b*c^7*h^
2*z^2 + 12288*a^6*b*c^8*f^2*z^2 + 61440*a^5*b*c^9*d^2*z^2 + 432*a*b^9*c^5*d
^2*z^2 - 49152*a^8*c^7*h*j*z^2 - 147456*a^7*c^8*d*j*z^2 - 65536*a^7*c^8*e*i
*z^2 - 16384*a^7*c^8*f*h*z^2 - 49152*a^6*c^9*d*f*z^2 + 516096*a^8*b^2*c^5*k
^2*z^2 - 288768*a^7*b^4*c^4*k^2*z^2 + 88576*a^6*b^6*c^3*k^2*z^2 - 15744*a^5
*b^8*c^2*k^2*z^2 - 61440*a^7*b^3*c^5*j^2*z^2 + 24064*a^6*b^5*c^4*j^2*z^2 -
4608*a^5*b^7*c^3*j^2*z^2 + 432*a^4*b^9*c^2*j^2*z^2 + 24576*a^7*b^2*c^6*i^2*
z^2 - 6144*a^6*b^4*c^5*i^2*z^2 + 512*a^5*b^6*c^4*i^2*z^2 - 8192*a^6*b^3*c^6
*h^2*z^2 + 1536*a^5*b^5*c^5*h^2*z^2 - 16*a^3*b^9*c^3*h^2*z^2 - 8192*a^6*b^2
*c^7*g^2*z^2 + 6144*a^5*b^4*c^6*g^2*z^2 - 1536*a^4*b^6*c^5*g^2*z^2 + 128*a^
3*b^8*c^4*g^2*z^2 - 8192*a^5*b^3*c^7*f^2*z^2 + 1536*a^4*b^5*c^6*f^2*z^2 - 1
6*a^2*b^9*c^4*f^2*z^2 + 24576*a^5*b^2*c^8*e^2*z^2 - 6144*a^4*b^4*c^7*e^2*z^
2 + 512*a^3*b^6*c^6*e^2*z^2 - 61440*a^4*b^3*c^8*d^2*z^2 + 24064*a^3*b^5*c^7
*d^2*z^2 - 4608*a^2*b^7*c^6*d^2*z^2 - 393216*a^9*c^6*k^2*z^2 - 64*a^3*b^12*
k^2*z^2 - 32768*a^8*c^7*i^2*z^2 - 32768*a^6*c^9*e^2*z^2 - 16*b^11*c^4*d^2*z
^2 - 16384*a^7*b*c^5*g*i*k*z - 10240*a^7*b*c^5*f*j*k*z + 4096*a^7*b*c^5*h*i
```

```
*j*z - 47104*a^6*b*c^6*d*h*k*z - 16384*a^6*b*c^6*e*g*k*z + 6144*a^6*b*c^6*f
*g*j*z + 4096*a^6*b*c^6*e*h*j*z + 32*a*b^10*c^2*d*f*k*z - 6144*a^5*b*c^7*d*
g*h*z - 4096*a^5*b*c^7*d*f*i*z - 32*a*b^8*c^4*d*f*g*z - 4096*a^4*b*c^8*d*e*
f*z + 64*a*b^7*c^5*d*e*f*z - 18432*a^7*b^2*c^4*h*j*k*z + 4608*a^6*b^4*c^3*h
*j*k*z - 384*a^5*b^6*c^2*h*j*k*z + 12288*a^6*b^3*c^4*g*i*k*z + 7680*a^6*b^3
*c^4*f*j*k*z - 3072*a^6*b^3*c^4*h*i*j*z - 3072*a^5*b^5*c^3*g*i*k*z - 1920*a
^5*b^5*c^3*f*j*k*z + 768*a^5*b^5*c^3*h*i*j*z + 256*a^4*b^7*c^2*g*i*k*z + 16
0*a^4*b^7*c^2*f*j*k*z - 64*a^4*b^7*c^2*h*i*j*z - 65536*a^6*b^2*c^5*d*j*k*z
- 24576*a^6*b^2*c^5*e*i*k*z + 21504*a^5*b^4*c^4*d*j*k*z + 9216*a^6*b^2*c^5*
f*i*j*z + 6144*a^5*b^4*c^4*e*i*k*z - 3072*a^5*b^4*c^4*f*h*k*z - 3072*a^4*b^
6*c^3*d*j*k*z - 2304*a^5*b^4*c^4*f*i*j*z - 2048*a^6*b^2*c^5*g*h*j*z + 1536*
a^5*b^4*c^4*g*h*j*z + 1024*a^4*b^6*c^3*f*h*k*z - 512*a^4*b^6*c^3*e*i*k*z -
384*a^4*b^6*c^3*g*h*j*z + 192*a^4*b^6*c^3*f*i*j*z + 160*a^3*b^8*c^2*d*j*k*z
 - 96*a^3*b^8*c^2*f*h*k*z + 32*a^3*b^8*c^2*g*h*j*z + 41472*a^5*b^3*c^5*d*h*
k*z - 13440*a^4*b^5*c^4*d*h*k*z + 12288*a^5*b^3*c^5*e*g*k*z - 4608*a^5*b^3*
c^5*f*g*j*z - 3072*a^5*b^3*c^5*e*h*j*z - 3072*a^4*b^5*c^4*e*g*k*z + 1888*a^
3*b^7*c^3*d*h*k*z + 1152*a^4*b^5*c^4*f*g*j*z + 768*a^4*b^5*c^4*e*h*j*z + 25
6*a^3*b^7*c^3*e*g*k*z - 96*a^3*b^7*c^3*f*g*j*z - 96*a^2*b^9*c^2*d*h*k*z - 6
4*a^3*b^7*c^3*e*h*j*z + 9216*a^5*b^2*c^6*e*f*j*z - 9216*a^5*b^2*c^6*d*h*i*z
 - 6656*a^4*b^4*c^5*d*f*k*z - 6144*a^5*b^2*c^6*d*f*k*z + 3456*a^3*b^6*c^4*d
*f*k*z - 2304*a^4*b^4*c^5*e*f*j*z + 2304*a^4*b^4*c^5*d*h*i*z - 576*a^2*b^8*
c^3*d*f*k*z + 192*a^3*b^6*c^4*e*f*j*z - 192*a^3...
```

3.59 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=1177

$$-\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+\left(2ac^3f-ab^3j-bc(c^2d+ach-3a^2j)\right)x^2\right)}{4ac^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{bc^3(ce+$$

```
[Out] -1/4*x*(c^2*(a*b*f-b^2*(d+a^2*j/c^2)+2*a*(c*d-a*h+a^2*j/c))+(2*a*c^3*f-a*b^
3*j-b*c*(-3*a^2*j+a*c*h+c^2*d))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1
/4*(-b*c^3*(a*i+c*e)+a*b^4*k-4*a^2*b^2*c*k+2*a*c^2*(a^2*k+c^2*g)-(2*c^5*e+b
^2*c^3*i-c^4*(2*a*i+b*g)-b^5*k+5*a*b^3*c*k-5*a^2*b*c^2*k)*x^2)/c^4/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)^2+1/8*x*(c*(a*b^3*f+8*a^2*b*c*f+4*a^2*(-9*a^2*j+a*c*h+
7*c^2*d)+b^4*(3*d-2*a^2*j/c^2)-a*b^2*(25*c*d+7*a*h-11*a^2*j/c))+(a*b^2*c^2*
f+20*a^2*c^3*f+b^3*(a^2*j+3*c^2*d)-4*a*b*c*(4*a^2*j+3*a*c*h+6*c^2*d))*x^2)/
a^2/c/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/4*(b^3*c^2*i+2*b*c^3*(a*i+3*c*e)+11*
a*b^4*k-b^6*k/c+32*a^3*c^2*k-3*b^2*(13*a^2*c*k+c^3*g)+2*(6*c^5*e+b^2*c^3*i-
c^4*(-2*a*i+3*b*g)+2*b^5*k-15*a*b^3*c*k+25*a^2*b*c^2*k)*x^2)/c^3/(-4*a*c+b^
2)^2/(c*x^4+b*x^2+a)-1/2*(12*c^5*e+2*b^2*c^3*i-c^4*(-4*a*i+6*b*g)-b^5*k+10*
a*b^3*c*k-30*a^2*b*c^2*k)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a
*c+b^2)^(5/2)+1/4*k*ln(c*x^4+b*x^2+a)/c^3+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*(a*b^2*c^2*f+20*a^2*c^3*f+b^3*(a^2*j+3*c^2*d)-4*
a*b*c*(4*a^2*j+3*a*c*h+6*c^2*d)+(a*b^3*c^2*f-52*a^2*b*c^3*f-6*a*b^2*c*(-3*a
^2*j-3*a*c*h+5*c^2*d)+b^4*(-a^2*j+3*c^2*d)+8*a^2*c^2*(5*a^2*j+3*a*c*h+21*c^
2*d))/(-4*a*c+b^2)^(1/2))/a^2/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*
(a*b^2*c^2*f+20*a^2*c^3*f+b^3*(a^2*j+3*c^2*d)-4*a*b*c*(4*a^2*j+3*a*c*h+6*c^
2*d)+(-a*b^3*c^2*f+52*a^2*b*c^3*f+6*a*b^2*c*(-3*a^2*j-3*a*c*h+5*c^2*d)-b^4*
(-a^2*j+3*c^2*d)-8*a^2*c^2*(5*a^2*j+3*a*c*h+21*c^2*d))/(-4*a*c+b^2)^(1/2))/
a^2/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 5.28, antiderivative size = 1179, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1687, 1692, 1180, 211, 1677, 1674, 648, 632, 212, 642}

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2 +
 c*x^4)^3,x]
```

```
[Out] -1/4*(x*(c^2*(a*b*f - b^2*(d + (a^2*j)/c^2) + 2*a*(c*d - a*h + (a^2*j)/c))
+ (2*a*c^3*f - a*b^3*j - b*c*(c^2*d + a*c*h - 3*a^2*j))*x^2))/(a*c^2*(b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)^2) - (b*c^3*(c*e + a*i) - a*b^4*k + 4*a^2*b^2*c
*k - 2*a*c^2*(c^2*g + a^2*k) + (2*c^5*e + b^2*c^3*i - c^4*(b*g + 2*a*i) - b
^5*k + 5*a*b^3*c*k - 5*a^2*b*c^2*k)*x^2)/(4*c^4*(b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)^2) + (x*(c*(a*b^3*f + 8*a^2*b*c*f + 4*a^2*(7*c^2*d + a*c*h - 9*a^2*j
) + b^4*(3*d - (2*a^2*j)/c^2) - a*b^2*(25*c*d + 7*a*h - (11*a^2*j)/c)) + (a
*b^2*c^2*f + 20*a^2*c^3*f + b^3*(3*c^2*d + a^2*j) - 4*a*b*c*(6*c^2*d + 3*a*
c*h + 4*a^2*j))*x^2))/(8*a^2*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^3*
c^2*i + 2*b*c^3*(3*c*e + a*i) + 11*a*b^4*k - (b^6*k)/c + 32*a^3*c^2*k - 3*b
^2*(c^3*g + 13*a^2*c*k) + 2*(6*c^5*e + b^2*c^3*i - c^4*(3*b*g - 2*a*i) + 2*
b^5*k - 15*a*b^3*c*k + 25*a^2*b*c^2*k)*x^2)/(4*c^3*(b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) + ((a*b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*h + 4*a^
2*j) + b^3*(3*c*d + (a^2*j)/c) + (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*a*b^2*c*
(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(21*c^2*d
 + 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((a*b^2*c*f + 20*a^2*c^2*f - 4*a*b*(6*c^2*d + 3*a*c*
h + 4*a^2*j) + b^3*(3*c*d + (a^2*j)/c) - (a*b^3*c^2*f - 52*a^2*b*c^3*f - 6*
a*b^2*c*(5*c^2*d - 3*a*c*h - 3*a^2*j) + b^4*(3*c^2*d - a^2*j) + 8*a^2*c^2*(
21*c^2*d + 3*a*c*h + 5*a^2*j))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((12*c^5*e + 2*b^2*c^3*i - c^4*(6*b*g - 4*a*
i) - b^5*k + 10*a*b^3*c*k - 30*a^2*b*c^2*k)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + (k*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
```

```
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+59x^5+jx^8+kx^{11}}{(a+bx^2+cx^4)^3}\,dx = \int \frac{d+fx^2+hx^4+jx^8}{(a+bx^2+cx^4)^3}\,dx + \int \frac{x(e+gx^2+59x^4+}{(a+bx^2+cx^4}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

$$= -\frac{x\left(c^2\left(abf-b^2\left(d+\frac{a^2j}{c^2}\right)+2a\left(cd-ah+\frac{a^2j}{c}\right)\right)+}{4ac^2\left(b^2-4ac\right)(a-}$$

Mathematica [A]

time = 6.85, size = 1649, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b
*x^2 + c*x^4)^3,x]
```

```
[Out] (a*b*c^4*e - 2*a^2*c^4*g + a^2*b*c^3*i - a^2*b^4*k + 4*a^3*b^2*c*k - 2*a^4*
c^2*k - b^2*c^4*d*x + 2*a*c^5*d*x + a*b*c^4*f*x - 2*a^2*c^4*h*x - a^2*b^2*c
^2*j*x + 2*a^3*c^3*j*x + 2*a*c^5*e*x^2 - a*b*c^4*g*x^2 + a*b^2*c^3*i*x^2 -
2*a^2*c^4*i*x^2 - a*b^5*k*x^2 + 5*a^2*b^3*c*k*x^2 - 5*a^3*b*c^2*k*x^2 - b*c
^5*d*x^3 + 2*a*c^5*f*x^3 - a*b*c^4*h*x^3 - a*b^3*c^2*j*x^3 + 3*a^2*b*c^3*j*
x^3)/(4*a*c^4*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (12*a^2*b*c^5*e - 6*a
^2*b^2*c^4*g + 2*a^2*b^3*c^3*i + 4*a^3*b*c^4*i - 2*a^2*b^6*k + 22*a^3*b^4*c
*k - 78*a^4*b^2*c^2*k + 64*a^5*c^3*k + 3*b^4*c^4*d*x - 25*a*b^2*c^5*d*x + 2
8*a^2*c^6*d*x + a*b^3*c^4*f*x + 8*a^2*b*c^5*f*x - 7*a^2*b^2*c^4*h*x + 4*a^3
*c^5*h*x - 2*a^2*b^4*c^2*j*x + 11*a^3*b^2*c^3*j*x - 36*a^4*c^4*j*x + 24*a^2
*c^6*e*x^2 - 12*a^2*b*c^5*g*x^2 + 4*a^2*b^2*c^4*i*x^2 + 8*a^3*c^5*i*x^2 + 8
*a^2*b^5*c*k*x^2 - 60*a^3*b^3*c^2*k*x^2 + 100*a^4*b*c^3*k*x^2 + 3*b^3*c^5*d
*x^3 - 24*a*b*c^6*d*x^3 + a*b^2*c^5*f*x^3 + 20*a^2*c^6*f*x^3 - 12*a^2*b*c^5
*h*x^3 + a^2*b^3*c^3*j*x^3 - 16*a^3*b*c^4*j*x^3)/(8*a^2*c^4*(-b^2 + 4*a*c)^
2*(a + b*x^2 + c*x^4)) + ((3*b^4*c^2*d - 30*a*b^2*c^3*d + 168*a^2*c^4*d + 3
*b^3*c^2*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d + a*b^3*c^2*f
 - 52*a^2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4
*a*c]*f + 18*a^2*b^2*c^2*h + 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*
h - a^2*b^4*j + 18*a^3*b^2*c*j + 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j
 - 16*a^3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt
[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + ((-3*b^4*c^2*d + 30*a*b^2*c^3*d - 168*a^2*c^4*d + 3*b^3*c^2
*Sqrt[b^2 - 4*a*c]*d - 24*a*b*c^3*Sqrt[b^2 - 4*a*c]*d - a*b^3*c^2*f + 52*a^
2*b*c^3*f + a*b^2*c^2*Sqrt[b^2 - 4*a*c]*f + 20*a^2*c^3*Sqrt[b^2 - 4*a*c]*f
 - 18*a^2*b^2*c^2*h - 24*a^3*c^3*h - 12*a^2*b*c^2*Sqrt[b^2 - 4*a*c]*h + a^2*
b^4*j - 18*a^3*b^2*c*j - 40*a^4*c^2*j + a^2*b^3*Sqrt[b^2 - 4*a*c]*j - 16*a^
3*b*c*Sqrt[b^2 - 4*a*c]*j)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(8*Sqrt[2]*a^2*c^(3/2)*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]]) + ((12*c^5*e - 6*b*c^4*g + 2*b^2*c^3*i + 4*a*c^4*i - b^5*k + 10*a*b^3
*c*k - 30*a^2*b*c^2*k + b^4*Sqrt[b^2 - 4*a*c]*k - 8*a*b^2*c*Sqrt[b^2 - 4*a*
c]*k + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]*k)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
])/(4*c^3*(b^2 - 4*a*c)^(5/2)) + ((-12*c^5*e + 6*b*c^4*g - 2*b^2*c^3*i - 4*
a*c^4*i + b^5*k - 10*a*b^3*c*k + 30*a^2*b*c^2*k + b^4*Sqrt[b^2 - 4*a*c]*k -
 8*a*b^2*c*Sqrt[b^2 - 4*a*c]*k + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]*k)*Log[b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2])/(4*c^3*(b^2 - 4*a*c)^(5/2))
```

<u>**Maple**</u> **[A]**
time = 0.82, size = 2058, normalized size = 1.75

method	result
risch	$\frac{-\frac{\left(16a^3bcj-a^2b^3j+12a^2b\,c^2h-20a^2c^3f-a\,b^2c^2f+24ab\,c^3d-3b^3c^2d\right)x^7}{8a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(25a^2b\,c^2k-15a\,b^3ck+2a\,c^4i+2b^5k+b^2c^3i-3b\,c^4g+6e\,c^5\right)x^6}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(36a^4c^2j\right.}{}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,method
=_RETURNVERBOSE)
```

```
[Out] (-1/8*(16*a^3*b*c*j-a^2*b^3*j+12*a^2*b*c^2*h-20*a^2*c^3*f-a*b^2*c^2*f+24*a*
b*c^3*d-3*b^3*c^2*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/2*(25*a^2*b*c^2*k
-15*a*b^3*c*k+2*a*c^4*i+2*b^5*k+b^2*c^3*i-3*b*c^4*g+6*c^5*e)/c^2/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^6-1/8/a^2*(36*a^4*c^2*j+5*a^3*b^2*c*j-4*a^3*c^3*h+a^2*b^
4*j+19*a^2*b^2*c^2*h-28*a^2*b*c^3*f-28*a^2*c^4*d-2*a*b^3*c^2*f+49*a*b^2*c^3
*d-6*b^4*c^2*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^5+1/4*(32*a^3*c^3*k+11*a^2*b
^2*c^2*k-19*a*b^4*c*k+6*a*b*c^4*i+3*b^6*k+3*b^3*c^3*i-9*b^2*c^4*g+18*b*c^5*
e)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^4-1/8/c*(28*a^4*b*c*j+2*a^3*b^3*j+16*a^
3*b*c^2*h-36*a^3*c^3*f+5*a^2*b^3*c*h-5*a^2*b^2*c^2*f+4*a^2*b*c^3*d-a*b^4*c*
f+20*a*b^3*c^2*d-3*b^5*c*d)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2/c^3*(31*
a^3*b*c^2*k-22*a^2*b^3*c*k-2*a^2*c^4*i+3*a*b^5*k+5*a*b^2*c^3*i-5*a*b*c^4*g+
10*a*c^5*e-b^3*c^3*g+2*b^2*c^4*e)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(20*a^
4*c*j+a^3*b^2*j+12*a^3*c^2*h+3*a^2*b^2*c*h-16*a^2*b*c^2*f-44*a^2*c^3*d+a*b^
3*c*f+37*a*b^2*c^2*d-5*b^4*c*d)/c/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/4*(24*a^
4*c^2*k-21*a^3*b^2*c*k+3*a^2*b^4*k+6*a^2*b*c^3*i-8*a^2*c^4*g-a*b^2*c^3*g+10
*a*b*c^4*e-b^3*c^3*e)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3)/(c*x^4+b*x^2+a)^2+1/2
/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/c*(1/4/c/(4*a*c-b^2)*(-1/4*(-240*(-4*a*c+b^
2)^(1/2)*a^4*b*c^2*k+80*(-4*a*c+b^2)^(1/2)*a^3*b^3*c*k+32*(-4*a*c+b^2)^(1/2
)*a^3*c^4*i-8*(-4*a*c+b^2)^(1/2)*a^2*b^5*k+16*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^
3*i-48*(-4*a*c+b^2)^(1/2)*a^2*b*c^4*g+96*(-4*a*c+b^2)^(1/2)*a^2*c^5*e-512*a
^5*c^3*k+384*a^4*b^2*c^2*k-96*a^3*b^4*c*k+8*a^2*b^6*k)/c*ln(-b-2*c*x^2+(-4*
a*c+b^2)^(1/2))+1/2*(40*(-4*a*c+b^2)^(1/2)*a^4*c^3*j+18*(-4*a*c+b^2)^(1/2)*
a^3*b^2*c^2*j+24*(-4*a*c+b^2)^(1/2)*a^3*c^4*h-(-4*a*c+b^2)^(1/2)*a^2*b^4*c*
j+18*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^3*h-52*(-4*a*c+b^2)^(1/2)*a^2*b*c^4*f+168
*(-4*a*c+b^2)^(1/2)*a^2*c^5*d+(-4*a*c+b^2)^(1/2)*a*b^3*c^3*f-30*(-4*a*c+b^2
)^(1/2)*a*b^2*c^4*d+3*(-4*a*c+b^2)^(1/2)*b^4*c^3*d+64*a^4*b*c^3*j-20*a^3*b^
3*c^2*j+48*a^3*b*c^4*h-80*a^3*c^5*f+a^2*b^5*c*j-12*a^2*b^3*c^3*h+16*a^2*b^2
*c^4*f+96*a^2*b*c^5*d+a*b^4*c^3*f-36*a*b^3*c^4*d+3*b^5*c^3*d)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)))+1/4/c/(4*a*c-b^2)*(1/4*(-240*(-4*a*c+b^2)^(1/2)*a^4*b*c^2*k+80*(-
4*a*c+b^2)^(1/2)*a^3*b^3*c*k+32*(-4*a*c+b^2)^(1/2)*a^3*c^4*i-8*(-4*a*c+b^2)
^(1/2)*a^2*b^5*k+16*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^3*i-48*(-4*a*c+b^2)^(1/2)*
```

```
a^2*b*c^4*g+96*(-4*a*c+b^2)^(1/2)*a^2*c^5*e+512*a^5*c^3*k-384*a^4*b^2*c^2*k
+96*a^3*b^4*c*k-8*a^2*b^6*k)/c*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(40*(-4
*a*c+b^2)^(1/2)*a^4*c^3*j+18*(-4*a*c+b^2)^(1/2)*a^3*b^2*c^2*j+24*(-4*a*c+b^
2)^(1/2)*a^3*c^4*h-(-4*a*c+b^2)^(1/2)*a^2*b^4*c*j+18*(-4*a*c+b^2)^(1/2)*a^2
*b^2*c^3*h-52*(-4*a*c+b^2)^(1/2)*a^2*b*c^4*f+168*(-4*a*c+b^2)^(1/2)*a^2*c^5
*d+(-4*a*c+b^2)^(1/2)*a*b^3*c^3*f-30*(-4*a*c+b^2)^(1/2)*a*b^2*c^4*d+3*(-4*a
*c+b^2)^(1/2)*b^4*c^3*d-64*a^4*b*c^3*j+20*a^3*b^3*c^2*j-48*a^3*b*c^4*h+80*a
^3*c^5*f-a^2*b^5*c*j+12*a^2*b^3*c^3*h-16*a^2*b^2*c^4*f-96*a^2*b*c^5*d-a*b^4
*c^3*f+36*a*b^3*c^4*d-3*b^5*c^3*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="maxima")
```

```
[Out] 1/8*(20*a^3*b*c^4*e - (12*a^2*b*c^5*h - 3*(b^3*c^5 - 8*a*b*c^6)*d - (a*b^2*
c^5 + 20*a^2*c^6)*f - (a^2*b^3*c^3 - 16*a^3*b*c^4)*j)*x^7 - 4*(3*a^2*b*c^5*
g - 6*a^2*c^6*e - I*a^2*b^2*c^4 - 2*I*a^3*c^5 - (2*a^2*b^5*c - 15*a^3*b^3*c
^2 + 25*a^4*b*c^3)*k)*x^6 + ((6*b^4*c^4 - 49*a*b^2*c^5 + 28*a^2*c^6)*d + 2*
(a*b^3*c^4 + 14*a^2*b*c^5)*f - (19*a^2*b^2*c^4 - 4*a^3*c^5)*h - (a^2*b^4*c^
2 + 5*a^3*b^2*c^3 + 36*a^4*c^4)*j)*x^5 - 2*(9*a^2*b^2*c^4*g - 18*a^2*b*c^5*
e - 3*I*a^2*b^3*c^3 - 6*I*a^3*b*c^4 - (3*a^2*b^6 - 19*a^3*b^4*c + 11*a^4*b^
2*c^2 + 32*a^5*c^3)*k)*x^4 - 2*(a^2*b^3*e - 6*I*a^4*b)*c^3 + ((3*b^5*c^3 -
20*a*b^3*c^4 - 4*a^2*b*c^5)*d + (a*b^4*c^3 + 5*a^2*b^2*c^4 + 36*a^3*c^5)*f
- (5*a^2*b^3*c^3 + 16*a^3*b*c^4)*h - 2*(a^3*b^3*c^2 + 14*a^4*b*c^3)*j)*x^3
+ 4*(10*a^3*c^5*e + 5*I*a^3*b^2*c^3 + 2*(a^2*b^2*e - I*a^4)*c^4 - (a^2*b^3*
c^3 + 5*a^3*b*c^4)*g + (3*a^3*b^5 - 22*a^4*b^3*c + 31*a^5*b*c^2)*k)*x^2 - 2
*(a^3*b^2*c^3 + 8*a^4*c^4)*g + 6*(a^4*b^4 - 7*a^5*b^2*c + 8*a^6*c^2)*k + ((
5*a*b^4*c^3 - 37*a^2*b^2*c^4 + 44*a^3*c^5)*d - (a^2*b^3*c^3 - 16*a^3*b*c^4)
*f - 3*(a^3*b^2*c^3 + 4*a^4*c^4)*h - (a^4*b^2*c^2 + 20*a^5*c^3)*j)*x)/(a^4*
b^4*c^3 - 8*a^5*b^2*c^4 + 16*a^6*c^5 + (a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^
4*c^7)*x^8 + 2*(a^2*b^5*c^4 - 8*a^3*b^3*c^5 + 16*a^4*b*c^6)*x^6 + (a^2*b^6*
c^3 - 6*a^3*b^4*c^4 + 32*a^5*c^6)*x^4 + 2*(a^3*b^5*c^3 - 8*a^4*b^3*c^4 + 16
*a^5*b*c^5)*x^2) + 1/8*integrate((8*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*k*
x^3 - (12*a^2*b*c^3*h - 3*(b^3*c^3 - 8*a*b*c^4)*d - (a*b^2*c^3 + 20*a^2*c^4
)*f - (a^2*b^3*c - 16*a^3*b*c^2)*j)*x^2 + 3*(b^4*c^2 - 9*a*b^2*c^3 + 28*a^2
*c^4)*d + (a*b^3*c^2 - 16*a^2*b*c^3)*f + 3*(a^2*b^2*c^2 + 4*a^3*c^3)*h + (a
^3*b^2*c + 20*a^4*c^2)*j - 8*(3*a^2*b*c^3*g - 6*a^2*c^4*e - I*a^2*b^2*c^2 -
 2*I*a^3*c^3 - (a^3*b^3 - 7*a^4*b*c)*k)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x**11+j*x**8+i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2
+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30016 vs. 2(1120) = 2240.
time = 12.63, size = 30016, normalized size = 25.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((k*x^11+j*x^8+i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^3,x,
 algorithm="giac")
```

```
[Out] 1/64*(3*(a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 256*a^7*b^2*c^8 +
256*a^8*c^9)^2*(2*b^5*c^4 - 24*a*b^3*c^5 + 64*a^2*b*c^6 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 16*(b^2 - 4*a*c)*a*b*
c^5)*d + (a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 256*a^7*b^2*c^8 +
 256*a^8*c^9)^2*(2*a*b^4*c^4 + 32*a^2*b^2*c^5 - 160*a^3*c^6 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 2*sqrt(2)*sqrt(b^2
```

- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 80*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 40*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^4 - 40*(b^2
- 4*a*c)*a^2*c^5)*f - 12*(a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 2
56*a^7*b^2*c^8 + 256*a^8*c^9)^2*(2*a^2*b^3*c^4 - 8*a^3*b*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b*c^
4)*h + (a^4*b^8*c^5 - 16*a^5*b^6*c^6 + 96*a^6*b^4*c^7 - 256*a^7*b^2*c^8 + 2
56*a^8*c^9)^2*(2*a^2*b^5*c^2 - 40*a^3*b^3*c^3 + 128*a^4*b*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 + 20*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 +
32*(b^2 - 4*a*c)*a^3*b*c^3)*j + 6*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^14*c^7 - 29*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^12*c^8 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^13*c^8 + 2*a^4*b^14*c^8 + 368
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^10*c^9 + 50*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^9 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b^12*c^9 - 58*a^5*b^12*c^9 - 2640*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^7*b^8*c^10 - 536*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c
^10 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^10*c^10 + 736*a^6*b^
10*c^10 + 11520*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c^11 + 3136
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^11 + 268*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^11 - 5280*a^7*b^8*c^11 - 30464*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^12 - 10496*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^8*b^5*c^12 - 1568*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^7*b^6*c^12 + 23040*a^8*b^6*c^12 + 45056*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^10*b^2*c^13 + 18944*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*
b^3*c^13 + 5248*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^13 - 6092
8*a^9*b^4*c^13 - 28672*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*c^14 -
14336*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^14 - 9472*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^14 + 90112*a^10*b^2*c^14 + 7168*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*c^15 - 57344*a^11*c^15 - 2*(b^2 -
4*a*c)*a^4*b^12*c^8 + 50*(b^2 - 4*a*c)*a^5*b^10*c^9 - 536*(b^2 - 4*a*c)*a^
6*b^8*c^10 + 3136*(b^2 - 4*a*c)*a^7*b^6*c^11 - 10496*(b^2 - 4*a*c)*a^8*b^4*
c^12 + 18944*(b^2 - 4*a*c)*a^9*b^2*c^13 - 14336*(b^2 - 4*a*c)*a^10*c^14)*d*
abs(-a^4*b^8*c^5 + 16*a^5*b^6*c^6 - 96*a^6*b^4*c^7 + 256*a^7*b^2*c^8 - 256*
a^8*c^9) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^13*c^7 - 36*sqr

```
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^11*c^8 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b^12*c^8 + 2*a^5*b^13*c^8 + 480*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^9*c^9 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^6*b^10*c^9 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^11*c^9 - 72*
a^6*b^11*c^9 - 3200*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^7*c^10 -
704*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8*c^10 - 32*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9*c^10 + 960*a^7*b^9*c^10 + 11520*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c^11 + 3584*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^8*b^6*c^11 + 352*sqrt(2)*sqr...
```

Mupad [B]
time = 17.18, size = 2500, normalized size = 2.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5 + j*x^8 + k*x^11)/(a + b*x^2 +
 c*x^4)^3,x)
```

```
[Out] ((x^7*(3*b^3*c^2*d + 20*a^2*c^3*f + a^2*b^3*j - 24*a*b*c^3*d - 16*a^3*b*c*j
 + a*b^2*c^2*f - 12*a^2*b*c^2*h))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) -
(b^3*c^3*e + 8*a^2*c^4*g - 3*a^2*b^4*k - 24*a^4*c^2*k - 10*a*b*c^4*e + a*b^
2*c^3*g - 6*a^2*b*c^3*i + 21*a^3*b^2*c*k)/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (x^4*(3*b^6*k - 9*b^2*c^4*g + 3*b^3*c^3*i + 32*a^3*c^3*k + 18*b*c^5
*e + 11*a^2*b^2*c^2*k + 6*a*b*c^4*i - 19*a*b^4*c*k))/(4*c^3*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)) + (x^2*(2*b^2*c^4*e - b^3*c^3*g - 2*a^2*c^4*i + 10*a*c^5*e
 + 3*a*b^5*k - 5*a*b*c^4*g + 5*a*b^2*c^3*i - 22*a^2*b^3*c*k + 31*a^3*b*c^2*
k))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^6*(6*c^5*e + 2*b^5*k + b^2*
c^3*i - 3*b*c^4*g + 2*a*c^4*i - 15*a*b^3*c*k + 25*a^2*b*c^2*k))/(2*c^2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(2*a^3*b^3*j - 36*a^3*c^3*f - 3*b^5*c*d
- 5*a^2*b^2*c^2*f - a*b^4*c*f + 28*a^4*b*c*j + 20*a*b^3*c^2*d + 4*a^2*b*c^3
*d + 5*a^2*b^3*c*h + 16*a^3*b*c^2*h))/(8*a^2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)) + (x^5*(28*a^2*c^4*d + 6*b^4*c^2*d + 4*a^3*c^3*h - a^2*b^4*j - 36*a^4*c
^2*j - 19*a^2*b^2*c^2*h - 49*a*b^2*c^3*d + 2*a*b^3*c^2*f + 28*a^2*b*c^3*f -
 5*a^3*b^2*c*j))/(8*a^2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(12*a^3*c^2*
h - 44*a^2*c^3*d + a^3*b^2*j - 5*b^4*c*d + 20*a^4*c*j + a*b^3*c*f + 37*a*b^
2*c^2*d - 16*a^2*b*c^2*f + 3*a^2*b^2*c*h))/(8*a*c*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + symsu
m(log((10368*a*b^5*c^10*d^3 - 8000*a^5*c^11*f^3 - 567*b^7*c^9*d^3 + 169344*
a^3*b*c^12*d^3 + 193536*a^4*c^12*d*e^2 - 141120*a^4*c^12*d^2*f + 1728*a^6*b
*c^9*h^3 + 315*b^8*c^8*d^2*f + 6400*a^9*b*c^6*j^3 + 27648*a^5*c^11*e^2*h +
21504*a^6*c^10*d*i^2 - 135*b^9*c^7*d^2*h + 192*a^2*b^14*d*k^2 - 2880*a^6*c^
10*f*h^2 + 46080*a^6*c^10*e^2*j - 1376256*a^9*c^7*d*k^2 + 9*b^11*c^5*d^2*j
+ 64*a^3*b^13*f*k^2 - 8000*a^8*c^8*f*j^2 + 3072*a^7*c^9*h*i^2 + 192*a^4*b^1
2*h*k^2 + 5120*a^8*c^8*i^2*j - 196608*a^10*c^6*h*k^2 + 2240*a^6*b^10*j*k^2
```

```
- 327680*a^11*c^5*j*k^2 - 67824*a^2*b^3*c^11*d^3 + 35*a^2*b^6*c^8*f^3 + 84*
a^3*b^4*c^9*f^3 - 12720*a^4*b^2*c^10*f^3 + 540*a^4*b^5*c^7*h^3 + 4320*a^5*b
^3*c^8*h^3 + 35*a^6*b^7*c^3*j^3 - 1176*a^7*b^5*c^4*j^3 + 9456*a^8*b^3*c^5*j
^3 + 129024*a^5*c^11*d*e*i - 40320*a^5*c^11*d*f*h - 67200*a^6*c^10*d*f*j +
18432*a^6*c^10*e*h*i + 245760*a^7*c^9*e*f*k + 30720*a^7*c^9*e*i*j - 9600*a^
7*c^9*f*h*j + 81920*a^8*c^8*f*i*k - 6237*a*b^6*c^9*d^2*f + 210*a*b^7*c^8*d*
f^2 + 116160*a^4*b*c^11*d*f^2 - 36864*a^4*b*c^11*e^2*f + 2430*a*b^7*c^8*d^2
*h + 133056*a^4*b*c^11*d^2*h + 27648*a^5*b*c^10*d*h^2 - 324*a*b^9*c^6*d^2*j
 + 193536*a^5*b*c^10*d^2*j + 26880*a^5*b*c^10*f^2*h + 63360*a^7*b*c^8*d*j^2
 - 5568*a^3*b^12*c*d*k^2 - 4096*a^6*b*c^9*f*i^2 + 40000*a^6*b*c^9*f^2*j - 2
304*a^4*b^11*c*f*k^2 - 352256*a^9*b*c^6*f*k^2 + 8064*a^7*b*c^8*h^2*j + 1248
0*a^8*b*c^7*h*j^2 - 2112*a^5*b^10*c*h*k^2 - 41664*a^7*b^8*c*j*k^2 + 6912*a^
2*b^4*c^10*d*e^2 - 62208*a^3*b^2*c^11*d*e^2 + 42372*a^2*b^4*c^10*d^2*f - 17
64*a^2*b^5*c^9*d*f^2 - 96048*a^3*b^2*c^11*d^2*f - 4608*a^3*b^3*c^10*d*f^2 +
 1728*a^2*b^6*c^8*d*g^2 + 2304*a^3*b^3*c^10*e^2*f - 15552*a^3*b^4*c^9*d*g^2
 + 48384*a^4*b^2*c^10*d*g^2 - 13716*a^2*b^5*c^9*d^2*h + 405*a^2*b^7*c^7*d*h
^2 + 12096*a^3*b^3*c^10*d^2*h - 5400*a^3*b^5*c^8*d*h^2 + 28944*a^4*b^3*c^9*
d*h^2 + 192*a^2*b^8*c^6*d*i^2 + 576*a^3*b^5*c^8*f*g^2 - 960*a^3*b^6*c^7*d*i
^2 + 6912*a^4*b^2*c^10*e^2*h - 9216*a^4*b^3*c^9*f*g^2 - 768*a^4*b^4*c^8*d*i
^2 + 14592*a^5*b^2*c^9*d*i^2 + 3717*a^2*b^7*c^7*d^2*j - 15*a^2*b^7*c^7*f^2*
h + 3*a^2*b^11*c^3*d*j^2 - 15192*a^3*b^5*c^8*d^2*j - 360*a^3*b^5*c^8*f^2*h
+ 135*a^3*b^6*c^7*f*h^2 - 132*a^3*b^9*c^4*d*j^2 - 7920*a^4*b^3*c^9*d^2*j +
15696*a^4*b^3*c^9*f^2*h - 5580*a^4*b^4*c^8*f*h^2 + 2079*a^4*b^7*c^5*d*j^2 -
 20592*a^5*b^2*c^9*f*h^2 - 14448*a^5*b^5*c^6*d*j^2 + 37104*a^6*b^3*c^7*d*j^
2 + 64*a^3*b^7*c^6*f*i^2 + 1728*a^4*b^4*c^8*g^2*h - 768*a^4*b^5*c^7*f*i^2 +
 70656*a^4*b^10*c^2*d*k^2 + 2304*a^5*b^2*c^9*e^2*j + 6912*a^5*b^2*c^9*g^2*h
 - 3840*a^5*b^3*c^8*f*i^2 - 499008*a^5*b^8*c^3*d*k^2 + 2071104*a^6*b^6*c^4*
d*k^2 - 4853952*a^7*b^4*c^5*d*k^2 + 5399808*a^8*b^2*c^6*d*k^2 + a^2*b^9*c^5
*f^2*j + 20*a^3*b^7*c^6*f^2*j + a^3*b^10*c^3*f*j^2 - 1596*a^4*b^5*c^7*f^2*j
 - 51*a^4*b^8*c^4*f*j^2 + 16736*a^5*b^3*c^8*f^2*j + 875*a^5*b^6*c^5*f*j^2 -
 2716*a^6*b^4*c^6*f*j^2 - 39600*a^7*b^2*c^7*f*j^2 + 192*a^4*b^6*c^6*h*i^2 +
 1536*a^5*b^4*c^7*h*i^2 + 576*a^5*b^4*c^7*g^2*j + 28480*a^5*b^9*c^2*f*k^2 +
 3840*a^6*b^2*c^8*h*i^2 + 11520*a^6*b^2*c^8*g^2*j - 164096*a^6*b^7*c^3*f*k^
2 + 436800*a^7*b^5*c^4*f*k^2 - 338944*a^8*b^3*c^5*f*k^2 - 81*a^4*b^7*c^5*h^
2*j + 3*a^4*b^9*c^3*h*j^2 + 720*a^5*b^5*c^6*h^2*j - 78*a^5*b^7*c^4*h*j^2 +
17136*a^6*b^3*c^7*h^2*j - 900*a^6*b^5*c^5*h*j^2 + 22272*a^7*b^3*c^6*h*j^2 +
 64*a^5*b^6*c^5*i^2*j + 1536*a^6*b^4*c^6*i^2*j - 960*a^6*b^8*c^2*h*k^2 + 53
76*a^7*b^2*c^7*i^2*j + 108672*a^7*b^6*c^3*h*k^2 - 548160*a^8*b^4*c^4*h*k^2
+ 922368*a^9*b^2*c^5*h*k^2 + 305024*a^8*b^6*c^2*j*k^2 - 1042880*a^9*b^4*c^3
*j*k^2 + 1479936*a^10*b^2*c^4*j*k^2 - 193536*a^4*b*c^11*d*e*g - 90*a*b^8*c^
7*d*f*h + 6*a*b^10*c^5*d*f*j - 64512*a^5*b*c^10...
```

3.60 $\int (a+bx^2+cx^4)^3 (ad+aex+(bd+af)x^2+bex^3+($

Optimal. Leaf size=416

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3(4bd+af)x^3+a^3bex^4+\frac{2}{5}a^2\left(3b^2d+2acd+2abf\right)x^5+\frac{1}{3}a^2\left(3b^2+2ac\right)ex^6+\frac{2}{7}a\left(2b^3d+6abc\right.$$

```
[Out] a^4*d*x+1/2*a^4*e*x^2+1/3*a^3*(a*f+4*b*d)*x^3+a^3*b*e*x^4+2/5*a^2*(2*a*b*f+
2*a*c*d+3*b^2*d)*x^5+1/3*a^2*(2*a*c+3*b^2)*e*x^6+2/7*a*(2*a^2*c*f+3*a*b^2*f
+6*a*b*c*d+2*b^3*d)*x^7+1/2*a*b*(3*a*c+b^2)*e*x^8+1/9*(12*a^2*b*c*f+6*a^2*c
^2*d+4*a*b^3*f+12*a*b^2*c*d+b^4*d)*x^9+1/10*(6*a^2*c^2+12*a*b^2*c+b^4)*e*x^
10+1/11*(6*a^2*c^2*f+12*a*b^2*c*f+12*a*b*c^2*d+b^4*f+4*b^3*c*d)*x^11+1/3*b*
c*(3*a*c+b^2)*e*x^12+2/13*c*(6*a*b*c*f+2*a*c^2*d+2*b^3*f+3*b^2*c*d)*x^13+1/
7*c^2*(2*a*c+3*b^2)*e*x^14+2/15*c^2*(2*a*c*f+3*b^2*f+2*b*c*d)*x^15+1/4*b*c^
3*e*x^16+1/17*c^3*(4*b*f+c*d)*x^17+1/18*c^4*e*x^18+1/19*c^4*f*x^19
```

Rubi [A]
time = 0.42, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.016$, Rules used = {1685}

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3x^3(af+4bd)+a^3bex^4+\frac{2}{5}a^2x^5(2abf+2acd+3b^2d)+\frac{1}{3}a^2ex^6(2ac+3b^2)+\frac{1}{10}ex^{10}(6a^2c^2+12ab^2c+b^4)+\frac{2}{7}ax^7(2a^2cf+3ab^2f+6abcd+2b^3d)+\frac{1}{11}x^{11}(6a^2c^2f+12ab^2cf+12abc^2d+b^4f+4b^3cd)+\frac{1}{9}x^9(12a^2bcf+6a^2c^2d+4ab^3f+12ab^2cd+b^4d)+\frac{2}{15}c^2x^{15}(2acf+3b^2f+2bcd)+\frac{1}{7}c^2ex^{14}(2ac+3b^2)+\frac{1}{3}bcex^{12}(3ac+b^2)+\frac{1}{2}abex^8(3ac+b^2)+\frac{2}{13}cx^{13}(6abcf+2ac^2d+2b^3f+3b^2cd)+\frac{1}{17}c^3x^{17}(4bf+cd)+\frac{1}{4}bc^3ex^{16}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d +
 b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2*
(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2*a
*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*c)*
e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)
*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b
*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*
e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/13 + (
c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f)*x^15)/
15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)/18 + (c^
4*f*x^19)/19
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+bx^2+cx^4)^3 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right) dx = \int \left(a^4d+a^4ex+\right.$$
$$= a^4dx+\frac{1}{2}a^4ex^2+$$

Mathematica [**A**]
time = 0.08, size = 416, normalized size = 1.00

$$a^4dx+\frac{1}{2}a^4ex^2+\frac{1}{3}a^3(4bd+af)x^3+a^3bex^4+\frac{2}{5}a^2\left(3b^2d+2acd+2abf\right)x^5+\frac{1}{3}a^2\left(3b^2+2ac\right)ex^6+\frac{2}{7}a\left(2b^3d+6abcd+3ab^2f+2a^2cf\right)x^7+\frac{1}{2}ab\left(b^2+3ac\right)ex^8+\frac{1}{9}\left(b^4d+12ab^2cd+6a^2c^2d+4ab^3f+12a^2bcf\right)x^9+\frac{1}{10}\left(b^4+12ab^2c+6a^2c^2\right)ex^{10}+\frac{1}{11}\left(4b^3cd+12abc^2d+b^4f+12ab^2cf+6a^2c^2f\right)x^{11}+\frac{1}{3}bc\left(b^2+3ac\right)ex^{12}+\frac{2}{13}c\left(3b^2cd+2ac^2d+2b^3f+6abcf\right)x^{13}+\frac{1}{7}c^2\left(3b^2+2ac\right)ex^{14}+\frac{2}{15}c^2\left(2bcd+3b^2f+2acf\right)x^{15}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}c^3(cd+4bf)x^{17}+\frac{1}{18}c^4ex^{18}+\frac{1}{19}c^4fx^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^3*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
(c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^4*d*x + (a^4*e*x^2)/2 + (a^3*(4*b*d + a*f)*x^3)/3 + a^3*b*e*x^4 + (2*a^2*
(3*b^2*d + 2*a*c*d + 2*a*b*f)*x^5)/5 + (a^2*(3*b^2 + 2*a*c)*e*x^6)/3 + (2*a
*(2*b^3*d + 6*a*b*c*d + 3*a*b^2*f + 2*a^2*c*f)*x^7)/7 + (a*b*(b^2 + 3*a*c)*
e*x^8)/2 + ((b^4*d + 12*a*b^2*c*d + 6*a^2*c^2*d + 4*a*b^3*f + 12*a^2*b*c*f)
*x^9)/9 + ((b^4 + 12*a*b^2*c + 6*a^2*c^2)*e*x^10)/10 + ((4*b^3*c*d + 12*a*b
*c^2*d + b^4*f + 12*a*b^2*c*f + 6*a^2*c^2*f)*x^11)/11 + (b*c*(b^2 + 3*a*c)*
e*x^12)/3 + (2*c*(3*b^2*c*d + 2*a*c^2*d + 2*b^3*f + 6*a*b*c*f)*x^13)/13 + (
c^2*(3*b^2 + 2*a*c)*e*x^14)/7 + (2*c^2*(2*b*c*d + 3*b^2*f + 2*a*c*f)*x^15)/
15 + (b*c^3*e*x^16)/4 + (c^3*(c*d + 4*b*f)*x^17)/17 + (c^4*e*x^18)/18 + (c^
4*f*x^19)/19
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(382) = 764$.
time = 16.24, size = 829, normalized size = 1.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^
5+c*f*x^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*c^4*f*x^19+1/18*c^4*e*x^18+1/17*(3*b*c^3*f+c^3*(b*f+c*d))*x^17+1/4*b*c
^3*e*x^16+1/15*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*c*f+3*b*c^2*(b*f+c*d)+c^3*(a*
f+b*d))*x^15+1/14*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*c*e+3*b^2*c^2*e+a*c^3*e)*x
^14+1/13*((4*a*b*c+b*(2*a*c+b^2))*c*f+(c^2*a+2*b^2*c+c*(2*a*c+b^2))*(b*f+c*
d)+3*b*c^2*(a*f+b*d)+c^3*a*d)*x^13+1/12*((4*a*b*c+b*(2*a*c+b^2))*c*e+(c^2*a
+2*b^2*c+c*(2*a*c+b^2))*e*b+3*a*b*c^2*e)*x^12+1/11*((a*(2*a*c+b^2)+2*a*b^2+
a^2*c)*c*f+(4*a*b*c+b*(2*a*c+b^2))*(b*f+c*d)+(c^2*a+2*b^2*c+c*(2*a*c+b^2))*
(a*f+b*d)+3*a*b*c^2*d)*x^11+1/10*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*c*e+(4*a*b*
c+b*(2*a*c+b^2))*e*b+(c^2*a+2*b^2*c+c*(2*a*c+b^2))*a*e)*x^10+1/9*(3*a^2*b*c
*f+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*(b*f+c*d)+(4*a*b*c+b*(2*a*c+b^2))*(a*f+b*d
```

```
)+(c^2*a+2*b^2*c+c*(2*a*c+b^2))*a*d)*x^9+1/8*(3*a^2*b*c*e+(a*(2*a*c+b^2)+2*
a*b^2+a^2*c)*e*b+(4*a*b*c+b*(2*a*c+b^2))*a*e)*x^8+1/7*(a^3*c*f+3*a^2*b*(b*f
+c*d)+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*(a*f+b*d)+(4*a*b*c+b*(2*a*c+b^2))*a*d)*
x^7+1/6*(a^3*c*e+3*a^2*b^2*e+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*a*e)*x^6+1/5*(a^
3*(b*f+c*d)+3*a^2*b*(a*f+b*d)+(a*(2*a*c+b^2)+2*a*b^2+a^2*c)*a*d)*x^5+a^3*b*
e*x^4+1/3*(a^3*(a*f+b*d)+3*a^3*b*d)*x^3+1/2*a^4*e*x^2+a^4*d*x
```

Maxima [A]
time = 0.28, size = 440, normalized size = 1.06

$$\frac{1}{19}c^4fx^{19}+\frac{1}{18}c^4x^{18}e+\frac{1}{4}bc^3x^{16}e+\frac{1}{17}\left(c^4d+4\,bc^3f\right)x^{17}+\frac{2}{15}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)f\right)x^{15}+\frac{1}{7}\left(3\,b^2c^2e+2\,ac^3e\right)x^{14}+\frac{2}{13}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)f\right)x^{13}+\frac{1}{3}\left(b^3ce+3\,abc^2e\right)x^{12}+\frac{1}{11}\left(4\left(b^3c+3\,abc^2\right)d+\left(b^4+12\,ab^2c+6\,a^2c^2\right)f\right)x^{11}+\frac{1}{10}\left(b^4e+12\,ab^2ce+6\,a^2c^2e\right)x^{10}+\frac{1}{9}\left(\left(b^4+12\,ab^2c+6\,a^2c^2\right)d+4\left(ab^3+3\,a^2bc\right)f\right)x^9+\frac{1}{2}\left(ab^3e+3\,a^2bce\right)x^8+a^3bx^4e+\frac{2}{7}\left(2\left(ab^3+3\,a^2bc\right)d+\left(3\,a^2b^2+2\,a^3c\right)f\right)x^7+\frac{1}{3}\left(3\,a^2b^2e+2\,a^3ce\right)x^6+\frac{1}{2}a^4x^2e+a^4dx+\frac{2}{5}\left(2\,a^3bf+\left(3\,a^2b^2+2\,a^3c\right)d\right)x^5+\frac{1}{3}\left(4\,a^3bd+a^4f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*x^18*e + 1/4*b*c^3*x^16*e + 1/17*(c^4*d + 4*b*c^
3*f)*x^17 + 2/15*(2*b*c^3*d + (3*b^2*c^2 + 2*a*c^3)*f)*x^15 + 1/7*(3*b^2*c^
2*e + 2*a*c^3*e)*x^14 + 2/13*((3*b^2*c^2 + 2*a*c^3)*d + 2*(b^3*c + 3*a*b*c^
2)*f)*x^13 + 1/3*(b^3*c*e + 3*a*b*c^2*e)*x^12 + 1/11*(4*(b^3*c + 3*a*b*c^2)
*d + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*f)*x^11 + 1/10*(b^4*e + 12*a*b^2*c*e +
6*a^2*c^2*e)*x^10 + 1/9*((b^4 + 12*a*b^2*c + 6*a^2*c^2)*d + 4*(a*b^3 + 3*a^
2*b*c)*f)*x^9 + 1/2*(a*b^3*e + 3*a^2*b*c*e)*x^8 + a^3*b*x^4*e + 2/7*(2*(a*b
^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*f)*x^7 + 1/3*(3*a^2*b^2*e + 2*a^3
*c*e)*x^6 + 1/2*a^4*x^2*e + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*b^2 + 2*a^3*c
)*d)*x^5 + 1/3*(4*a^3*b*d + a^4*f)*x^3
```

Fricas [A]
time = 0.58, size = 418, normalized size = 1.00

$$\frac{1}{19}c^4fx^{19}+\frac{1}{18}c^4ex^{18}+\frac{1}{4}bc^3ex^{16}+\frac{1}{17}\left(c^4d+4\,bc^3f\right)x^{17}+\frac{1}{7}\left(3\,b^2c^2+2\,ac^3\right)ex^{14}+\frac{2}{15}\left(2\,bc^3d+\left(3\,b^2c^2+2\,ac^3\right)f\right)x^{15}+\frac{1}{3}\left(b^3c+3\,abc^2\right)ex^{12}+\frac{2}{13}\left(\left(3\,b^2c^2+2\,ac^3\right)d+2\left(b^3c+3\,abc^2\right)f\right)x^{13}+\frac{1}{10}\left(b^4+12\,ab^2c+6\,a^2c^2\right)ex^{10}+\frac{1}{11}\left(4\left(b^3c+3\,abc^2\right)d+\left(b^4+12\,ab^2c+6\,a^2c^2\right)f\right)x^{11}+\frac{1}{2}\left(ab^3+3\,a^2bc\right)ex^8+\frac{1}{9}\left(\left(b^4+12\,ab^2c+6\,a^2c^2\right)d+4\left(ab^3+3\,a^2bc\right)f\right)x^9+a^3bex^4+\frac{1}{3}\left(3\,a^2b^2+2\,a^3c\right)ex^6+\frac{2}{7}\left(2\left(ab^3+3\,a^2bc\right)d+\left(3\,a^2b^2+2\,a^3c\right)f\right)x^7+\frac{1}{2}a^4ex^2+a^4dx+\frac{2}{5}\left(2\,a^3bf+\left(3\,a^2b^2+2\,a^3c\right)d\right)x^5+\frac{1}{3}\left(4\,a^3bd+a^4f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*e*x^18 + 1/4*b*c^3*e*x^16 + 1/17*(c^4*d + 4*b*c^
3*f)*x^17 + 1/7*(3*b^2*c^2 + 2*a*c^3)*e*x^14 + 2/15*(2*b*c^3*d + (3*b^2*c^2
 + 2*a*c^3)*f)*x^15 + 1/3*(b^3*c + 3*a*b*c^2)*e*x^12 + 2/13*((3*b^2*c^2 + 2
*a*c^3)*d + 2*(b^3*c + 3*a*b*c^2)*f)*x^13 + 1/10*(b^4 + 12*a*b^2*c + 6*a^2*
c^2)*e*x^10 + 1/11*(4*(b^3*c + 3*a*b*c^2)*d + (b^4 + 12*a*b^2*c + 6*a^2*c^2
)*f)*x^11 + 1/2*(a*b^3 + 3*a^2*b*c)*e*x^8 + 1/9*((b^4 + 12*a*b^2*c + 6*a^2*
c^2)*d + 4*(a*b^3 + 3*a^2*b*c)*f)*x^9 + a^3*b*e*x^4 + 1/3*(3*a^2*b^2 + 2*a^
3*c)*e*x^6 + 2/7*(2*(a*b^3 + 3*a^2*b*c)*d + (3*a^2*b^2 + 2*a^3*c)*f)*x^7 +
1/2*a^4*e*x^2 + a^4*d*x + 2/5*(2*a^3*b*f + (3*a^2*b^2 + 2*a^3*c)*d)*x^5 + 1
/3*(4*a^3*b*d + a^4*f)*x^3
```

Sympy [A]
time = 0.04, size = 503, normalized size = 1.21

$$a^4dx + \frac{a^4ex^2}{2} + a^3bex^4 + \frac{bc^3ex^{16}}{4} + \frac{c^4ex^{18}}{18} + \frac{c^4fx^{19}}{19} + x^{17}\cdot\left(\frac{4bc^3f}{17} + \frac{c^4d}{17}\right) + x^{15}\cdot\left(\frac{4ac^3f}{15} + \frac{2b^2c^2f}{5} + \frac{4bc^3d}{15}\right) + x^{14}\cdot\left(\frac{2ac^3e}{7} + \frac{3b^2c^2e}{7}\right) + x^{13}\cdot\left(\frac{12abc^2f}{13} + \frac{4ac^3d}{13} + \frac{4b^3cf}{13} + \frac{6b^2c^2d}{13}\right) + x^{12}\left(abc^2e + \frac{b^3ce}{3}\right) + x^{11}\cdot\left(\frac{6a^2c^2f}{11} + \frac{12ab^2cf}{11} + \frac{12abc^2d}{11} + \frac{b^4f}{11} + \frac{4b^3cd}{11}\right) + x^{10}\cdot\left(\frac{3a^2c^2e}{5} + \frac{6ab^2ce}{5} + \frac{b^4e}{10}\right) + x^9\cdot\left(\frac{4a^2bcf}{3} + \frac{2a^2c^2d}{3} + \frac{4ab^3f}{9} + \frac{4ab^2cd}{3} + \frac{b^4d}{9}\right) + x^8\cdot\left(\frac{3a^2bce}{2} + \frac{ab^3e}{2}\right) + x^7\cdot\left(\frac{4a^3cf}{7} + \frac{6a^2b^2f}{7} + \frac{12a^2bcd}{7} + \frac{4ab^3d}{7}\right) + x^6\cdot\left(\frac{2a^3ce}{3} + a^2b^2e\right) + x^5\cdot\left(\frac{4a^3bf}{5} + \frac{4a^3cd}{5} + \frac{6a^2b^2d}{5}\right) + x^3\left(\frac{a^4f}{3} + \frac{4a^3bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**3*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)
*x**4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**4*d*x + a**4*e*x**2/2 + a**3*b*e*x**4 + b*c**3*e*x**16/4 + c**4*e*x**18/
18 + c**4*f*x**19/19 + x**17*(4*b*c**3*f/17 + c**4*d/17) + x**15*(4*a*c**3*
f/15 + 2*b**2*c**2*f/5 + 4*b*c**3*d/15) + x**14*(2*a*c**3*e/7 + 3*b**2*c**2
*e/7) + x**13*(12*a*b*c**2*f/13 + 4*a*c**3*d/13 + 4*b**3*c*f/13 + 6*b**2*c*
*2*d/13) + x**12*(a*b*c**2*e + b**3*c*e/3) + x**11*(6*a**2*c**2*f/11 + 12*a
*b**2*c*f/11 + 12*a*b*c**2*d/11 + b**4*f/11 + 4*b**3*c*d/11) + x**10*(3*a**
2*c**2*e/5 + 6*a*b**2*c*e/5 + b**4*e/10) + x**9*(4*a**2*b*c*f/3 + 2*a**2*c*
*2*d/3 + 4*a*b**3*f/9 + 4*a*b**2*c*d/3 + b**4*d/9) + x**8*(3*a**2*b*c*e/2 +
 a*b**3*e/2) + x**7*(4*a**3*c*f/7 + 6*a**2*b**2*f/7 + 12*a**2*b*c*d/7 + 4*a
*b**3*d/7) + x**6*(2*a**3*c*e/3 + a**2*b**2*e) + x**5*(4*a**3*b*f/5 + 4*a**
3*c*d/5 + 6*a**2*b**2*d/5) + x**3*(a**4*f/3 + 4*a**3*b*d/3)
```

Giac [A]
time = 4.32, size = 478, normalized size = 1.15

$$\frac{1}{19}c^4fx^{19} + \frac{1}{18}c^4x^{18}e + \frac{1}{17}c^4dx^{17} + \frac{4}{17}bc^3fx^{17} + \frac{1}{4}bc^3x^{16}e + \frac{4}{15}bc^3dx^{15} + \frac{2}{5}b^2c^2fx^{15} + \frac{4}{15}ac^3fx^{15} + \frac{3}{7}b^2c^2x^{14}e + \frac{2}{7}ac^3x^{14}e + \frac{6}{13}b^2c^2dx^{13} + \frac{4}{13}ac^3dx^{13} + \frac{4}{13}b^3cfx^{13} + \frac{12}{13}abc^2fx^{13} + \frac{1}{3}b^3cx^{12}e + abc^2x^{12}e + \frac{4}{11}b^3cdx^{11} + \frac{12}{11}abc^2dx^{11} + \frac{1}{11}b^4fx^{11} + \frac{12}{11}ab^2cfx^{11} + \frac{6}{11}a^2c^2fx^{11} + \frac{1}{10}b^4x^{10}e + \frac{6}{5}ab^2cx^{10}e + \frac{3}{5}a^2c^2x^{10}e + \frac{1}{9}b^4dx^9 + \frac{4}{3}ab^2cdx^9 + \frac{2}{3}a^2c^2dx^9 + \frac{4}{9}ab^3fx^9 + \frac{4}{3}a^2bcfx^9 + \frac{1}{2}ab^3x^8e + \frac{3}{2}a^2bcx^8e + \frac{4}{7}ab^3dx^7 + \frac{12}{7}a^2bcdx^7 + \frac{6}{7}a^2b^2fx^7 + \frac{4}{7}a^3cfx^7 + a^2b^2x^6e + \frac{2}{3}a^3cx^6e + \frac{6}{5}a^2b^2dx^5 + \frac{4}{5}a^3cdx^5 + \frac{4}{5}a^3bfx^5 + a^3bx^4e + \frac{4}{3}a^3bdx^3 + \frac{1}{3}a^4fx^3 + \frac{1}{2}a^4x^2e + a^4dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^3*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/19*c^4*f*x^19 + 1/18*c^4*x^18*e + 1/17*c^4*d*x^17 + 4/17*b*c^3*f*x^17 + 1
/4*b*c^3*x^16*e + 4/15*b*c^3*d*x^15 + 2/5*b^2*c^2*f*x^15 + 4/15*a*c^3*f*x^1
5 + 3/7*b^2*c^2*x^14*e + 2/7*a*c^3*x^14*e + 6/13*b^2*c^2*d*x^13 + 4/13*a*c^
3*d*x^13 + 4/13*b^3*c*f*x^13 + 12/13*a*b*c^2*f*x^13 + 1/3*b^3*c*x^12*e + a*
b*c^2*x^12*e + 4/11*b^3*c*d*x^11 + 12/11*a*b*c^2*d*x^11 + 1/11*b^4*f*x^11 +
 12/11*a*b^2*c*f*x^11 + 6/11*a^2*c^2*f*x^11 + 1/10*b^4*x^10*e + 6/5*a*b^2*c
*x^10*e + 3/5*a^2*c^2*x^10*e + 1/9*b^4*d*x^9 + 4/3*a*b^2*c*d*x^9 + 2/3*a^2*
c^2*d*x^9 + 4/9*a*b^3*f*x^9 + 4/3*a^2*b*c*f*x^9 + 1/2*a*b^3*x^8*e + 3/2*a^2
*b*c*x^8*e + 4/7*a*b^3*d*x^7 + 12/7*a^2*b*c*d*x^7 + 6/7*a^2*b^2*f*x^7 + 4/7
*a^3*c*f*x^7 + a^2*b^2*x^6*e + 2/3*a^3*c*x^6*e + 6/5*a^2*b^2*d*x^5 + 4/5*a^
3*c*d*x^5 + 4/5*a^3*b*f*x^5 + a^3*b*x^4*e + 4/3*a^3*b*d*x^3 + 1/3*a^4*f*x^3
 + 1/2*a^4*x^2*e + a^4*d*x
```

Mupad [B]
time = 0.38, size = 398, normalized size = 0.96

$$x^3\left(\frac{f\,a^4}{3}+\frac{4\,b\,d\,a^3}{3}\right)+x^{17}\left(\frac{d\,c^4}{17}+\frac{4\,b\,f\,c^3}{17}\right)+x^5\left(\frac{4\,f\,a^3\,b}{5}+\frac{4\,c\,d\,a^3}{5}+\frac{6\,d\,a^2\,b^2}{5}\right)+x^{15}\left(\frac{2\,f\,b^2\,c^2}{5}+\frac{4\,d\,b\,c^3}{15}+\frac{4\,a\,f\,c^3}{15}\right)+x^9\left(\frac{4\,f\,a^2\,b\,c}{3}+\frac{2\,d\,a^2\,c^2}{3}+\frac{4\,f\,a\,b^3}{9}+\frac{4\,d\,a\,b^2\,c}{3}+\frac{d\,b^4}{9}\right)+x^{11}\left(\frac{6\,f\,a^2\,c^2}{11}+\frac{12\,f\,a\,b^2\,c}{11}+\frac{12\,d\,a\,b\,c^2}{11}+\frac{f\,b^4}{11}+\frac{4\,d\,b^3\,c}{11}\right)+x^7\left(\frac{4\,c\,f\,a^3}{7}+\frac{6\,f\,a^2\,b^2}{7}+\frac{12\,c\,d\,a^2\,b}{7}+\frac{4\,d\,a\,b^3}{7}\right)+x^{13}\left(\frac{4\,f\,b^3\,c}{13}+\frac{6\,d\,b^2\,c^2}{13}+\frac{12\,a\,f\,b\,c^2}{13}+\frac{4\,a\,d\,c^3}{13}\right)+\frac{a^4\,e\,x^2}{2}+\frac{c^4\,e\,x^{18}}{18}+\frac{c^4\,f\,x^{19}}{19}+\frac{e\,x^{10}\,\left(6\,a^2\,c^2+12\,a\,b^2\,c+b^4\right)}{10}+a^4\,d\,x+\frac{a^2\,e\,x^6\,\left(3\,b^2+2\,a\,c\right)}{3}+\frac{c^2\,e\,x^{14}\,\left(3\,b^2+2\,a\,c\right)}{7}+a^3\,b\,e\,x^4+\frac{b\,c^3\,e\,x^{16}}{4}+\frac{a\,b\,e\,x^8\,\left(b^2+3\,a\,c\right)}{2}+\frac{b\,c\,e\,x^{12}\,\left(b^2+3\,a\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^3*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x
+ b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

```
[Out] x^3*((a^4*f)/3 + (4*a^3*b*d)/3) + x^17*((c^4*d)/17 + (4*b*c^3*f)/17) + x^5*
((6*a^2*b^2*d)/5 + (4*a^3*c*d)/5 + (4*a^3*b*f)/5) + x^15*((2*b^2*c^2*f)/5 +
 (4*b*c^3*d)/15 + (4*a*c^3*f)/15) + x^9*((b^4*d)/9 + (2*a^2*c^2*d)/3 + (4*a
*b^3*f)/9 + (4*a*b^2*c*d)/3 + (4*a^2*b*c*f)/3) + x^11*((b^4*f)/11 + (6*a^2*
c^2*f)/11 + (4*b^3*c*d)/11 + (12*a*b*c^2*d)/11 + (12*a*b^2*c*f)/11) + x^7*(
(6*a^2*b^2*f)/7 + (4*a*b^3*d)/7 + (4*a^3*c*f)/7 + (12*a^2*b*c*d)/7) + x^13*
((6*b^2*c^2*d)/13 + (4*a*c^3*d)/13 + (4*b^3*c*f)/13 + (12*a*b*c^2*f)/13) +
(a^4*e*x^2)/2 + (c^4*e*x^18)/18 + (c^4*f*x^19)/19 + (e*x^10*(b^4 + 6*a^2*c^
2 + 12*a*b^2*c))/10 + a^4*d*x + (a^2*e*x^6*(2*a*c + 3*b^2))/3 + (c^2*e*x^14
*(2*a*c + 3*b^2))/7 + a^3*b*e*x^4 + (b*c^3*e*x^16)/4 + (a*b*e*x^8*(3*a*c +
b^2))/2 + (b*c*e*x^12*(3*a*c + b^2))/3
```

3.61 $\int (a+bx^2+cx^4)^2 \left(ad+aex+(bd+af)x^2+bex^3+\right.$

Optimal. Leaf size=259

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bd+af)x^3+\frac{3}{4}a^2bex^4+\frac{3}{5}a\left(b^2d+acd+abf\right)x^5+\frac{1}{2}a\left(b^2+ac\right)ex^6+\frac{1}{7}\left(b^3d+6abcd+3ab\right.$$

```
[Out] a^3*d*x+1/2*a^3*e*x^2+1/3*a^2*(a*f+3*b*d)*x^3+3/4*a^2*b*e*x^4+3/5*a*(a*b*f+
a*c*d+b^2*d)*x^5+1/2*a*(a*c+b^2)*e*x^6+1/7*(3*a^2*c*f+3*a*b^2*f+6*a*b*c*d+b
^3*d)*x^7+1/8*b*(6*a*c+b^2)*e*x^8+1/9*(6*a*b*c*f+3*a*c^2*d+b^3*f+3*b^2*c*d)
*x^9+3/10*c*(a*c+b^2)*e*x^10+3/11*c*(a*c*f+b^2*f+b*c*d)*x^11+1/4*b*c^2*e*x^
12+1/13*c^2*(3*b*f+c*d)*x^13+1/14*c^3*e*x^14+1/15*c^3*f*x^15
```

Rubi [A]
time = 0.22, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.016$,
Rules used = {1685}

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{7}x^7\left(3a^2cf+3ab^2f+6abcd+b^3d\right)+\frac{1}{3}a^2x^3(af+3bd)+\frac{3}{4}a^2bex^4+\frac{3}{11}cx^{11}\left(acf+b^2f+bcd\right)+\frac{3}{5}ax^5\left(abf+acd+b^2d\right)+\frac{3}{10}cex^{10}\left(ac+b^2\right)+\frac{1}{8}bex^8\left(6ac+b^2\right)+\frac{1}{2}aex^6\left(ac+b^2\right)+\frac{1}{9}x^9\left(6abcf+3ac^2d+b^3f+3b^2cd\right)+\frac{1}{13}c^2x^{13}(3bf+cd)+\frac{1}{4}bc^2ex^{12}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d +
 b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 + (
3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d + 6*
a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((3*b
^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/1
0 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d +
3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)^2 \left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx=\int \left(a^3d+a^3ex+\right.$$

$$=a^3dx+\frac{1}{2}a^3ex^2+$$

Mathematica [A]

time = 0.03, size = 259, normalized size = 1.00

$$a^3dx+\frac{1}{2}a^3ex^2+\frac{1}{3}a^2(3bd+af)x^3+\frac{3}{4}a^2bex^4+\frac{3}{5}a\left(b^2d+acd+abf\right)x^5+\frac{1}{2}a\left(b^2+ac\right)ex^6+\frac{1}{7}\left(b^3d+6abcd+3ab^2f+3a^2cf\right)x^7+\frac{1}{8}b\left(b^2+6ac\right)ex^8+\frac{1}{9}\left(3b^2cd+3ac^2d+b^3f+6abcf\right)x^9+\frac{3}{10}c\left(b^2+ac\right)ex^{10}+\frac{3}{11}c\left(bcd+b^2f+acf\right)x^{11}+\frac{1}{4}bc^2ex^{12}+\frac{1}{13}c^2(cd+3bf)x^{13}+\frac{1}{14}c^3ex^{14}+\frac{1}{15}c^3fx^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 +
(c*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^3*d*x + (a^3*e*x^2)/2 + (a^2*(3*b*d + a*f)*x^3)/3 + (3*a^2*b*e*x^4)/4 + (
3*a*(b^2*d + a*c*d + a*b*f)*x^5)/5 + (a*(b^2 + a*c)*e*x^6)/2 + ((b^3*d + 6*
a*b*c*d + 3*a*b^2*f + 3*a^2*c*f)*x^7)/7 + (b*(b^2 + 6*a*c)*e*x^8)/8 + ((3*b
^2*c*d + 3*a*c^2*d + b^3*f + 6*a*b*c*f)*x^9)/9 + (3*c*(b^2 + a*c)*e*x^10)/1
0 + (3*c*(b*c*d + b^2*f + a*c*f)*x^11)/11 + (b*c^2*e*x^12)/4 + (c^2*(c*d +
3*b*f)*x^13)/13 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15
```

Maple [A]

time = 15.15, size = 354, normalized size = 1.37

method	result
norman	$\left(\frac{1}{3}a^3f+a^2bd\right)x^3+\left(\frac{3}{10}a\,c^2e+\frac{3}{10}b^2ce\right)x^{10}+\left(\frac{1}{2}a^2ce+\frac{1}{2}a\,b^2e\right)x^6+\left(\frac{3}{13}b\,c^2f+\frac{1}{13}c^3d\right)x^{13}+\left(\frac{3}{4}abc\right.$
risch	$\frac{3}{4}a^2be\,x^4+\frac{1}{4}b\,c^2e\,x^{12}+\frac{1}{2}a^3e\,x^2+\frac{3}{13}x^{13}b\,c^2f+\frac{3}{11}x^{11}a\,c^2f+\frac{3}{11}x^{11}b^2cf+\frac{3}{11}x^{11}b\,c^2d+\frac{3}{10}x^{10}a\,c^2e+$
gosper	$x\left(24024c^3f\,x^{14}+25740c^3e\,x^{13}+83160b\,c^2f\,x^{12}+27720c^3d\,x^{12}+90090b\,c^2e\,x^{11}+98280a\,c^2f\,x^{10}+98280b^2cf\,x^{10}+98280b\,c^2d\,x^{10}+1081\right.$
default	$\frac{c^3f\,x^{15}}{15}+\frac{c^3e\,x^{14}}{14}+\frac{(2b\,c^2f+c^2(bf+cd))x^{13}}{13}+\frac{b\,c^2e\,x^{12}}{4}+\frac{((2ac+b^2)cf+2bc(bf+cd)+c^2(fa+bd))x^{11}}{11}+\frac{((2ac+b^2)ce+2b^2ce}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^
5+c*f*x^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*c^3*f*x^15+1/14*c^3*e*x^14+1/13*(2*b*c^2*f+c^2*(b*f+c*d))*x^13+1/4*b*c
^2*e*x^12+1/11*((2*a*c+b^2)*c*f+2*b*c*(b*f+c*d)+c^2*(a*f+b*d))*x^11+1/10*((
2*a*c+b^2)*c*e+2*b^2*c*e+a*c^2*e)*x^10+1/9*(2*a*b*c*f+(2*a*c+b^2)*(b*f+c*d)
+2*b*c*(a*f+b*d)+a*c^2*d)*x^9+1/8*(4*a*b*c*e+(2*a*c+b^2)*e*b)*x^8+1/7*(a^2*
c*f+2*a*b*(b*f+c*d)+(2*a*c+b^2)*(a*f+b*d)+2*a*b*c*d)*x^7+1/6*(a^2*c*e+2*a*b
^2*e+(2*a*c+b^2)*a*e)*x^6+1/5*(a^2*(b*f+c*d)+2*a*b*(a*f+b*d)+(2*a*c+b^2)*a*
d)*x^5+3/4*a^2*b*e*x^4+1/3*(a^2*(a*f+b*d)+2*a^2*b*d)*x^3+1/2*a^3*e*x^2+a^3*
d*x
```

Maxima [A]

time = 0.28, size = 265, normalized size = 1.02

$$\frac{1}{15}c^3fx^{15}+\frac{1}{14}c^3x^{14}e+\frac{1}{4}bc^2x^{12}e+\frac{1}{13}\left(c^3d+3\,bc^2f\right)x^{13}+\frac{3}{11}\left(bc^2d+\left(b^2c+ac^2\right)f\right)x^{11}+\frac{3}{10}\left(b^2ce+ac^2e\right)x^{10}+\frac{1}{9}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)f\right)x^9+\frac{1}{8}\left(b^3e+6\,abce\right)x^8+\frac{1}{7}\left(\left(b^3+6\,abc\right)d+3\left(ab^2+a^2c\right)f\right)x^7+\frac{3}{4}a^2bx^4e+\frac{1}{2}\left(ab^2e+a^2ce\right)x^6+\frac{3}{5}\left(a^2bf+\left(ab^2+a^2c\right)d\right)x^5+\frac{1}{2}a^3x^2e+a^3dx+\frac{1}{3}\left(3\,a^2bd+a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*x^14*e + 1/4*b*c^2*x^12*e + 1/13*(c^3*d + 3*b*c^
2*f)*x^13 + 3/11*(b*c^2*d + (b^2*c + a*c^2)*f)*x^11 + 3/10*(b^2*c*e + a*c^2
*e)*x^10 + 1/9*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*f)*x^9 + 1/8*(b^3*e +
 6*a*b*c*e)*x^8 + 1/7*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*f)*x^7 + 3/4*a
^2*b*x^4*e + 1/2*(a*b^2*e + a^2*c*e)*x^6 + 3/5*(a^2*b*f + (a*b^2 + a^2*c)*d
)*x^5 + 1/2*a^3*x^2*e + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Fricas [A]
time = 0.74, size = 251, normalized size = 0.97

$$\frac{1}{15}c^3fx^{15} + \frac{1}{14}c^3ex^{14} + \frac{1}{4}bc^2ex^{12} + \frac{1}{13}\left(c^3d + 3\,bc^2f\right)x^{13} + \frac{3}{10}\left(b^2c + ac^2\right)ex^{10} + \frac{3}{11}\left(bc^2d + \left(b^2c + ac^2\right)f\right)x^{11} + \frac{1}{8}\left(b^3 + 6\,abc\right)ex^8 + \frac{1}{9}\left(3\left(b^2c + ac^2\right)d + \left(b^3 + 6\,abc\right)f\right)x^9 + \frac{3}{4}a^2bex^4 + \frac{1}{2}\left(ab^2 + a^2c\right)ex^6 + \frac{1}{7}\left(\left(b^3 + 6\,abc\right)d + 3\left(ab^2 + a^2c\right)f\right)x^7 + \frac{1}{2}a^3ex^2 + \frac{3}{5}\left(a^2bf + \left(ab^2 + a^2c\right)d\right)x^5 + a^3dx + \frac{1}{3}\left(3\,a^2bd + a^3f\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*e*x^14 + 1/4*b*c^2*e*x^12 + 1/13*(c^3*d + 3*b*c^
2*f)*x^13 + 3/10*(b^2*c + a*c^2)*e*x^10 + 3/11*(b*c^2*d + (b^2*c + a*c^2)*f
)*x^11 + 1/8*(b^3 + 6*a*b*c)*e*x^8 + 1/9*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*f)*x^9 + 3/4*a^2*b*e*x^4 + 1/2*(a*b^2 + a^2*c)*e*x^6 + 1/7*((b^3 + 6*a
*b*c)*d + 3*(a*b^2 + a^2*c)*f)*x^7 + 1/2*a^3*e*x^2 + 3/5*(a^2*b*f + (a*b^2
+ a^2*c)*d)*x^5 + a^3*d*x + 1/3*(3*a^2*b*d + a^3*f)*x^3
```

Sympy [A]
time = 0.03, size = 309, normalized size = 1.19

$$a^3dx + \frac{a^3ex^2}{2} + \frac{3a^2bex^4}{4} + \frac{bc^2ex^{12}}{4} + \frac{c^3ex^{14}}{14} + \frac{c^3fx^{15}}{15} + x^{13}\cdot\left(\frac{3bc^2f}{13} + \frac{c^3d}{13}\right) + x^{11}\cdot\left(\frac{3ac^2f}{11} + \frac{3b^2cf}{11} + \frac{3bc^2d}{11}\right) + x^{10}\cdot\left(\frac{3ac^2e}{10} + \frac{3b^2ce}{10}\right) + x^9\cdot\left(\frac{2abcf}{3} + \frac{ac^2d}{3} + \frac{b^3f}{9} + \frac{b^2cd}{3}\right) + x^8\cdot\left(\frac{3abce}{4} + \frac{b^3e}{8}\right) + x^7\cdot\left(\frac{3a^2cf}{7} + \frac{3ab^2f}{7} + \frac{6abcd}{7} + \frac{b^3d}{7}\right) + x^6\left(\frac{a^2ce}{2} + \frac{ab^2e}{2}\right) + x^5\cdot\left(\frac{3a^2bf}{5} + \frac{3a^2cd}{5} + \frac{3ab^2d}{5}\right) + x^3\left(\frac{a^3f}{3} + a^2bd\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)
*x**4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**3*d*x + a**3*e*x**2/2 + 3*a**2*b*e*x**4/4 + b*c**2*e*x**12/4 + c**3*e*x*
*14/14 + c**3*f*x**15/15 + x**13*(3*b*c**2*f/13 + c**3*d/13) + x**11*(3*a*c
**2*f/11 + 3*b**2*c*f/11 + 3*b*c**2*d/11) + x**10*(3*a*c**2*e/10 + 3*b**2*c
*e/10) + x**9*(2*a*b*c*f/3 + a*c**2*d/3 + b**3*f/9 + b**2*c*d/3) + x**8*(3*
a*b*c*e/4 + b**3*e/8) + x**7*(3*a**2*c*f/7 + 3*a*b**2*f/7 + 6*a*b*c*d/7 + b
**3*d/7) + x**6*(a**2*c*e/2 + a*b**2*e/2) + x**5*(3*a**2*b*f/5 + 3*a**2*c*d
/5 + 3*a*b**2*d/5) + x**3*(a**3*f/3 + a**2*b*d)
```

Giac [A]
time = 4.42, size = 295, normalized size = 1.14

$$\frac{1}{15}c^3fx^{15} + \frac{1}{14}c^3x^{14}e + \frac{1}{13}c^3dx^{13} + \frac{3}{13}bc^2fx^{13} + \frac{1}{4}bc^2x^{12}e + \frac{3}{11}bc^2dx^{11} + \frac{3}{11}b^2cfx^{11} + \frac{3}{11}ac^2fx^{11} + \frac{3}{10}b^2cx^{10}e + \frac{3}{10}ac^2x^{10}e + \frac{1}{3}b^2cdx^9 + \frac{1}{3}ac^2dx^9 + \frac{1}{9}b^3fx^9 + \frac{2}{3}abcfx^9 + \frac{1}{8}b^3x^8e + \frac{3}{4}abcx^8e + \frac{1}{7}b^3dx^7 + \frac{6}{7}abcdx^7 + \frac{3}{7}ab^2fx^7 + \frac{3}{7}a^2cfx^7 + \frac{1}{2}ab^2x^6e + \frac{1}{2}a^2cx^6e + \frac{3}{5}ab^2dx^5 + \frac{3}{5}a^2cdx^5 + \frac{3}{5}a^2bfx^5 + \frac{3}{4}a^2bx^4e + a^2bdx^3 + \frac{1}{3}a^3fx^3 + \frac{1}{2}a^3x^2e + a^3dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+
c*e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/15*c^3*f*x^15 + 1/14*c^3*x^14*e + 1/13*c^3*d*x^13 + 3/13*b*c^2*f*x^13 + 1
/4*b*c^2*x^12*e + 3/11*b*c^2*d*x^11 + 3/11*b^2*c*f*x^11 + 3/11*a*c^2*f*x^11
 + 3/10*b^2*c*x^10*e + 3/10*a*c^2*x^10*e + 1/3*b^2*c*d*x^9 + 1/3*a*c^2*d*x^
9 + 1/9*b^3*f*x^9 + 2/3*a*b*c*f*x^9 + 1/8*b^3*x^8*e + 3/4*a*b*c*x^8*e + 1/7
*b^3*d*x^7 + 6/7*a*b*c*d*x^7 + 3/7*a*b^2*f*x^7 + 3/7*a^2*c*f*x^7 + 1/2*a*b^
2*x^6*e + 1/2*a^2*c*x^6*e + 3/5*a*b^2*d*x^5 + 3/5*a^2*c*d*x^5 + 3/5*a^2*b*f
*x^5 + 3/4*a^2*b*x^4*e + a^2*b*d*x^3 + 1/3*a^3*f*x^3 + 1/2*a^3*x^2*e + a^3*
d*x
```

Mupad [B]
time = 0.95, size = 246, normalized size = 0.95

$$x^3\left(\frac{f\,a^3}{3}+b\,d\,a^2\right)+x^{13}\left(\frac{d\,c^3}{13}+\frac{3\,b\,f\,c^2}{13}\right)+x^5\left(\frac{3\,f\,a^2\,b}{5}+\frac{3\,c\,d\,a^2}{5}+\frac{3\,d\,a\,b^2}{5}\right)+x^{11}\left(\frac{3\,f\,b^2\,c}{11}+\frac{3\,d\,b\,c^2}{11}+\frac{3\,a\,f\,c^2}{11}\right)+x^7\left(\frac{3\,c\,f\,a^2}{7}+\frac{3\,f\,a\,b^2}{7}+\frac{6\,c\,d\,a\,b}{7}+\frac{d\,b^3}{7}\right)+x^9\left(\frac{f\,b^3}{9}+\frac{d\,b^2\,c}{3}+\frac{2\,a\,f\,b\,c}{3}+\frac{a\,d\,c^2}{3}\right)+\frac{a^3\,e\,x^2}{2}+\frac{c^3\,e\,x^{14}}{14}+\frac{c^3\,f\,x^{15}}{15}+a^3\,d\,x+\frac{a\,e\,x^6\,\left(b^2+a\,c\right)}{2}+\frac{b\,e\,x^8\,\left(b^2+6\,a\,c\right)}{8}+\frac{3\,c\,e\,x^{10}\,\left(b^2+a\,c\right)}{10}+\frac{3\,a^2\,b\,e\,x^4}{4}+\frac{b\,c^2\,e\,x^{12}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x
+ b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

```
[Out] x^3*((a^3*f)/3 + a^2*b*d) + x^13*((c^3*d)/13 + (3*b*c^2*f)/13) + x^5*((3*a*
b^2*d)/5 + (3*a^2*c*d)/5 + (3*a^2*b*f)/5) + x^11*((3*b*c^2*d)/11 + (3*a*c^2
*f)/11 + (3*b^2*c*f)/11) + x^7*((b^3*d)/7 + (3*a*b^2*f)/7 + (3*a^2*c*f)/7 +
 (6*a*b*c*d)/7) + x^9*((b^3*f)/9 + (a*c^2*d)/3 + (b^2*c*d)/3 + (2*a*b*c*f)/
3) + (a^3*e*x^2)/2 + (c^3*e*x^14)/14 + (c^3*f*x^15)/15 + a^3*d*x + (a*e*x^6
*(a*c + b^2))/2 + (b*e*x^8*(6*a*c + b^2))/8 + (3*c*e*x^10*(a*c + b^2))/10 +
 (3*a^2*b*e*x^4)/4 + (b*c^2*e*x^12)/4
```

3.62 $\int \left(a+bx^2+cx^4\right)\left(ad+aex+(bd+af)x^2+bex^3+\right.$

Optimal. Leaf size=154

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right.$$

```
[Out] a^2*d*x+1/2*a^2*e*x^2+1/3*a*(a*f+2*b*d)*x^3+1/2*a*b*e*x^4+1/5*(2*a*b*f+2*a*
c*d+b^2*d)*x^5+1/6*(2*a*c+b^2)*e*x^6+1/7*(2*a*c*f+b^2*f+2*b*c*d)*x^7+1/4*b*
c*e*x^8+1/9*c*(2*b*f+c*d)*x^9+1/10*c^2*e*x^10+1/11*c^2*f*x^11
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.016, Rules used = {1685}

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{7}x^7\left(2acf+b^2f+2bcd\right)+\frac{1}{5}x^5\left(2abf+2acd+b^2d\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3(af+2bd)+\frac{1}{2}abex^4+\frac{1}{9}cx^9(2bf+cd)+\frac{1}{4}bcex^8+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b
*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Rule 1685

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Pq, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+bx^2+cx^4\right)\left(ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6\right)\,dx=\int \left(a^2d+a^2ex+a\right.$$

$$=a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}$$

Mathematica [A]
time = 0.02, size = 154, normalized size = 1.00

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{3}a(2bd+af)x^3+\frac{1}{2}abex^4+\frac{1}{5}\left(b^2d+2acd+2abf\right)x^5+\frac{1}{6}\left(b^2+2ac\right)ex^6+\frac{1}{7}\left(2bcd+b^2f+2acf\right)x^7+\frac{1}{4}bcex^8+\frac{1}{9}c(cd+2bf)x^9+\frac{1}{10}c^2ex^{10}+\frac{1}{11}c^2fx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)*(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c
*d + b*f)*x^4 + c*e*x^5 + c*f*x^6),x]
```

```
[Out] a^2*d*x + (a^2*e*x^2)/2 + (a*(2*b*d + a*f)*x^3)/3 + (a*b*e*x^4)/2 + ((b^2*d
 + 2*a*c*d + 2*a*b*f)*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + ((2*b*c*d + b^2*f
+ 2*a*c*f)*x^7)/7 + (b*c*e*x^8)/4 + (c*(c*d + 2*b*f)*x^9)/9 + (c^2*e*x^10)/
10 + (c^2*f*x^11)/11
```

Maple [A]
time = 0.08, size = 161, normalized size = 1.05

method	result
norman	$\frac{c^2 f x^{11}}{11} + \frac{c^2 e x^{10}}{10} + (\frac{2}{9} fbc + \frac{1}{9} c^2 d) x^9 + \frac{bce x^8}{4} + (\frac{2}{7} acf + \frac{1}{7} b^2 f + \frac{2}{7} bcd) x^7 + (\frac{1}{3} ace + \frac{1}{6} b^2 e) x^6 + (\frac{2}{5} ab$
risch	$\frac{1}{11} c^2 f x^{11} + \frac{1}{10} c^2 e x^{10} + \frac{2}{9} x^9 fbc + \frac{1}{9} c^2 d x^9 + \frac{1}{4} bce x^8 + \frac{2}{7} x^7 acf + \frac{1}{7} x^7 b^2 f + \frac{2}{7} bcd x^7 + \frac{1}{3} x^6 ace + \frac{1}{6} x^6$
gosper	$\frac{x(1260c^2 f x^{10} + 1386 c^2 e x^9 + 3080 x^8 fbc + 1540 x^8 c^2 d + 3465 bce x^7 + 3960 x^6 acf + 1980 x^6 b^2 f + 3960 x^6 bcd + 4620 x^5 ace + 2310 x^5 b^2 e + 5544}{13860}$
default	$\frac{c^2 f x^{11}}{11} + \frac{c^2 e x^{10}}{10} + \frac{(fbc + c(bf + cd)) x^9}{9} + \frac{bce x^8}{4} + \frac{(acf + b(bf + cd) + c(fa + bd)) x^7}{7} + \frac{(2ace + b^2 e) x^6}{6} + \frac{(a(bf + cd) + b(fa + bd)}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+
c*f*x^6),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*f*x^11+1/10*c^2*e*x^10+1/9*(f*b*c+c*(b*f+c*d))*x^9+1/4*b*c*e*x^8+1
/7*(a*c*f+b*(b*f+c*d)+c*(a*f+b*d))*x^7+1/6*(2*a*c*e+b^2*e)*x^6+1/5*(a*(b*f+
c*d)+b*(a*f+b*d)+a*c*d)*x^5+1/2*a*b*e*x^4+1/3*(a*(a*f+b*d)+a*b*d)*x^3+1/2*a
^2*e*x^2+a^2*d*x
```

Maxima [A]
time = 0.29, size = 146, normalized size = 0.95

$$\frac{1}{11} c^2 f x^{11} + \frac{1}{10} c^2 x^{10} e + \frac{1}{4} bcx^8 e + \frac{1}{9} (c^2 d + 2 bcf) x^9 + \frac{1}{7} (2 bcd + (b^2 + 2 ac) f) x^7 + \frac{1}{6} (b^2 e + 2 ace) x^6 + \frac{1}{2} abx^4 e + \frac{1}{5} (2 abf + (b^2 + 2 ac) d) x^5 + \frac{1}{2} a^2 x^2 e + a^2 dx + \frac{1}{3} (2 abd + a^2 f) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="maxima")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/4*b*c*x^8*e + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/6*(b^2*e + 2*a*c*e)*x^6 + 1/2*
a*b*x^4*e + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*x^2*e + a^2*d*x +
 1/3*(2*a*b*d + a^2*f)*x^3
```

Fricas [A]
time = 0.39, size = 138, normalized size = 0.90

$$\frac{1}{11} c^2 f x^{11} + \frac{1}{10} c^2 e x^{10} + \frac{1}{4} bce x^8 + \frac{1}{9} (c^2 d + 2 bcf) x^9 + \frac{1}{6} (b^2 + 2 ac) e x^6 + \frac{1}{7} (2 bcd + (b^2 + 2 ac) f) x^7 + \frac{1}{2} abe x^4 + \frac{1}{5} (2 abf + (b^2 + 2 ac) d) x^5 + \frac{1}{2} a^2 e x^2 + a^2 dx + \frac{1}{3} (2 abd + a^2 f) x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="fricas")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*e*x^10 + 1/4*b*c*e*x^8 + 1/9*(c^2*d + 2*b*c*f)*x
^9 + 1/6*(b^2 + 2*a*c)*e*x^6 + 1/7*(2*b*c*d + (b^2 + 2*a*c)*f)*x^7 + 1/2*a*
b*e*x^4 + 1/5*(2*a*b*f + (b^2 + 2*a*c)*d)*x^5 + 1/2*a^2*e*x^2 + a^2*d*x + 1
/3*(2*a*b*d + a^2*f)*x^3
```

Sympy [A]
time = 0.02, size = 165, normalized size = 1.07

$$a^2dx + \frac{a^2ex^2}{2} + \frac{abex^4}{2} + \frac{bcex^8}{4} + \frac{c^2ex^{10}}{10} + \frac{c^2fx^{11}}{11} + x^9 \cdot \left(\frac{2bcf}{9} + \frac{c^2d}{9}\right) + x^7 \cdot \left(\frac{2acf}{7} + \frac{b^2f}{7} + \frac{2bcd}{7}\right) + x^6\left(\frac{ace}{3} + \frac{b^2e}{6}\right) + x^5 \cdot \left(\frac{2abf}{5} + \frac{2acd}{5} + \frac{b^2d}{5}\right) + x^3\left(\frac{a^2f}{3} + \frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)*(a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x*
*4+c*e*x**5+c*f*x**6),x)
```

```
[Out] a**2*d*x + a**2*e*x**2/2 + a*b*e*x**4/2 + b*c*e*x**8/4 + c**2*e*x**10/10 +
c**2*f*x**11/11 + x**9*(2*b*c*f/9 + c**2*d/9) + x**7*(2*a*c*f/7 + b**2*f/7
+ 2*b*c*d/7) + x**6*(a*c*e/3 + b**2*e/6) + x**5*(2*a*b*f/5 + 2*a*c*d/5 + b*
*2*d/5) + x**3*(a**2*f/3 + 2*a*b*d/3)
```

Giac [A]
time = 3.66, size = 157, normalized size = 1.02

$$\frac{1}{11}c^2fx^{11} + \frac{1}{10}c^2x^{10}e + \frac{1}{9}c^2dx^9 + \frac{2}{9}bcfx^9 + \frac{1}{4}bcx^8e + \frac{2}{7}bcdx^7 + \frac{1}{7}b^2fx^7 + \frac{2}{7}acfx^7 + \frac{1}{6}b^2x^6e + \frac{1}{3}acx^6e + \frac{1}{5}b^2dx^5 + \frac{2}{5}acdx^5 + \frac{2}{5}abfx^5 + \frac{1}{2}abx^4e + \frac{2}{3}abdx^3 + \frac{1}{3}a^2fx^3 + \frac{1}{2}a^2x^2e + a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)*(a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*
e*x^5+c*f*x^6),x, algorithm="giac")
```

```
[Out] 1/11*c^2*f*x^11 + 1/10*c^2*x^10*e + 1/9*c^2*d*x^9 + 2/9*b*c*f*x^9 + 1/4*b*c
*x^8*e + 2/7*b*c*d*x^7 + 1/7*b^2*f*x^7 + 2/7*a*c*f*x^7 + 1/6*b^2*x^6*e + 1/
3*a*c*x^6*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*f*x^5 + 1/2*a*b*x^4*e
 + 2/3*a*b*d*x^3 + 1/3*a^2*f*x^3 + 1/2*a^2*x^2*e + a^2*d*x
```

Mupad [B]
time = 0.09, size = 138, normalized size = 0.90

$$x^5\left(\frac{d\,b^2}{5} + \frac{2\,a\,f\,b}{5} + \frac{2\,a\,c\,d}{5}\right) + x^7\left(\frac{f\,b^2}{7} + \frac{2\,c\,d\,b}{7} + \frac{2\,a\,c\,f}{7}\right) + x^3\left(\frac{f\,a^2}{3} + \frac{2\,b\,d\,a}{3}\right) + x^9\left(\frac{d\,c^2}{9} + \frac{2\,b\,f\,c}{9}\right) + \frac{a^2\,e\,x^2}{2} + \frac{c^2\,e\,x^{10}}{10} + \frac{c^2\,f\,x^{11}}{11} + \frac{e\,x^6\,(b^2 + 2\,a\,c)}{6} + a^2\,d\,x + \frac{a\,b\,e\,x^4}{2} + \frac{b\,c\,e\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)*(a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x +
b*e*x^3 + c*e*x^5 + c*f*x^6),x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*c*d)/5 + (2*a*b*f)/5) + x^7*((b^2*f)/7 + (2*b*c*d)/7
+ (2*a*c*f)/7) + x^3*((a^2*f)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*f)
/9) + (a^2*e*x^2)/2 + (c^2*e*x^10)/10 + (c^2*f*x^11)/11 + (e*x^6*(2*a*c + b
^2))/6 + a^2*d*x + (a*b*e*x^4)/2 + (b*c*e*x^8)/4
```

3.63 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=20

$$dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

```
[Out] d*x+1/2*e*x^2+1/3*f*x^3
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.016$, Rules used = {1600}

$$dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{a+bx^2+cx^4}\,dx &= \int \left(d+ex+fx^2\right)\,dx\\ &= dx+\frac{ex^2}{2}+\frac{fx^3}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$dx+\frac{ex^2}{2}+\frac{fx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4),x]
```

[Out] d*x + (e*x^2)/2 + (f*x^3)/3

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
default	$dx + \frac{1}{2} e\, x^2 + \frac{1}{3} f\, x^3$	17
norman	$dx + \frac{1}{2} e\, x^2 + \frac{1}{3} f\, x^3$	17
risch	$dx + \frac{1}{2} e\, x^2 + \frac{1}{3} f\, x^3$	17
gosper	$\frac{x (2 f\, x^2 + 3 e x + 6 d)}{6}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a),x,method=_RETURNVERBOSE)
```

[Out] d*x+1/2*e*x^2+1/3*f*x^3

Maxima [A]
time = 0.27, size = 17, normalized size = 0.85

$$\frac{1}{3} f x^3 + \frac{1}{2} x^2 e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="maxima")
```

[Out] 1/3*f*x^3 + 1/2*x^2*e + d*x

Fricas [A]
time = 0.39, size = 16, normalized size = 0.80

$$\frac{1}{3} f x^3 + \frac{1}{2} e x^2 + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="fricas")
```

[Out] 1/3*f*x^3 + 1/2*e*x^2 + d*x

Sympy [A]
time = 0.02, size = 15, normalized size = 0.75

$$dx + \frac{e x^2}{2} + \frac{f x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a),x)
```

```
[Out] d*x + e*x**2/2 + f*x**3/3
```

Giac [A]
time = 4.05, size = 17, normalized size = 0.85

$$\frac{1}{3}fx^3 + \frac{1}{2}x^2e + dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*f*x^3 + 1/2*x^2*e + d*x
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.80

$$\frac{f\,x^3}{3} + \frac{e\,x^2}{2} + d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4),x)
```

```
[Out] d*x + (e*x^2)/2 + (f*x^3)/3
```

3.64 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=211

$$\frac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{e\tan\ldots}{}$$

```
[Out] -e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(f+(-b*f+2*c*d)/(-4*a*c+b^2)^
(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(f+(b*f-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.127$, Rules used = {1600, 1687, 1180, 211, 12, 1121, 632, 212}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-bf}{\sqrt{b^2-4ac}}+f\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(f-\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f + (2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((f -
(2*c*d - b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (e*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^2}\,dx = \int \frac{d+ex+fx^2}{a+bx^2+cx^4}\,dx$$

$$= \int \frac{ex}{a+bx^2+cx^4}\,dx + \int \frac{d+fx^2}{a+bx^2+c \cdots}$$

$= e\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\Big(f - \frac{2cd}{\sqrt{b^2 \cdots}}$

$= \dfrac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\frac{\sqrt{2} \cdots}{\sqrt{b-\sqrt{\cdots}}}}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} \cdots}$

$= \dfrac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\frac{\sqrt{2} \cdots}{\sqrt{b-\sqrt{\cdots}}}}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} \cdots}$

$= \dfrac{\left(f+\frac{2cd-bf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Big(\frac{\sqrt{2} \cdots}{\sqrt{b-\sqrt{\cdots}}}}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}} \cdots}$

Mathematica [A]
time = 0.14, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)-e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((Sqrt[2]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + e*Log[-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2] - e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 240, normalized size = 1.14

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)} \frac{\left(\_R^2 f+\_R e+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln\left(-b-2cx^2+\sqrt{-4ac+b^2}\right)}{2}+\frac{\left(-f\sqrt{-4ac+b^2}+bf-2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4a\right.}}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4c\left(4ac-b^2\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4*(-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(1/2*e*ln(-b-2*c*x^2+(-4*a*c+b^2
)^(1/2))+1/2*(-f*(-4*a*c+b^2)^(1/2)+b*f-2*c*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4*(
-4*a*c+b^2)^(1/2)/c/(4*a*c-b^2)*(-1/2*e*ln(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/
2*(f*(-4*a*c+b^2)^(1/2)+b*f-2*c*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*f*x^6 + c*x^5*e + (c*d + b*f)*x^4 + b*x^3*e + (b*d + a*f)*x^2
+ a*x*e + a*d)/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [C] Result contains complex when optimal does not.
time = 18.48, size = 723401, normalized size = 3428.44

too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*((-I*sqrt(3) + 1)*(3*(b^4*c*d^2 + (b^2 - 4*a*c)^(3/2)*b*c*d^2 + 16*(4
*sqrt(1/2)*c^3*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 - a*f^2)*sqr
t(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - c^2*f^2)*a^3 + 8*(2*c^3*d^2 - sqrt
(b^2 - 4*a*c)*c^2*e^2 - (4*sqrt(1/2)*c^2*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b
*f^2 - (c*d^2 - a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - c*f^2)*b
^2)*a^2 + ((4*sqrt(1/2)*c*e*sqrt(-(b*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 -
 a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c - 4*a^2*c^2)) - f^2)*b^4 + (b^2 - 4*a*c
)^(3/2)*b*f^2 - 4*(b^2 - 4*a*c)^(3/2)*(e^2 + d*f)*c - 2*(4*c^2*d^2 - sqrt(b
^2 - 4*a*c)*c*e^2)*b^2)*a)/((b^2 - 4*a*c)^(3/2)*a*b^2*c - 4*(b^2 - 4*a*c)^(
3/2)*a^2*c^2) - 2*(b^2*e - 4*a*c*e + sqrt(1/2)*(b^2 - 4*a*c)^(3/2)*sqrt(-(b
*c*d^2 - 4*a*c*d*f + a*b*f^2 - (c*d^2 - a*f^2)*sqrt(b^2 - 4*a*c))/(a*b^2*c
- 4*a^2*c^2)))^2/(b^2 - 4*a*c)^3)/(-1/32*(b^3*c*d^2*e - 3*sqrt(b^2 - 4*a*c)
*b^2*c*d^2*e - 2*sqrt(1/2)*(b^2 - 4*a*c)^(3/2)*b*c*d^2*sqrt(-(b*c*d^2 - 4*a
*c*d*f + a*b*f^2 - (c*d^2 ...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. 2(174) = 348.
time = 6.84, size = 1620, normalized size = 7.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*
(b + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b
*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sq
rt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt
```

```
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*
c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
+ 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*d
- 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2
*(b^2 - 4*a*c)*a*c^2)*f)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/
c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c
)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*f)*arctan(2*sqrt(1
/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c +
16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 1.17, size = 2500, normalized size = 11.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(c^2*d*e^2 - c^2*d^2*f + c^2*e^3*x - a*c*f^3 - 8*root(16*a*b^4*c*
z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 + 64*a^2*c
```

^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2 +
32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*e*f^2*z +
4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^2*f + 2*a
*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2 + a*c*e^
4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*b^3*c^2*x + b*c*d*f^2 - 16*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z
^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c
^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^
2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*
d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2
*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^3*d - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*c^3*d^2*x + 4*root(16
*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2 +
64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d
^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c*
e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e^
2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^2
+ a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*b^2*c^2*d + 32*root(1
6*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2*
d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*c
*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*e
^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f^
2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^3*a*b*c^3*x + 16*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*a*c^3*e*x + 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*a*c^2*f^2*x + 2*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
+ 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*

```
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*b*c^2*e^2*x - 2*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)*b^2*c*f^2*x - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z^2
 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c^2
*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^2*
c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*d*
e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2*f
^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^4, z, k)^2*b^2*c^2*e*x + 4*roo
t(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*a*b^2*c*d*f*z
^2 + 64*a^2*c^2*d*f*z^2 - 16*a^2*b*c*f^2*z^2 - 8*a*b^2*c*e^2*z^2 - 16*a*b*c
^2*d^2*z^2 + 32*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*a*b^3*f^2*z^2 + 16*a^
2*c*e*f^2*z + 4*b^2*c*d^2*e*z - 4*a*b^2*e*f^2*z - 16*a*c^2*d^2*e*z - 4*a*c*
d*e^2*f + 2*a*c*d^2*f^2 - 2*b*c*d^3*f - 2*a*b*d*f^3 + b*c*d^2*e^2 + a*b*e^2
*f^2 + a*c*e^4 + b^2*d^2*f^2 + c^2*d^4 + a^2*f^...
```

3.65 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=368

$$-\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(bd-2af+\frac{b^2d-12acd+4abf}{\sqrt{b^2-4ac}}\right)\text{ta}}{2\sqrt{2}\,a(b^2-4ac)\sqrt{b-}}$$

```
[Out] -1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(b^2*d-2*a*c*d-a*b*f+
c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*e*arctanh((2*c*x^2+b
)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*f+(4*a*b*f-12*a*c*d+b^2*d)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-2*a*f+(-4*a*
b*f+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1600, 1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abf-12acd+b^2d}{\sqrt{b^2-4ac}}-2af+bd\right)}{2\sqrt{2}\,a(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{2a(b^2-4ac)(a+bx^2+cx^4)}+\frac{2ce\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/2*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(b^2*d - 2*
a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b*d - 2*a*f + (b^2*d - 12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a*f - (b^2*d -
12*a*c*d + 4*a*b*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (2*c*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{\left(a+bx^2+cx^4\right)^3}\,dx = \int \frac{d+ex+fx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{ex}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{d+}{(a+bx^2}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2}{}$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2}{}$$

$$= -\frac{e(b+2cx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x(b^2}{}$$

Mathematica [A]

time = 0.76, size = 398, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2ab(e+fx)-2bdx(b+cx^2)+4acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}\,d+4af\right)-2a\left(6cd+\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}\,d-4abf-2a\sqrt{b^2-4ac}\,f\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}-\frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*a*b*(e + f*x) - 2*b*d*x*(b + c*x^2) + 4*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 -
4*a*c]*d + 4*a*f) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]
*d - 4*a*b*f - 2*a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) -
 (4*c*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Maple [A]

time = 0.15, size = 579, normalized size = 1.57

method	result
risch	$\frac{\frac{c(2fa-bd)x^3}{2a(4ac-b^2)}+\frac{c\,x^2 e}{4ac-b^2}+\frac{(abf+2acd-b^2d)x}{2a(4ac-b^2)}+\frac{eb}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{c(2fa-bd)\_R^2}{a(4ac-b^2)}+\frac{4ce\_R}{4ac-b^2}-\frac{abf-6acd+b^2d}{a(4ac-b^2)}\right)}{2c\_R^3+\_Rb}\right.}{4}$
default	$16c^2\left(\frac{\frac{\left(4\sqrt{-4ac+b^2}\,acd-\sqrt{-4ac+b^2}\,b^2d+8a^2cf-2a\,b^2f-4abcd+b^3d\right)x}{16ac}+\frac{e(4ac-b^2)}{8c}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(b+2c\,x^2+\right.}{}}{x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)^2*((1/16*(4*(-4*a*c+b^2)^(1/2)*a*c*d-(-4*a*c+b^2)
^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x+1/8*e*(4*a*c-b^2)/c
)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/8/a*(2*(-4*a*c+b^2)^(1/2)*a*e*ln
(b+2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(-4*(-4*a*c+b^2)^(1/2)*a*b*f+12*(-4*a*c+
b^2)^(1/2)*a*c*d-(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3
*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))))-1/4/c/(4*a*c-b^2)^2*((-1/16*(-4*(-4*a*c+b^2)^(1/2)
*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*b^2*f-4*a*b*c*d+b^3*d)/a/c*x-
1/8*e*(4*a*c-b^2)/c)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))+1/8/a*(2*(-4*a*
c+b^2)^(1/2)*a*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(4*(-4*a*c+b^2)^(1/2
)*a*b*f-12*(-4*a*c+b^2)^(1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+8*a^2*c*f-2*a*
b^2*f-4*a*b*c*d+b^3*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*c*x^2*e - (b*c*d - 2*a*c*f)*x^3 + a*b*e + (a*b*f - (b^2 - 2*a*c)*
d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2) - 1/2*integrate((4*a*c*x*e - a*b*f - (b*c*d - 2*a*c*f)*x^2 - (b^2 - 6*
a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5164 vs. $2(322) = 644$.
time = 8.96, size = 5164, normalized size = 14.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*f*x^3 - 2*a*c*x^2*e + b^2*d*x - 2*a*c*d*x - a*b*f*x
- a*b*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^2 - 8*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b
^2 - 4*a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a
*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 +
24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 -
 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*
d*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3
*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 +
 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*f*abs(a*b^2 - 4*a^2
*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*s
```

```
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^
2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d +
4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4
*a*c)*a^4*b^2*c^3)*f)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((
a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2
*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 +
 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*
a^2*c)^2*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c
+ 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4...
```

Mupad [B]
time = 1.52, size = 2500, normalized size = 6.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] symsum(log((5*b^3*c^4*d^3 + 8*a^3*c^4*f^3 - 96*a^2*c^5*d*e^2 + 72*a^2*c^5*d
^2*f - 3*b^4*c^3*d^2*f + 6*a^2*b^2*c^3*f^3 - 36*a*b*c^5*d^3 + 16*a*b^2*c^4*
d*e^2 + 18*a*b^2*c^4*d^2*f + 3*a*b^3*c^3*d*f^2 - 60*a^2*b*c^4*d*f^2 + 16*a^
2*b*c^4*e^2*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) -
 root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3
*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 -
256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2 - 3072
*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f^2*z^2 +
 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 81
92*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2*c^4*e^2*z
^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4
*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32*a*b^10
*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 - 4
096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*d*e*f*z - 768
*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^2*e*z + 1536*
a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*c^4*d^2*e*z +
 4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a^4*c^5*d^2*e*z +
32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^2*f - 192*a^3
*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d*f^3 - 960*a^2*b*
c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 + 768*a^3*c^4*d*e^2*f + 2016*a^2*b*c^4*
d^3*f - 496*a*b^3*c^3*d^3*f + 224*a^3*b*c^3*d*f^3 - 16*a^2*b^3*c^2*e^2*f^2
- 960*a^2*b^2*c^3*d^2*f^2 - 18*a*b^5*c*d*f^3 - 288*a^3*c^4*d^2*f^2 - 16*b^5
*c^2*d^2*e^2 - 24*a^3*b^2*c^2*f^4 + 30*b^5*c^2*d^3*f - 9*b^6*c*d^2*f^2 - 9*
a^2*b^4*c*f^4 + 360*a*b^2*c^4*d^4 - 16*a^4*c^3*f^4 - 256*a^3*c^4*e^4 - 25*b
^4*c^3*d^4 - 1296*a^2*c^5*d^4, z, k)*((32*a*b^5*c^3*d*e - 512*a^4*c^5*e*f +
 1024*a^3*b*c^5*d*e - 384*a^2*b^3*c^4*d*e + 32*a^2*b^4*c^3*e*f)/(8*(a^2*b^6
 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + root(1572864*a^8*b^2*c^5*
z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z
^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*a^2
*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d*f*z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 204
8*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*c^4*f^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 -
 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c*d^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1
536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*
z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c
^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e
^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^11*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64
*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*
a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*
a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z
- 2048*a^5*c^4*e*f^2*z + 18432*a^4*c^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^
4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4
*c^2*d^2*f^2 + 144*a^2*b^3*c^2*d*f^3 - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^
3*d^2*e^2 + 768*a^3*c^4*d*e^2*f + 2016*a^2*b*c^4*d^3*f - 496*a*b^3*c^3*d^3*
f + 224*a^3*b*c^3*d*f^3 - 16*a^2*b^3*c^2*e^2*f^2 - 960*a^2*b^2*c^3*d^2*f^2
- 18*a*b^5*c*d*f^3 - 288*a^3*c^4*d^2*f^2 - 16*b^5*c^2*d^2*e^2 - 24*a^3*b^2*
```

```
c^2*f^4 + 30*b^5*c^2*d^3*f - 9*b^6*c*d^2*f^2 - 9*a^2*b^4*c*f^4 + 360*a*b^2*
c^4*d^4 - 16*a^4*c^3*f^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*d^4 - 1296*a^2*c^5*
d^4, z, k)*((x*(1024*a^5*c^6*e - 16*a^2*b^6*c^3*e + 192*a^3*b^4*c^4*e - 768
*a^4*b^2*c^5*e))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))
 - (6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*
c^5*d + 16*a^2*b^7*c^2*f - 192*a^3*b^5*c^3*f + 768*a^4*b^3*c^4*f + 16*a*b^8
*c^2*d - 1024*a^5*b*c^5*f)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 32768
0*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a
^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*a^2*b^8*c*d*f*z^2 + 24576*a^5*b^2*c^4*d
*f*z^2 - 3072*a^3*b^6*c^2*d*f*z^2 + 2048*a^4*b^4*c^3*d*f*z^2 + 12288*a^6*b*
c^4*f^2*z^2 + 61440*a^5*b*c^5*d^2*z^2 - 49152*a^6*c^5*d*f*z^2 + 432*a*b^9*c
*d^2*z^2 - 8192*a^5*b^3*c^3*f^2*z^2 + 1536*a^4*b^5*c^2*f^2*z^2 + 24576*a^5*
b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 - 6144
0*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^
2 - 32*a*b^10*d*f*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*a^2*b^9*f^2*z^2 - 16*b^1
1*d^2*z^2 - 4096*a^4*b*c^4*d*e*f*z + 64*a*b^7*c*d*e*f*z + 3072*a^3*b^3*c^3*
d*e*f*z - 768*a^2*b^5*c^2*d*e*f*z + 32*a^2*b^6*c*e*f^2*z - 672*a*b^6*c^2*d^
2*e*z + 1536*a^4*b^2*c^3*e*f^2*z - 384*a^3*b^4*c^2*e*f^2*z - 15872*a^3*b^2*
c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z - 2048*a^5*c^4*e*f^2*z + 18432*a^4*c
^5*d^2*e*z + 32*b^8*c*d^2*e*z - 32*a*b^4*c^2*d*e^2*f + 192*a^2*b^2*c^3*d*e^
2*f - 192*a^3*b*c^3*e^2*f^2 + 198*a*b^4*c^2*d^2...
```

3.66 $\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx$

Optimal. Leaf size=621

$$-\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x(3b^4$$

```
[Out] -1/4*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*x*(b^2*d-2*a*c*d-a*b*
f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/2*c*e*(2*c*x^2+b)/
(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/8*x*(3*b^4*d-25*a*b^2*c*d+28*a^2*c^2*d+a*b
^3*f+8*a^2*b*c*f+c*(20*a^2*c*f+a*b^2*f-24*a*b*c*d+3*b^3*d)*x^2)/a^2/(-4*a*c
+b^2)^2/(c*x^4+b*x^2+a)-6*c^2*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(5/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*c^(1/2)*(3*b^4*d+b^3*(a*f+3*d*(-4*a*c+b^2)^(1/2))-4*a*b*c*(13*a*f+6*d*(-4*
a*c+b^2)^(1/2))-a*b^2*(30*c*d-f*(-4*a*c+b^2)^(1/2))+4*a^2*c*(42*c*d+5*f*(-4
*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b
^3*d-24*a*b*c*d+a*b^2*f+20*a^2*c*f+(52*a^2*b*c*f-168*a^2*c^2*d-a*b^3*f+30*a
*b^2*c*d-3*b^4*d)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.29, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 63, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {1600, 1687, 1192, 1180, 211, 12, 1121, 628, 632, 212}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-52a^2bcf+168a^2c^2d+ab^3f-30ab^2cd+3b^4d}{\sqrt{b^2-4ac}}+20a^2cf+ab^2f-24abcd+3b^3d\right)}{8\sqrt{2}\,a^2(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(4a^2c\left(5f\sqrt{b^2-4ac}+42cd\right)-ab^2\left(30cd-f\sqrt{b^2-4ac}\right)-4abc\left(6d\sqrt{b^2-4ac}+13af\right)+b^3\left(3d\sqrt{b^2-4ac}+af\right)+3b^4d\right)}{8\sqrt{2}\,a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{x(cx^2(20a^2cf+ab^2f-24abcd+3b^3d)+8a^2bcf+28a^2c^2d+ab^3f-25ab^2cd+3b^4d)}{8a^2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{6c^2e\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3ce(b+2cx^2)}{2(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

```
[Out] -1/4*(e*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(b^2*d -
2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)^2) + (3*c*e*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (x
*(3*b^4*d - 25*a*b^2*c*d + 28*a^2*c^2*d + a*b^3*f + 8*a^2*b*c*f + c*(3*b^3*
d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f)*x^2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*
x^2 + c*x^4)) + (Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a
*b*c*(6*Sqrt[b^2 - 4*a*c]*d + 13*a*f) - a*b^2*(30*c*d - Sqrt[b^2 - 4*a*c]*f
) + 4*a^2*c*(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqr
```

```
t[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3*d - 24*a*b*c*d + a*b^2*f + 20*a^2*c*f -
(3*b^4*d - 30*a*b^2*c*d + 168*a^2*c^2*d + a*b^3*f - 52*a^2*b*c*f)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[
2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (6*c^2*e*ArcTanh[(b +
 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{ad+aex+(bd+af)x^2+bex^3+(cd+bf)x^4+cex^5+cfx^6}{(a+bx^2+cx^4)^4}\,dx = \int \frac{d+ex+fx^2}{(a+bx^2+cx^4)^3}\,dx$$

$$= \int \frac{ex}{(a+bx^2+cx^4)^3}\,dx + \int \frac{d+}{(a+bx^2}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b}{}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b}{}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b}{}$$

$$= -\frac{e(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(b}{}$$

Mathematica [A]
time = 2.22, size = 625, normalized size = 1.01

$$\frac{1}{16}\left(\frac{4ab(e+fx)-4bdx(b+cx^2)+8acx(d+x(e+fx))}{a(-b^2+4ac)(a+bx^2+cx^4)^2}+\frac{6b^3dx(b+cx^2)+2abx(-25bcd+b^2f-24c^2dx^2+bcfx^2)+8a^2c(b(3e+2fx)+cx(7d+6ex+5fx^2))}{a^2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d+af\right)-4abc\left(6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(-30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-3b^4d+b^3\left(3\sqrt{b^2-4ac}\,d-af\right)+4abc\left(-6\sqrt{b^2-4ac}\,d+13af\right)+ab^2\left(30cd+\sqrt{b^2-4ac}\,f\right)+4a^2c\left(-42cd+5\sqrt{b^2-4ac}\,f\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^2(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{48c^2e\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{5/2}}-\frac{48c^2e\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d + a*e*x + (b*d + a*f)*x^2 + b*e*x^3 + (c*d + b*f)*x^4 + c*e*
x^5 + c*f*x^6)/(a + b*x^2 + c*x^4)^4,x]
```

```
[Out] ((4*a*b*(e + f*x) - 4*b*d*x*(b + c*x^2) + 8*a*c*x*(d + x*(e + f*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (6*b^3*d*x*(b + c*x^2) + 2*a*b*x*(-25*
b*c*d + b^2*f - 24*c^2*d*x^2 + b*c*f*x^2) + 8*a^2*c*(b*(3*e + 2*f*x) + c*x*
```

```
(7*d + 6*e*x + 5*f*x^2)))/(a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d + a*f) - 4*a*b*c*(6*Sqrt[
b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(-30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*c*
(42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[2]*Sqrt[c]*(-3*b^4*d + b^3*(3*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-6*S
qrt[b^2 - 4*a*c]*d + 13*a*f) + a*b^2*(30*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2
*c*(-42*c*d + 5*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 (48*c^2*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - (48
*c^2*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(561) = 1122$.
time = 0.33, size = 1311, normalized size = 2.11

method	result
risch	$\frac{\frac{c^{2}\left(20a^{2}cf+a\,b^{2}f-24abcd+3b^{3}d\right)x^{7}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{3c^{3}e\,x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{c\left(28a^{2}bcf+28a^{2}c^{2}d+2a\,b^{3}f-49a\,b^{2}cd+6b^{4}d\right)x^{5}}{8a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{9b\,c^{2}e\,x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{(36a^{3}c^{2}f \cdots}{\cdots}}{(c\,x^{4}+b\,x^{2} \cdots}$
default	$64c^{3}\left(-\frac{-\frac{\left(720\sqrt{-4ac+b^{2}}\,a^{2}c^{2}d-312\sqrt{-4ac+b^{2}}\,a\,b^{2}cd+33\sqrt{-4ac+b^{2}}\,b^{4}d+800a^{3}c^{2}f-112a^{2}b^{2}cf-1104a^{2}b\,c^{2}d-22a \cdots\right.}{64a^{2}c^{2}\left(100ac+11b^{2}\right)}}{\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(c*x^4+
b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)*((-1/64*(720*(-4*a*c
+b^2)^(1/2)*a^2*c^2*d-312*(-4*a*c+b^2)^(1/2)*a*b^2*c*d+33*(-4*a*c+b^2)^(1/2
)*b^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b*c^2*d-22*a*b^4*f+408*a*b^3
*c*d-33*b^5*d)*(-b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)/a^2/c^2/(100*a*c+11*b^2)*
```

```
x^3-3/4*e*(4*a*c-b^2)/c*x^2-1/32*(1232*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-568*(-4
*a*c+b^2)^(1/2)*a*b^2*c*d+65*(-4*a*c+b^2)^(1/2)*b^4*d+1568*a^3*c^2*f-496*a^
2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*c*d-65*b^5*d)*(7*(-4*a*c+b^
2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/8*e*(16*a*c*(-4*a*c+b^2)^(1/2)-4*b
^2*(-4*a*c+b^2)^(1/2)+12*a*b*c-3*b^3)/c^2)/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/
2*b/c)^2+1/16/a^2/c*(-24*(-4*a*c+b^2)^(1/2)*a^2*c*e*ln(b+2*c*x^2+(-4*a*c+b^
2)^(1/2))+1/2*(52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f-168*(-4*a*c+b^2)^(1/2)*a^2*c
^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*f+30*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-3*(-4*a*c+b
^2)^(1/2)*b^4*d-80*a^3*c^2*f+16*a^2*b^2*c*f+96*a^2*b*c^2*d+a*b^4*f-36*a*b^3
*c*d+3*b^5*d)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/32/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c
-b^2)*((-1/64*(-720*(-4*a*c+b^2)^(1/2)*a^2*c^2*d+312*(-4*a*c+b^2)^(1/2)*a*b
^2*c*d-33*(-4*a*c+b^2)^(1/2)*b^4*d+800*a^3*c^2*f-112*a^2*b^2*c*f-1104*a^2*b
*c^2*d-22*a*b^4*f+408*a*b^3*c*d-33*b^5*d)*(b*(-4*a*c+b^2)^(1/2)+20*a*c+b^2)
/a^2/c^2/(100*a*c+11*b^2)*x^3-3/4*e*(4*a*c-b^2)/c*x^2-1/32*(-1232*(-4*a*c+b
^2)^(1/2)*a^2*c^2*d+568*(-4*a*c+b^2)^(1/2)*a*b^2*c*d-65*(-4*a*c+b^2)^(1/2)*
b^4*d+1568*a^3*c^2*f-496*a^2*b^2*c*f-1616*a^2*b*c^2*d+26*a*b^4*f+664*a*b^3*
c*d-65*b^5*d)*(-7*(-4*a*c+b^2)^(1/2)+6*b)/c^2/(196*a*c-13*b^2)/a*x-1/8*e*(-
16*a*c*(-4*a*c+b^2)^(1/2)+4*b^2*(-4*a*c+b^2)^(1/2)+12*a*b*c-3*b^3)/c^2)/(x^
2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2+1/16/a^2/c*(24*(-4*a*c+b^2)^(1/2)*a^2
*c*e*ln(-b-2*c*x^2+(-4*a*c+b^2)^(1/2))+1/2*(52*(-4*a*c+b^2)^(1/2)*a^2*b*c*f
-168*(-4*a*c+b^2)^(1/2)*a^2*c^2*d-(-4*a*c+b^2)^(1/2)*a*b^3*f+30*(-4*a*c+b^2
)^(1/2)*a*b^2*c*d-3*(-4*a*c+b^2)^(1/2)*b^4*d+80*a^3*c^2*f-16*a^2*b^2*c*f-96
*a^2*b*c^2*d-a*b^4*f+36*a*b^3*c*d-3*b^5*d)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/8*(24*a^2*c^3*x^6*e + 36*a^2*b*c^2*x^4*e + (3*(b^3*c^2 - 8*a*b*c^3)*d + (
a*b^2*c^2 + 20*a^2*c^3)*f)*x^7 + ((6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d +
 2*(a*b^3*c + 14*a^2*b*c^2)*f)*x^5 - 2*a^2*b^3*e + 20*a^3*b*c*e + ((3*b^5 -
 20*a*b^3*c - 4*a^2*b*c^2)*d + (a*b^4 + 5*a^2*b^2*c + 36*a^3*c^2)*f)*x^3 +
8*(a^2*b^2*c*e + 5*a^3*c^2*e)*x^2 + ((5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*
d - (a^2*b^3 - 16*a^3*b*c)*f)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 +
 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) + 1/8*integrate((48*a^2*c^2*x*e + (3*(b^
3*c - 8*a*b*c^2)*d + (a*b^2*c + 20*a^2*c^2)*f)*x^2 + 3*(b^4 - 9*a*b^2*c + 2
```

```
8*a^2*c^2)*d + (a*b^3 - 16*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x**2+b*e*x**3+(b*f+c*d)*x**4+c*e*x**5+c*f*x*
*6)/(c*x**4+b*x**2+a)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5288 vs. $2(564) = 1128$.
time = 6.09, size = 5288, normalized size = 8.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d+a*e*x+(a*f+b*d)*x^2+b*e*x^3+(b*f+c*d)*x^4+c*e*x^5+c*f*x^6)/(
c*x^4+b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*a*c)*e*log(x^2 + 1/2*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3))/((b^8 - 16*a*b^
6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*c^2 - 256*a^3*b^2*c^3 -
 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a^3*b*c^4 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)*c^2) + 3*(b^2*c^4 - 4*a*c^5 - 2*b*c^5 + c^6)*sqrt(b^2 - 4*
a*c)*e*log(x^2 + 1/2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 - sqrt((a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3))/((b^8 - 16*a*b^6*c - 2*b^7*c + 96*a^2*b^4*c^2 + 24*a*b^5*c^2 + b^6*
```

c^2 - 256*a^3*b^2*c^3 - 96*a^2*b^3*c^3 - 12*a*b^4*c^3 + 256*a^4*c^4 + 128*a
^3*b*c^4 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*c^2) + 1/32*(3*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6
*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 2*b^8*c + 116*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2
 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 232*a^2*b^4*c^3 - 30*a*b
^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^6*c - 26*(b^2 - 4*a*c)
*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(
b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b
*c^4)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48*a^2*b^5*c^2 + 2*a*b^6*c^2 -
 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*c^4 + 64*a^3*b^2*c^4 + 320*a
^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 +
22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 32*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 36*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 160*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 18*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 40*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^5
c - 40(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 128*(b^2 -

```
4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3 + 80*(b^2 - 4*a*c)*a^3*c^4)
*f)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + sqrt(
(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/(a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*a^3*b^7*c + 96*a^5*b^4*c^2 +
 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3 - 96*a^5*b^3*c^3 - 12*a^4*b
^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*abs(c))
 + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*...
```

Mupad [B]
time = 3.16, size = 2500, normalized size = 4.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d + x^2*(b*d + a*f) + x^4*(c*d + b*f) + a*e*x + b*e*x^3 + c*e*x^5 +
c*f*x^6)/(a + b*x^2 + c*x^4)^4,x)
```

```
[Out] ((x^2*(5*a*c^2*e + b^2*c*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3*e - 10*a
*b*c*e)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*a^2*c^3*d + 6*b^4*c*d
 + 2*a*b^3*c*f - 49*a*b^2*c^2*d + 28*a^2*b*c^2*f))/(8*a^2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x*(5*b^4*d + 44*a^2*c^2*d - a*b^3*f - 37*a*b^2*c*d + 16*a
^2*b*c*f))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e*x^6)/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^3*(3*b^5*d + 36*a^3*c^2*f + a*b^4*f - 20*a*b^3*c*d
 - 4*a^2*b*c^2*d + 5*a^2*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) +
 (9*b*c^2*e*x^4)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*a^2*c^2*f
+ 3*b^3*c*d - 24*a*b*c^2*d + a*b^2*c*f))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + symsum(
log(root(56371445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 471859
20*a^7*b^16*c^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4
*c^8*z^4 - 128849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 +
3523215360*a^9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^1
0*z^4 + 65536*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321205760*a^9*b^2*
c^8*d*f*z^2 + 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2
 - 330301440*a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 15175680
*a^4*b^12*c^3*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 440401920*a^10*b*c^8
*f^2*z^2 + 1761607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*c*f^2*z^2 + 6936330
240*a^8*b^3*c^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b
*c^9*d^2*z^2 - 1509949440*a^9*b^2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*
z^2 - 94464*a*b^17*c*d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^
5*b^9*c^5*d^2*z^2 + 477102080*a^9*b^3*c^7*f^2*z^2 - 174325760*a^8*b^5*c^6*f
^2*z^2 - 188743680*a^7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 1
```

```
1206656*a^7*b^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^2*z^2 + 23592960*a^6*b^
8*c^5*e^2*z^2 - 2600960*a^5*b^11*c^3*f^2*z^2 + 291840*a^4*b^13*c^2*f^2*z^2
- 19860480*a^3*b^13*c^3*d^2*z^2 - 1179648*a^5*b^10*c^4*e^2*z^2 + 1771776*a^
2*b^15*c^2*d^2*z^2 + 1536*a*b^18*d*f*z^2 + 1207959552*a^10*c^9*e^2*z^2 + 25
6*a^2*b^17*f^2*z^2 + 2304*b^19*d^2*z^2 + 169869312*a^7*b*c^8*d*e*f*z + 9216
*a*b^13*c^2*d*e*f*z - 221773824*a^6*b^3*c^7*d*e*f*z + 117964800*a^5*b^5*c^6
*d*e*f*z - 32440320*a^4*b^7*c^5*d*e*f*z + 4792320*a^3*b^9*c^4*d*e*f*z - 350
208*a^2*b^11*c^3*d*e*f*z - 428544*a*b^12*c^3*d^2*e*z + 1022754816*a^6*b^2*c
^8*d^2*e*z - 642318336*a^5*b^4*c^7*d^2*e*z + 223395840*a^4*b^6*c^6*d^2*e*z
- 50724864*a^7*b^2*c^7*e*f^2*z + 26542080*a^6*b^4*c^6*e*f^2*z - 46725120*a^
3*b^8*c^5*d^2*e*z - 7127040*a^5*b^6*c^5*e*f^2*z + 1013760*a^4*b^8*c^4*e*f^2
*z - 69120*a^3*b^10*c^3*e*f^2*z + 1536*a^2*b^12*c^2*e*f^2*z + 5930496*a^2*b
^10*c^4*d^2*e*z - 693633024*a^7*c^9*d^2*e*z + 39321600*a^8*c^8*e*f^2*z + 13
824*b^14*c^2*d^2*e*z + 13824*a*b^8*c^4*d*e^2*f - 7741440*a^4*b^2*c^7*d*e^2*
f + 2903040*a^3*b^4*c^6*d*e^2*f - 387072*a^2*b^6*c^5*d*e^2*f + 37310976*a^3
*b^3*c^7*d^3*f + 3870720*a^5*b*c^7*e^2*f^2 + 34836480*a^4*b*c^8*d^2*e^2 - 8
068032*a^2*b^5*c^6*d^3*f - 5623296*a^4*b^3*c^6*d*f^3 + 1737792*a^3*b^5*c^5*
d*f^3 - 260190*a*b^8*c^4*d^2*f^2 - 211680*a^2*b^7*c^4*d*f^3 - 435456*a*b^7*
c^5*d^2*e^2 - 75188736*a^4*b*c^8*d^3*f - 15482880*a^5*c^8*d*e^2*f - 4262400
*a^5*b*c^7*d*f^3 + 852768*a*b^7*c^5*d^3*f + 7350*a*b^9*c^3*d*f^3 + 35525376
*a^4*b^2*c^7*d^2*f^2 + 645120*a^4*b^3*c^6*e^2*f^2 - 80640*a^3*b^5*c^5*e^2*f
^2 + 2304*a^2*b^7*c^4*e^2*f^2 - 15269184*a^3*b^4*c^6*d^2*f^2 + 2870784*a^2*
b^6*c^5*d^2*f^2 - 17418240*a^3*b^3*c^7*d^2*e^2 + 3919104*a^2*b^5*c^6*d^2*e^
2 + 11025*b^10*c^3*d^2*f^2 + 5644800*a^5*c^8*d^2*f^2 + 20736*b^9*c^4*d^2*e^
2 + 492800*a^5*b^2*c^6*f^4 + 351456*a^4*b^4*c^5*f^4 - 43120*a^3*b^6*c^4*f^4
 + 1225*a^2*b^8*c^3*f^4 - 27433728*a^3*b^2*c^8*d^4 + 6446304*a^2*b^4*c^7*d^
4 - 39690*b^9*c^4*d^3*f - 734832*a*b^6*c^6*d^4 + 49787136*a^4*c^9*d^4 + 160
000*a^6*c^7*f^4 + 5308416*a^5*c^8*e^4 + 35721*b^8*c^5*d^4, z, k)*(root(5637
1445760*a^11*b^8*c^6*z^4 - 503316480*a^8*b^14*c^3*z^4 + 47185920*a^7*b^16*c
^2*z^4 - 171798691840*a^14*b^2*c^9*z^4 + 193273528320*a^13*b^4*c^8*z^4 - 12
8849018880*a^12*b^6*c^7*z^4 - 16911433728*a^10*b^10*c^5*z^4 + 3523215360*a^
9*b^12*c^4*z^4 - 2621440*a^6*b^18*c*z^4 + 68719476736*a^15*c^10*z^4 + 65536
*a^5*b^20*z^4 - 73728*a^2*b^16*c*d*f*z^2 - 1321205760*a^9*b^2*c^8*d*f*z^2 +
 732168192*a^7*b^6*c^6*d*f*z^2 - 366280704*a^6*b^8*c^5*d*f*z^2 - 330301440*
a^8*b^4*c^7*d*f*z^2 + 96583680*a^5*b^10*c^4*d*f*z^2 - 15175680*a^4*b^12*c^3
*d*f*z^2 + 1428480*a^3*b^14*c^2*d*f*z^2 - 440401920*a^10*b*c^8*f^2*z^2 + 17
61607680*a^10*c^9*d*f*z^2 - 14080*a^3*b^15*c*f^2*z^2 + 6936330240*a^8*b^3*c
^8*d^2*z^2 + 2464874496*a^6*b^7*c^6*d^2*z^2 - 3963617280*a^9*b*c^9*d^2*z^2
- 1509949440*a^9*b^2*c^8*e^2*z^2 - 5400428544*a^7*b^5*c^7*d^2*z^2 - 94464*a
*b^17*c*d^2*z^2 + 754974720*a^8*b^4*c^7*e^2*z^2 - 730054656*a^5*b^9*c^5*d^2
*z^2 + 477102080*a^9*b^3*c^7*f^2*z^2 - 174325760*a^8*b^5*c^6*f^2*z^2 - 1887
43680*a^7*b^6*c^6*e^2*z^2 + 146165760*a^4*b^11*c^4*d^2*z^2 + 11206656*a^7*b
^7*c^5*f^2*z^2 + 8929280*a^6*b^9*c^4*f^2*z^2 + ...
```

3.67 $\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4} dx$

Optimal. Leaf size=4

$$\log(2+x)$$

[Out] ln(2+x)

Rubi [A]
time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1600, 31}

$$\log(x+2)$$

Antiderivative was successfully verified.

[In] Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]

[Out] Log[2 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x-2x^2+x^3}{4-5x^2+x^4}\,dx &= \int \frac{1}{2+x}\,dx\\ &= \log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 4, normalized size = 1.00

$$\log(2+x)$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4),x]

```
[Out] Log[2 + x]
```

Maple [A]
time = 0.01, size = 5, normalized size = 1.25

method	result	size
default	$\ln(x+2)$	5
norman	$\ln(x+2)$	5
risch	$\ln(x+2)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] ln(x+2)
```

Maxima [A]
time = 0.27, size = 4, normalized size = 1.00

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] log(x + 2)
```

Fricas [A]
time = 0.40, size = 4, normalized size = 1.00

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] log(x + 2)
```

Sympy [A]
time = 0.01, size = 3, normalized size = 0.75

$$\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
[Out] log(x + 2)
```

Giac [**A**]
time = 4.60, size = 5, normalized size = 1.25

$$\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] log(abs(x + 2))
```

Mupad [**B**]
time = 0.02, size = 4, normalized size = 1.00

$$\ln(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2*x^2 - x^3 - 2)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 2)
```

3.68 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=14

$$ex+(d-2e)\log(2+x)$$

```
[Out] e*x+(d-2*e)*ln(2+x)
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1600, 45}

$$(d-2e)\log(x+2)+ex$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] e*x + (d - 2*e)*Log[2 + x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(2-x-2x^2+x^3\right)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex}{2+x}\,dx\\ &= \int \left(e+\frac{d-2e}{2+x}\right)dx\\ &= ex+(d-2e)\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.14

$$e(2+x)+(d-2e)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
[Out] e*(2 + x) + (d - 2*e)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 15, normalized size = 1.07

method	result	size
default	$ex + (d - 2e)\ln(x+2)$	15
norman	$ex + (d - 2e)\ln(x+2)$	15
risch	$ex + \ln(x+2)\,d - 2\ln(x+2)\,e$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] e*x+(d-2*e)*ln(x+2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 1.14

$$xe + (d - 2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] x*e + (d - 2*e)*log(x + 2)
```

Fricas [A]
time = 0.41, size = 14, normalized size = 1.00

$$ex + (d - 2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] e*x + (d - 2*e)*log(x + 2)
```

Sympy [A]
time = 0.04, size = 12, normalized size = 0.86

$$ex + (d - 2e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] e*x + (d - 2*e)*log(x + 2)
```

Giac [**A**]
time = 4.59, size = 17, normalized size = 1.21

$$xe + (d - 2e)\log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] x*e + (d - 2*e)*log(abs(x + 2))
```

Mupad [**B**]
time = 0.73, size = 14, normalized size = 1.00

$$\ln(x + 2)\,(d - 2e) + e\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x)
[Out] log(x + 2)*(d - 2*e) + e*x
```

3.69 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4} dx$

Optimal. Leaf size=31

$$(e-4f)x+\frac{1}{2}f(2+x)^2+(d-2e+4f)\log(2+x)$$

```
[Out] (e-4*f)*x+1/2*f*(2+x)^2+(d-2*e+4*f)*ln(2+x)
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1600, 712}

$$\log(x+2)(d-2e+4f)+x(e-4f)+\frac{1}{2}f(x+2)^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f)*x + (f*(2 + x)^2)/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2}{2+x}\,dx\\ &= \int \left(e-4f+\frac{d-2e+4f}{2+x}+f(2+x)\right)dx\\ &= (e-4f)x+\frac{1}{2}f(2+x)^2+(d-2e+4f)\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.97

$$\frac{1}{2}(2e+f(-6+x))(2+x)+(d-2e+4f)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] ((2*e + f*(-6 + x))*(2 + x))/2 + (d - 2*e + 4*f)*Log[2 + x]
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.90

method	result	size
default	$\frac{f\,x^2}{2}+ex-2fx+(d-2e+4f)\ln(x+2)$	28
norman	$(e-2f)\,x+\frac{f\,x^2}{2}+(d-2e+4f)\ln(x+2)$	28
risch	$\frac{f\,x^2}{2}+ex-2fx+\ln(x+2)\,d-2\ln(x+2)\,e+4\ln(x+2)\,f$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f*x^2+e*x-2*f*x+(d-2*e+4*f)*ln(x+2)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.03

$$\frac{1}{2}\,fx^2-(2\,f-e)x+(d+4\,f-2\,e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima"
)
```

```
[Out] 1/2*f*x^2 - (2*f - e)*x + (d + 4*f - 2*e)*log(x + 2)
```

Fricas [A]
time = 0.39, size = 27, normalized size = 0.87

$$\frac{1}{2}\,fx^2+(e-2\,f)x+(d-2\,e+4\,f)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas"
)
```

```
[Out] 1/2*f*x^2 + (e - 2*f)*x + (d - 2*e + 4*f)*log(x + 2)
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.84

$$\frac{fx^2}{2} + x(e - 2f) + (d - 2e + 4f)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4),x)
[Out] f*x**2/2 + x*(e - 2*f) + (d - 2*e + 4*f)*log(x + 2)
```

Giac [A]
time = 4.53, size = 30, normalized size = 0.97

$$\frac{1}{2} fx^2 - 2 fx + xe + (d + 4 f - 2 e)\log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*f*x^2 - 2*f*x + x*e + (d + 4*f - 2*e)*log(abs(x + 2))
```

Mupad [B]
time = 0.04, size = 27, normalized size = 0.87

$$x(e - 2 f) + \frac{f x^2}{2} + \ln(x + 2)\ (d - 2 e + 4 f)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x + f*x^2)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x)
[Out] x*(e - 2*f) + (f*x^2)/2 + log(x + 2)*(d - 2*e + 4*f)
```

3.70 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=51

$$(e-4f+12g)x+\frac{1}{2}(f-6g)(2+x)^2+\frac{1}{3}g(2+x)^3+(d-2e+4f-8g)\log(2+x)$$

```
[Out] (e-4*f+12*g)*x+1/2*(f-6*g)*(2+x)^2+1/3*g*(2+x)^3+(d-2*e+4*f-8*g)*ln(2+x)
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {1600, 1864}

$$\log(x+2)(d-2e+4f-8g)+x(e-4f+12g)+\frac{1}{2}(x+2)^2(f-6g)+\frac{1}{3}g(x+2)^3$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 4*f + 12*g)*x + ((f - 6*g)*(2 + x)^2)/2 + (g*(2 + x)^3)/3 + (d - 2*e +
 4*f - 8*g)*Log[2 + x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3}{2+x}\,dx\\ &= \int \left(e-4f+12g+\frac{d-2e+4f-8g}{2+x}+(f-6g)(2+x)\right.\\ &= (e-4f+12g)x+\frac{1}{2}(f-6g)(2+x)^2+\frac{1}{3}g(2+x)^3+(d\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.88

$$\frac{1}{6}(2+x)\left(6e+3f(-6+x)+2g\left(22-5x+x^2\right)\right)+(d-2e+4f-8g)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^
4),x]
```

```
[Out] ((2 + x)*(6*e + 3*f*(-6 + x) + 2*g*(22 - 5*x + x^2)))/6 + (d - 2*e + 4*f -
8*g)*Log[2 + x]
```

Maple [A]
time = 0.02, size = 47, normalized size = 0.92

method	result
norman	$\left(\frac{f}{2}-g\right)x^{2}+\left(e-2f+4g\right)x+\frac{g\,x^{3}}{3}+\left(d-2e+4f-8g\right)\ln\left(x+2\right)$
default	$\frac{g\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+ex-2fx+4gx+\left(d-2e+4f-8g\right)\ln\left(x+2\right)$
risch	$\frac{g\,x^{3}}{3}+\frac{f\,x^{2}}{2}-g\,x^{2}+ex-2fx+4gx+\ln\left(x+2\right)d-2\ln\left(x+2\right)e+4\ln\left(x+2\right)f-8\ln\left(x+2\right)g$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/3*g*x^3+1/2*f*x^2-g*x^2+e*x-2*f*x+4*g*x+(d-2*e+4*f-8*g)*ln(x+2)
```

Maxima [A]
time = 0.29, size = 48, normalized size = 0.94

$$\frac{1}{3}\,gx^3+\frac{1}{2}\,(f-2\,g)x^2-(2\,f-4\,g-e)x+(d+4\,f-8\,g-2\,e)\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="m
axima")
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 - (2*f - 4*g - e)*x + (d + 4*f - 8*g - 2*e)*l
og(x + 2)
```

Fricas [A]
time = 0.40, size = 43, normalized size = 0.84

$$\frac{1}{3}\,gx^3+\frac{1}{2}\,(f-2\,g)x^2+(e-2\,f+4\,g)x+(d-2\,e+4\,f-8\,g)\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="f
ricas")
```

```
[Out] 1/3*g*x^3 + 1/2*(f - 2*g)*x^2 + (e - 2*f + 4*g)*x + (d - 2*e + 4*f - 8*g)*l
og(x + 2)
```

Sympy [A]
time = 0.07, size = 41, normalized size = 0.80

$$\frac{gx^3}{3} + x^2\left(\frac{f}{2} - g\right) + x(e - 2f + 4g) + (d - 2e + 4f - 8g)\log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**3/3 + x**2*(f/2 - g) + x*(e - 2*f + 4*g) + (d - 2*e + 4*f - 8*g)*log(x
 + 2)
```

Giac [A]
time = 5.37, size = 49, normalized size = 0.96

$$\frac{1}{3}\,gx^3 + \frac{1}{2}\,fx^2 - gx^2 - 2\,fx + 4\,gx + xe + (d + 4\,f - 8\,g - 2\,e)\log(|x + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="g
iac")
```

```
[Out] 1/3*g*x^3 + 1/2*f*x^2 - g*x^2 - 2*f*x + 4*g*x + x*e + (d + 4*f - 8*g - 2*e)
*log(abs(x + 2))
```

Mupad [B]
time = 0.04, size = 44, normalized size = 0.86

$$x^2\left(\frac{f}{2} - g\right) + x\,(e - 2\,f + 4\,g) + \frac{g\,x^3}{3} + \ln(x + 2)\,\left(d - 2\,e + 4\,f - 8\,g\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x + f*x^2 + g*x^3)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] x^2*(f/2 - g) + x*(e - 2*f + 4*g) + (g*x^3)/3 + log(x + 2)*(d - 2*e + 4*f -
 8*g)
```

3.71 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=68

$$(e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2h)x^3+\frac{hx^4}{4}+(d-2e+4f-8g+16h)\log(2+x)$$

```
[Out] (e-2*f+4*g-8*h)*x+1/2*(f-2*g+4*h)*x^2+1/3*(g-2*h)*x^3+1/4*h*x^4+(d-2*e+4*f-
8*g+16*h)*ln(2+x)
```

Rubi [A]
time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1600, 1864}

$$\log(x+2)(d-2e+4f-8g+16h)+x(e-2f+4g-8h)+\frac{1}{2}x^2(f-2g+4h)+\frac{1}{3}x^3(g-2h)+\frac{hx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h*
x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3+hx^4}{2+x}\,dx\\ &= \int \left(e\left(1-\frac{2(f-2g+4h)}{e}\right)+(f-2g+4h)x+(g\right.\\ &= (e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2\end{aligned}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 1.00

$$(e-2f+4g-8h)x+\frac{1}{2}(f-2g+4h)x^2+\frac{1}{3}(g-2h)x^3+\frac{hx^4}{4}+(d-2e+4f-8g+16h)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*
x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h)*x + ((f - 2*g + 4*h)*x^2)/2 + ((g - 2*h)*x^3)/3 + (h*
x^4)/4 + (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 72, normalized size = 1.06

method	result
norman	$\left(\frac{g}{3}-\frac{2h}{3}\right)x^3+\left(\frac{f}{2}-g+2h\right)x^2+(e-2f+4g-8h)x+\frac{h x^4}{4}+(d-2e+4f-8g+16h)\ln(x -$
default	$\frac{h x^4}{4}+\frac{g x^3}{3}-\frac{2h x^3}{3}+\frac{f x^2}{2}-g x^2+2h x^2+ex-2fx+4gx-8hx+(d-2e+4f-8g+16h)\ln$
risch	$\frac{h x^4}{4}+\frac{g x^3}{3}-\frac{2h x^3}{3}+\frac{f x^2}{2}-g x^2+2h x^2+ex-2fx+4gx-8hx+\ln(x+2)d-2\ln(x+2)e +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETUR
NVERBOSE)
```

```
[Out] 1/4*h*x^4+1/3*g*x^3-2/3*h*x^3+1/2*f*x^2-g*x^2+2*h*x^2+e*x-2*f*x+4*g*x-8*h*x
+(d-2*e+4*f-8*g+16*h)*ln(x+2)
```

Maxima [A]
time = 0.30, size = 67, normalized size = 0.99

$$\frac{1}{4}hx^4+\frac{1}{3}(g-2h)x^3+\frac{1}{2}(f-2g+4h)x^2-(2f-4g+8h-e)x+(d+4f-8g+16h-2e)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="maxima")
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 - (2*f - 4*g + 8*h
- e)*x + (d + 4*f - 8*g + 16*h - 2*e)*log(x + 2)
```

Fricas [A]
time = 0.40, size = 62, normalized size = 0.91

$$\frac{1}{4}hx^4+\frac{1}{3}(g-2h)x^3+\frac{1}{2}(f-2g+4h)x^2+(e-2f+4g-8h)x+(d-2e+4f-8g+16h)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="fricas")
```

```
[Out] 1/4*h*x^4 + 1/3*(g - 2*h)*x^3 + 1/2*(f - 2*g + 4*h)*x^2 + (e - 2*f + 4*g -
8*h)*x + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Sympy [A]
time = 0.09, size = 63, normalized size = 0.93

$$\frac{hx^4}{4}+x^3\left(\frac{g}{3}-\frac{2h}{3}\right)+x^2\left(\frac{f}{2}-g+2h\right)+x(e-2f+4g-8h)+(d-2e+4f-8g+16h)\log{(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**4/4 + x**3*(g/3 - 2*h/3) + x**2*(f/2 - g + 2*h) + x*(e - 2*f + 4*g - 8
*h) + (d - 2*e + 4*f - 8*g + 16*h)*log(x + 2)
```

Giac [A]
time = 3.45, size = 74, normalized size = 1.09

$$\frac{1}{4}\,hx^4+\frac{1}{3}\,gx^3-\frac{2}{3}\,hx^3+\frac{1}{2}\,fx^2-gx^2+2\,hx^2-2\,fx+4\,gx-8\,hx+xe+(d+4\,f-8\,g+16\,h-2\,e)\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algori
thm="giac")
```

```
[Out] 1/4*h*x^4 + 1/3*g*x^3 - 2/3*h*x^3 + 1/2*f*x^2 - g*x^2 + 2*h*x^2 - 2*f*x + 4
*g*x - 8*h*x + x*e + (d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2))
```

Mupad [B]
time = 0.03, size = 64, normalized size = 0.94

$$x^3\left(\frac{g}{3}-\frac{2\,h}{3}\right)+\ln\left(x+2\right)\,\left(d-2\,e+4\,f-8\,g+16\,h\right)+\frac{h\,x^4}{4}+x^2\,\left(\frac{f}{2}-g+2\,h\right)+x\,\left(e-2\,f+4\,g-8\,h\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2
 + 4),x)
```

```
[Out] x^3*(g/3 - (2*h)/3) + log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h) + (h*x^4)/4 +
 x^2*(f/2 - g + 2*h) + x*(e - 2*f + 4*g - 8*h)
```

3.72 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4} dx$

Optimal. Leaf size=92

$$(e-2f+4g-8h+16i)x+\frac{1}{2}(f-2g+4h-8i)x^2+\frac{1}{3}(g-2h+4i)x^3+\frac{1}{4}(h-2i)x^4+\frac{ix^5}{5}+(d-2e+4f-8g+16h-32i)$$

```
[Out] (e-2*f+4*g-8*h+16*i)*x+1/2*(f-2*g+4*h-8*i)*x^2+1/3*(g-2*h+4*i)*x^3+1/4*(h-2
*i)*x^4+1/5*i*x^5+(d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {1600, 1864}

$$\log(x+2)(d-2e+4f-8g+16h-32i)+x(e-2f+4g-8h+16i)+\frac{1}{2}x^2(f-2g+4h-8i)+\frac{1}{3}x^3(g-2h+4i)+\frac{1}{4}x^4(h-2i)+\frac{ix^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
+ 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*Log[2 + x]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+72x^5)}{4-5x^2+x^4} dx &= \int \frac{d+ex+fx^2+gx^3+hx^4+72x^5}{2+x} dx\\ &= \int \left(1152\left(1+\frac{e-2f+4g-8h}{1152}\right)+(-57\right.\\ &= (1152+e-2f+4g-8h)x-\frac{1}{2}(576-f+\end{aligned}$$

Mathematica [A]
time = 0.02, size = 92, normalized size = 1.00

$$(e-2f+4g-8h+16i)x+\frac{1}{2}(f-2g+4h-8i)x^2+\frac{1}{3}(g-2h+4i)x^3+\frac{1}{4}(h-2i)x^4+\frac{ix^5}{5}+(d-2e+4f-8g+16h-32i)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))
/(4 - 5*x^2 + x^4),x]
```

```
[Out] (e - 2*f + 4*g - 8*h + 16*i)*x + ((f - 2*g + 4*h - 8*i)*x^2)/2 + ((g - 2*h
+ 4*i)*x^3)/3 + ((h - 2*i)*x^4)/4 + (i*x^5)/5 + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 103, normalized size = 1.12

method	result
norman	$\left(\frac{h}{4}-\frac{i}{2}\right)x^4+\left(\frac{g}{3}-\frac{2h}{3}+\frac{4i}{3}\right)x^3+\left(\frac{f}{2}-g+2h-4i\right)x^2+(e-2f+4g-8h+16i)x+\frac{i\,x^5}{5}+(d-$
default	$\frac{i\,x^5}{5}+\frac{h\,x^4}{4}-\frac{i\,x^4}{2}+\frac{g\,x^3}{3}-\frac{2h\,x^3}{3}+\frac{4i\,x^3}{3}+\frac{f\,x^2}{2}-g\,x^2+2h\,x^2-4i\,x^2+ex-2fx+4gx-8hx+16i$
risch	$\frac{i\,x^5}{5}+\frac{h\,x^4}{4}-\frac{i\,x^4}{2}+\frac{g\,x^3}{3}-\frac{2h\,x^3}{3}+\frac{4i\,x^3}{3}+\frac{f\,x^2}{2}-g\,x^2+2h\,x^2-4i\,x^2+ex-2fx+4gx-8hx+16i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=
_RETURNVERBOSE)
```

```
[Out] 1/5*i*x^5+1/4*h*x^4-1/2*i*x^4+1/3*g*x^3-2/3*h*x^3+4/3*i*x^3+1/2*f*x^2-g*x^2
+2*h*x^2-4*i*x^2+e*x-2*f*x+4*g*x-8*h*x+16*i*x+(d-2*e+4*f-8*g+16*h-32*i)*ln(
x+2)
```

Maxima [A]
time = 0.28, size = 78, normalized size = 0.85

$$\frac{1}{4}(h-2i)x^4+\frac{1}{5}i\,x^5+\frac{1}{3}(g-2\,h+4i)x^3+\frac{1}{2}(f-2\,g+4\,h-8i)x^2-(2\,f-4\,g+8\,h-e-16i)x+(d+4\,f-8\,g+16\,h-2\,e-32i)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="maxima")
```

```
[Out] 1/4*(h - 2*I)*x^4 + 1/5*I*x^5 + 1/3*(g - 2*h + 4*I)*x^3 + 1/2*(f - 2*g + 4*
h - 8*I)*x^2 - (2*f - 4*g + 8*h - e - 16*I)*x + (d + 4*f - 8*g + 16*h - 2*e
 - 32*I)*log(x + 2)
```

Fricas [A]
time = 0.41, size = 84, normalized size = 0.91

$$\frac{1}{5}ix^5+\frac{1}{4}(h-2\,i)x^4+\frac{1}{3}(g-2\,h+4\,i)x^3+\frac{1}{2}(f-2\,g+4\,h-8\,i)x^2+(e-2\,f+4\,g-8\,h+16\,i)x+(d-2\,e+4\,f-8\,g+16\,h-32\,i)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="fricas")
```

```
[Out] 1/5*i*x^5 + 1/4*(h - 2*i)*x^4 + 1/3*(g - 2*h + 4*i)*x^3 + 1/2*(f - 2*g + 4*
h - 8*i)*x^2 + (e - 2*f + 4*g - 8*h + 16*i)*x + (d - 2*e + 4*f - 8*g + 16*h
 - 32*i)*log(x + 2)
```

Sympy [A]
time = 0.11, size = 88, normalized size = 0.96

$$\frac{ix^5}{5} + x^4\left(\frac{h}{4} - \frac{i}{2}\right) + x^3\left(\frac{g}{3} - \frac{2h}{3} + \frac{4i}{3}\right) + x^2\left(\frac{f}{2} - g + 2h - 4i\right) + x(e - 2f + 4g - 8h + 16i) + (d - 2e + 4f - 8g + 16h - 32i)\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**
2+4),x)
```

```
[Out] i*x**5/5 + x**4*(h/4 - i/2) + x**3*(g/3 - 2*h/3 + 4*i/3) + x**2*(f/2 - g +
2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + (d - 2*e + 4*f - 8*g + 16*h -
 32*i)*log(x + 2)
```

Giac [A]
time = 3.70, size = 98, normalized size = 1.07

$$\frac{1}{4}hx^4 + \frac{1}{5}ix^5 + \frac{1}{3}gx^3 - \frac{2}{3}hx^3 - \frac{1}{2}ix^4 + \frac{1}{2}fx^2 - gx^2 + 2hx^2 + \frac{4}{3}ix^3 - 2fx + 4gx - 8hx - 4ix^2 + xe + (d + 4f - 8g + 16h - 2e - 32i)\log(|x+2|) + 16ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,
algorithm="giac")
```

```
[Out] 1/4*h*x^4 + 1/5*I*x^5 + 1/3*g*x^3 - 2/3*h*x^3 - 1/2*I*x^4 + 1/2*f*x^2 - g*x
^2 + 2*h*x^2 + 4/3*I*x^3 - 2*f*x + 4*g*x - 8*h*x - 4*I*x^2 + x*e + (d + 4*f
 - 8*g + 16*h - 2*e - 32*I)*log(abs(x + 2)) + 16*I*x
```

Mupad [B]
time = 0.04, size = 87, normalized size = 0.95

$$x^4\left(\frac{h}{4} - \frac{i}{2}\right) + \ln(x+2)\,(d - 2e + 4f - 8g + 16h - 32i) + \frac{ix^5}{5} + x^2\left(\frac{f}{2} - g + 2h - 4i\right) + x\,(e - 2f + 4g - 8h + 16i) + x^3\left(\frac{g}{3} - \frac{2h}{3} + \frac{4i}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4
 - 5*x^2 + 4),x)
```

```
[Out] x^4*(h/4 - i/2) + log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h - 32*i) + (i*x^5)/
5 + x^2*(f/2 - g + 2*h - 4*i) + x*(e - 2*f + 4*g - 8*h + 16*i) + x^3*(g/3 -
 (2*h)/3 + (4*i)/3)
```

3.73 $\int \frac{2-3x+x^2}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\log(1+x)-\log(2+x)$$

```
[Out] ln(1+x)-ln(2+x)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1600, 630, 31}

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{2-3x+x^2}{4-5x^2+x^4} \, dx &= \int \frac{1}{2+3x+x^2} \, dx \\ &= \int \frac{1}{1+x} \, dx - \int \frac{1}{2+x} \, dx \\ &= \log(1+x)-\log(2+x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\log(1+x) - \log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4),x]
[Out] Log[1 + x] - Log[2 + x]
```

Maple [A]
time = 0.01, size = 12, normalized size = 1.09

method	result	size
default	$\ln(1+x) - \ln(x+2)$	12
norman	$\ln(1+x) - \ln(x+2)$	12
risch	$\ln(1+x) - \ln(x+2)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] ln(1+x)-ln(x+2)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 1.00

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] -log(x + 2) + log(x + 1)
```

Fricas [A]
time = 0.42, size = 11, normalized size = 1.00

$$-\log(x+2) + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$\log(x+1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

Giac [A]
time = 3.74, size = 13, normalized size = 1.18

$$-\log(|x+2|)+\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -log(abs(x + 2)) + log(abs(x + 1))
```

Mupad [B]
time = 0.08, size = 8, normalized size = 0.73

$$-2\,\mathrm{atanh}(2\,x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] -2*atanh(2*x + 3)
```

3.74 $\int \frac{(d+ex)(2-3x+x^2)}{4-5x^2+x^4} dx$

Optimal. Leaf size=22

$$(d-e)\log(1+x)-(d-2e)\log(2+x)$$

```
[Out] (d-e)*ln(1+x)-(d-2*e)*ln(2+x)
```

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1600, 646, 31}

$$(d-e)\log(x+1)-(d-2e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (d - e)*Log[1 + x] - (d - 2*e)*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)\left(2-3x+x^2\right)}{4-5x^2+x^4} dx &= \int \frac{d+ex}{2+3x+x^2} dx \\
&= -\left((d-2e)\int \frac{1}{2+x} dx\right)+(d-e)\int \frac{1}{1+x} dx \\
&= (d-e)\log(1+x)-(d-2e)\log(2+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.05

$$(d-e)\log(1+x)+(-d+2e)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4),x]
[Out] (d - e)*Log[1 + x] + (-d + 2*e)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 24, normalized size = 1.09

method	result	size
default	$(-d+2e)\ln(x+2)+(d-e)\ln(1+x)$	24
norman	$(-d+2e)\ln(x+2)+(d-e)\ln(1+x)$	24
risch	$-\ln(x+2)\,d+2\ln(x+2)\,e+\ln(-1-x)\,d-\ln(-1-x)\,e$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] (-d+2*e)*ln(x+2)+(d-e)*ln(1+x)
```

Maxima [A]
time = 0.29, size = 24, normalized size = 1.09

$$-(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Fricas [A]
time = 0.39, size = 22, normalized size = 1.00

$$-(d-2\,e)\log(x+2)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] -(d - 2*e)*log(x + 2) + (d - e)*log(x + 1)
```

Sympy [A]
time = 0.14, size = 29, normalized size = 1.32

$$(-d+2e)\log\left(x+\frac{4d-6e}{2d-3e}\right)+(d-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4),x)
```

```
[Out] (-d + 2*e)*log(x + (4*d - 6*e)/(2*d - 3*e)) + (d - e)*log(x + 1)
```

Giac [A]
time = 3.85, size = 26, normalized size = 1.18

$$-(d-2e)\log(|x+2|)+(d-e)\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -(d - 2*e)*log(abs(x + 2)) + (d - e)*log(abs(x + 1))
```

Mupad [B]
time = 0.80, size = 22, normalized size = 1.00

$$\ln(x+1)\ (d-e)-\ln(x+2)\ (d-2e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x^2 - 3*x + 2))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)*(d - e) - log(x + 2)*(d - 2*e)
```

3.75 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=29

$$fx + (d-e+f)\log(1+x) - (d-2e+4f)\log(2+x)$$

```
[Out] f*x+(d-e+f)*ln(1+x)-(d-2*e+4*f)*ln(2+x)
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1600, 1671, 646, 31}

$$\log(x+1)(d-e+f) - \log(x+2)(d-2e+4f) + fx$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] f*x + (d - e + f)*Log[1 + x] - (d - 2*e + 4*f)*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-3x+x^2)(d+ex+fx^2)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2}{2+3x+x^2}\,dx \\ &= \int \left(f+\frac{d-2f+(e-3f)x}{2+3x+x^2}\right)dx \\ &= fx+\int \frac{d-2f+(e-3f)x}{2+3x+x^2}\,dx \\ &= fx+(d-e+f)\int \frac{1}{1+x}\,dx-(d-2e+4f)\int \frac{1}{2+x}\,dx \\ &= fx+(d-e+f)\log(1+x)-(d-2e+4f)\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.03

$$fx+(d-e+f)\log(1+x)+(-d+2e-4f)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
[Out] f*x + (d - e + f)*Log[1 + x] + (-d + 2*e - 4*f)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 31, normalized size = 1.07

method	result
default	$fx+(-d+2e-4f)\ln(x+2)+(d-e+f)\ln(1+x)$
norman	$fx+(-d+2e-4f)\ln(x+2)+(d-e+f)\ln(1+x)$
risch	$fx+\ln(-1-x)d-\ln(-1-x)e+\ln(-1-x)f-\ln(x+2)d+2\ln(x+2)e-4\ln(x+2)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] f*x+(-d+2*e-4*f)*ln(x+2)+(d-e+f)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 31, normalized size = 1.07

$$fx-(d+4f-2e)\log(x+2)+(d+f-e)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] f*x - (d + 4*f - 2*e)*log(x + 2) + (d + f - e)*log(x + 1)
```

Fricas [A]
time = 0.39, size = 29, normalized size = 1.00

$$fx - (d - 2e + 4f)\log(x + 2) + (d - e + f)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] f*x - (d - 2*e + 4*f)*log(x + 2) + (d - e + f)*log(x + 1)
```

Sympy [A]
time = 0.28, size = 44, normalized size = 1.52

$$fx + (-d + 2e - 4f)\log\left(x + \frac{4d - 6e + 10f}{2d - 3e + 5f}\right) + (d - e + f)\log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
[Out] f*x + (-d + 2*e - 4*f)*log(x + (4*d - 6*e + 10*f)/(2*d - 3*e + 5*f)) + (d -
 e + f)*log(x + 1)
```

Giac [A]
time = 3.60, size = 33, normalized size = 1.14

$$fx - (d + 4f - 2e)\log(|x + 2|) + (d + f - e)\log(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] f*x - (d + 4*f - 2*e)*log(abs(x + 2)) + (d + f - e)*log(abs(x + 1))
```

Mupad [B]
time = 0.07, size = 29, normalized size = 1.00

$$f\,x + \ln(x + 1)\,(d - e + f) - \ln(x + 2)\,(d - 2\,e + 4\,f)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4),x)
[Out] f*x + log(x + 1)*(d - e + f) - log(x + 2)*(d - 2*e + 4*f)
```

3.76 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4} dx$

Optimal. Leaf size=47

$$(f-3g)x+\frac{gx^2}{2}+(d-e+f-g)\log(1+x)-(d-2e+4f-8g)\log(2+x)$$

[Out] (f-3*g)*x+1/2*g*x^2+(d-e+f-g)*ln(1+x)-(d-2*e+4*f-8*g)*ln(2+x)

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1600, 1671, 646, 31}

$$\log(x+1)(d-e+f-g)-\log(x+2)(d-2e+4f-8g)+x(f-3g)+\frac{gx^2}{2}$$

Antiderivative was successfully verified.

[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]

[Out] (f - 3*g)*x + (g*x^2)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)*Log[2 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3}{2+3x+x^2}\,dx\\ &= \int \left(f-3g+gx+\frac{d-2f+6g+(e-3f+7g)x}{2+3x+x^2}\right)dx\\ &= (f-3g)x+\frac{gx^2}{2}+\int \frac{d-2f+6g+(e-3f+7g)x}{2+3x+x^2}\,dx\\ &= (f-3g)x+\frac{gx^2}{2}-(d-2e+4f-8g)\int \frac{1}{2+x}\,dx+(d-e+f\\ &= (f-3g)x+\frac{gx^2}{2}+(d-e+f-g)\log(1+x)-(d-2e+4f-\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.94

$$fx+\frac{1}{2}g(-6+x)x+(d-e+f-g)\log(1+x)-(d-2e+4f-8g)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
[Out] f*x + (g*(-6 + x)*x)/2 + (d - e + f - g)*Log[1 + x] - (d - 2*e + 4*f - 8*g)
*Log[2 + x]
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.00

method	result
default	$\frac{g x^2}{2}+fx-3gx+(-d+2e-4f+8g)\ln(x+2)+(d-e+f-g)\ln(1+x)$
norman	$(f-3g)\,x+\frac{g x^2}{2}+(-d+2e-4f+8g)\ln(x+2)+(d-e+f-g)\ln(1+x)$
risch	$\frac{g x^2}{2}+fx-3gx+\ln(-1-x)\,d-\ln(-1-x)\,e+\ln(-1-x)\,f-\ln(-1-x)\,g-\ln(x+2)\,d+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/2*g*x^2+f*x-3*g*x+(-d+2*e-4*f+8*g)*ln(x+2)+(d-e+f-g)*ln(1+x)
```

Maxima [A]
time = 0.30, size = 47, normalized size = 1.00

$$\frac{1}{2}\,gx^{2}+(f-3\,g)x-(d+4\,f-8\,g-2\,e)\log\left(x+2\right)+(d+f-g-e)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d + 4*f - 8*g - 2*e)*log(x + 2) + (d + f - g - e
)*log(x + 1)
```

Fricas [A]
time = 0.38, size = 45, normalized size = 0.96

$$\frac{1}{2}\,gx^{2}+(f-3\,g)x-(d-2\,e+4\,f-8\,g)\log\left(x+2\right)+(d-e+f-g)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*g*x^2 + (f - 3*g)*x - (d - 2*e + 4*f - 8*g)*log(x + 2) + (d - e + f - g
)*log(x + 1)
```

Sympy [A]
time = 0.54, size = 66, normalized size = 1.40

$$\frac{gx^{2}}{2}+x(f-3g)+(-d+2e-4f+8g)\log\left(x+\frac{4d-6e+10f-18g}{2d-3e+5f-9g}\right)+(d-e+f-g)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x**2/2 + x*(f - 3*g) + (-d + 2*e - 4*f + 8*g)*log(x + (4*d - 6*e + 10*f -
 18*g)/(2*d - 3*e + 5*f - 9*g)) + (d - e + f - g)*log(x + 1)
```

Giac [A]
time = 4.97, size = 49, normalized size = 1.04

$$\frac{1}{2}\,gx^{2}+fx-3\,gx-(d+4\,f-8\,g-2\,e)\log\left(|x+2|\right)+(d+f-g-e)\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] 1/2*g*x^2 + f*x - 3*g*x - (d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + (d + f -
g - e)*log(abs(x + 1))
```

Mupad [B]
time = 0.76, size = 45, normalized size = 0.96

$$\ln(x+1)\,(d-e+f-g)+x\,(f-3\,g)+\frac{g\,x^2}{2}-\ln(x+2)\,(d-2\,e+4\,f-8\,g)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)*(d - e + f - g) + x*(f - 3*g) + (g*x^2)/2 - log(x + 2)*(d - 2*e
+ 4*f - 8*g)
```

3.77 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=66

$$(f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\log(1+x)-(d-2e+4f-8g+16h)\log(2+x)$$

```
[Out] (f-3*g+7*h)*x+1/2*(g-3*h)*x^2+1/3*h*x^3+(d-e+f-g+h)*ln(1+x)-(d-2*e+4*f-8*g+
16*h)*ln(2+x)
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {1600, 1671, 646, 31}

$$\log(x+1)(d-e+f-g+h)-\log(x+2)(d-2e+4f-8g+16h)+x(f-3g+7h)+\frac{1}{2}x^2(g-3h)+\frac{hx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x
]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Log
[1 + x] - (d - 2*e + 4*f - 8*g + 16*h)*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
```

```
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{2+3x+x^2}\,dx$$
$$= \int \left(f-3g+7h+(g-3h)x+hx^2+\frac{d-2f+6g-14h}{2\cdots}\right.$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+\int \frac{d-2f+6g-\cdots}{\cdots}$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+\cdots$$
$$= (f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+\cdots$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 1.02

$$(f-3g+7h)x+\frac{1}{2}(g-3h)x^2+\frac{hx^3}{3}+(d-e+f-g+h)\log(1+x)+(-d+2e-4f+8g-16h)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (f - 3*g + 7*h)*x + ((g - 3*h)*x^2)/2 + (h*x^3)/3 + (d - e + f - g + h)*Log
[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h)*Log[2 + x]
```

Maple [A]
time = 0.03, size = 67, normalized size = 1.02

method	result
norman	$\left(\frac{g}{2}-\frac{3h}{2}\right)x^2+(f-3g+7h)x+\frac{hx^3}{3}+(-d+2e-4f+8g-16h)\ln(x+2)+(d-e+f-g+\cdots$
default	$\frac{hx^3}{3}+\frac{gx^2}{2}-\frac{3hx^2}{2}+fx-3gx+7hx+(-d+2e-4f+8g-16h)\ln(x+2)+(d-e+f-g+\cdots$
risch	$\frac{hx^3}{3}+\frac{gx^2}{2}-\frac{3hx^2}{2}+fx-3gx+7hx+\ln(-1-x)d-\ln(-1-x)e+\ln(-1-x)f-\ln(-1-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/3*h*x^3+1/2*g*x^2-3/2*h*x^2+f*x-3*g*x+7*h*x+(-d+2*e-4*f+8*g-16*h)*ln(x+2)
+(d-e+f-g+h)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.97

$$\frac{1}{3}\,hx^3+\frac{1}{2}\,(g-3\,h)x^2+(f-3\,g+7\,h)x-(d+4\,f-8\,g+16\,h-2\,e)\log\left(x+2\right)+(d+f-g+h-e)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"maxima")
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d + 4*f - 8*g + 16*h -
 2*e)*log(x + 2) + (d + f - g + h - e)*log(x + 1)
```

Fricas [A]
time = 0.46, size = 62, normalized size = 0.94

$$\frac{1}{3}\,hx^3+\frac{1}{2}\,(g-3\,h)x^2+(f-3\,g+7\,h)x-(d-2\,e+4\,f-8\,g+16\,h)\log\left(x+2\right)+(d-e+f-g+h)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"fricas")
```

```
[Out] 1/3*h*x^3 + 1/2*(g - 3*h)*x^2 + (f - 3*g + 7*h)*x - (d - 2*e + 4*f - 8*g +
16*h)*log(x + 2) + (d - e + f - g + h)*log(x + 1)
```

Sympy [A]
time = 0.89, size = 94, normalized size = 1.42

$$\frac{hx^3}{3}+x^2\left(\frac{g}{2}-\frac{3h}{2}\right)+x(f-3g+7h)+(-d+2e-4f+8g-16h)\log\left(x+\frac{4d-6e+10f-18g+34h}{2d-3e+5f-9g+17h}\right)+(d-e+f-g+h)\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] h*x**3/3 + x**2*(g/2 - 3*h/2) + x*(f - 3*g + 7*h) + (-d + 2*e - 4*f + 8*g -
 16*h)*log(x + (4*d - 6*e + 10*f - 18*g + 34*h)/(2*d - 3*e + 5*f - 9*g + 17
*h)) + (d - e + f - g + h)*log(x + 1)
```

Giac [A]
time = 2.86, size = 69, normalized size = 1.05

$$\frac{1}{3}\,hx^3+\frac{1}{2}\,gx^2-\frac{3}{2}\,hx^2+fx-3\,gx+7\,hx-(d+4\,f-8\,g+16\,h-2\,e)\log\left(|x+2|\right)+(d+f-g+h-e)\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"giac")
```

```
[Out] 1/3*h*x^3 + 1/2*g*x^2 - 3/2*h*x^2 + f*x - 3*g*x + 7*h*x - (d + 4*f - 8*g +
16*h - 2*e)*log(abs(x + 2)) + (d + f - g + h - e)*log(abs(x + 1))
```

Mupad [B]
time = 0.07, size = 63, normalized size = 0.95

$$x^2\left(\frac{g}{2}-\frac{3\,h}{2}\right)+x\,(f-3\,g+7\,h)-\ln(x+2)\,(d-2\,e+4\,f-8\,g+16\,h)+\frac{h\,x^3}{3}+\ln(x+1)\,(d-e+f-g+h)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4),x
)
```

```
[Out] x^2*(g/2 - (3*h)/2) + x*(f - 3*g + 7*h) - log(x + 2)*(d - 2*e + 4*f - 8*g +
 16*h) + (h*x^3)/3 + log(x + 1)*(d - e + f - g + h)
```

3.78 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=90

$$(f-3g+7h-15i)x+\frac{1}{2}(g-3h+7i)x^2+\frac{1}{3}(h-3i)x^3+\frac{ix^4}{4}+(d-e+f-g+h-i)\log(1+x)-(d-2e+4f-8g+16h-$$

```
[Out] (f-3*g+7*h-15*i)*x+1/2*(g-3*h+7*i)*x^2+1/3*(h-3*i)*x^3+1/4*i*x^4+(d-e+f-g+h
-i)*ln(1+x)-(d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1600, 1671, 646, 31}

$$\log(x+1)(d-e+f-g+h-i)-\log(x+2)(d-2e+4f-8g+16h-32i)+x(f-3g+7h-15i)+\frac{1}{2}x^2(g-3h+7i)+\frac{1}{3}x^3(h-3i)+\frac{ix^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
+ x^4),x]
```

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (i
*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] - (d - 2*e + 4*f - 8*g + 16*h
- 32*i)*Log[2 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1671

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4+78x^5)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+78x^5}{2+3x+x^2}\,dx$$
$$= \int \Big(-1170+f-3g+7h+(546+g-3h)x-($$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-$$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-$$
$$= -(1170-f+3g-7h)x+\frac{1}{2}(546+g-3h)x^2-$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.01

$$(f-3g+7h-15i)x+\frac{1}{2}(g-3h+7i)x^2+\frac{1}{3}(h-3i)x^3+\frac{ix^4}{4}+(d-e+f-g+h-i)\log(1+x)+(-d+2e-4f+8g-16h+32i)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4),x]
```

```
[Out] (f - 3*g + 7*h - 15*i)*x + ((g - 3*h + 7*i)*x^2)/2 + ((h - 3*i)*x^3)/3 + (i
*x^4)/4 + (d - e + f - g + h - i)*Log[1 + x] + (-d + 2*e - 4*f + 8*g - 16*h
 + 32*i)*Log[2 + x]
```

Maple [A]
time = 0.04, size = 95, normalized size = 1.06

method	result
norman	$\left(\frac{h}{3}-i\right)x^3+\left(\frac{g}{2}-\frac{3h}{2}+\frac{7i}{2}\right)x^2+(f-3g+7h-15i)\,x+\frac{i\,x^4}{4}+(-d+2e-4f+8g-16h+32i)$
default	$\frac{i\,x^4}{4}+\frac{h\,x^3}{3}-i\,x^3+\frac{g\,x^2}{2}-\frac{3h\,x^2}{2}+\frac{7i\,x^2}{2}+fx-3gx+7hx-15ix+(-d+2e-4f+8g-16h+32$
risch	$\frac{i\,x^4}{4}+\frac{h\,x^3}{3}-i\,x^3+\frac{g\,x^2}{2}-\frac{3h\,x^2}{2}+\frac{7i\,x^2}{2}+fx-3gx+7hx-15ix+\ln(-1-x)\,d-\ln(-1-x)\,e+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/4*i*x^4+1/3*h*x^3-i*x^3+1/2*g*x^2-3/2*h*x^2+7/2*i*x^2+f*x-3*g*x+7*h*x-15*
i*x+(-d+2*e-4*f+8*g-16*h+32*i)*ln(x+2)+(d-e+f-g+h-i)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.83

$$\frac{1}{3}(h-3i)x^3+\frac{1}{4}ix^4+\frac{1}{2}(g-3h+7i)x^2+(f-3g+7h-15i)x-(d+4f-8g+16h-2e-32i)\log(x+2)+(d+f-g+h-e-i)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="maxima")
```

```
[Out] 1/3*(h - 3*I)*x^3 + 1/4*I*x^4 + 1/2*(g - 3*h + 7*I)*x^2 + (f - 3*g + 7*h -
15*I)*x - (d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(x + 2) + (d + f - g + h -
 e - I)*log(x + 1)
```

Fricas [A]
time = 0.58, size = 84, normalized size = 0.93

$$\frac{1}{4}ix^4+\frac{1}{3}(h-3i)x^3+\frac{1}{2}(g-3h+7i)x^2+(f-3g+7h-15i)x-(d-2e+4f-8g+16h-32i)\log(x+2)+(d-e+f-g+h-i)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="fricas")
```

```
[Out] 1/4*i*x^4 + 1/3*(h - 3*i)*x^3 + 1/2*(g - 3*h + 7*i)*x^2 + (f - 3*g + 7*h -
15*i)*x - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*log(x + 2) + (d - e + f - g +
 h - i)*log(x + 1)
```

Sympy [A]
time = 1.60, size = 122, normalized size = 1.36

$$\frac{ix^4}{4}+x^3\left(\frac{h}{3}-i\right)+x^2\left(\frac{g}{2}-\frac{3h}{2}+\frac{7i}{2}\right)+x(f-3g+7h-15i)+(-d+2e-4f+8g-16h+32i)\log\left(x+\frac{4d-6e+10f-18g+34h-66i}{2d-3e+5f-9g+17h-33i}\right)+(d-e+f-g+h-i)\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),
x)
```

```
[Out] i*x**4/4 + x**3*(h/3 - i) + x**2*(g/2 - 3*h/2 + 7*i/2) + x*(f - 3*g + 7*h -
 15*i) + (-d + 2*e - 4*f + 8*g - 16*h + 32*i)*log(x + (4*d - 6*e + 10*f - 1
8*g + 34*h - 66*i)/(2*d - 3*e + 5*f - 9*g + 17*h - 33*i)) + (d - e + f - g
+ h - i)*log(x + 1)
```

Giac [A]
time = 4.42, size = 89, normalized size = 0.99

$$\frac{1}{3}hx^3+\frac{1}{4}i\,x^4+\frac{1}{2}gx^2-\frac{3}{2}hx^2-i\,x^3+fx-3\,gx+7\,hx+\frac{7}{2}i\,x^2-(d+4\,f-8\,g+16\,h-2\,e-32i)\log\left(|x+2|\right)+(d+f-g+h-e-i)\log\left(|x+1|\right)-15i\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algo
rithm="giac")
```

```
[Out] 1/3*h*x^3 + 1/4*I*x^4 + 1/2*g*x^2 - 3/2*h*x^2 - I*x^3 + f*x - 3*g*x + 7*h*x
 + 7/2*I*x^2 - (d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(abs(x + 2)) + (d + f
 - g + h - e - I)*log(abs(x + 1)) - 15*I*x
```

Mupad [B]
time = 0.08, size = 86, normalized size = 0.96

$$x^3\left(\frac{h}{3}-i\right)-\ln\left(x+2\right)\,\left(d-2\,e+4\,f-8\,g+16\,h-32\,i\right)+\ln\left(x+1\right)\,\left(d-e+f-g+h-i\right)+\frac{i\,x^4}{4}+x^2\left(\frac{g}{2}-\frac{3\,h}{2}+\frac{7\,i}{2}\right)+x\,\left(f-3\,g+7\,h-15\,i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^
2 + 4),x)
```

```
[Out] x^3*(h/3 - i) - log(x + 2)*(d - 2*e + 4*f - 8*g + 16*h - 32*i) + log(x + 1)
*(d - e + f - g + h - i) + (i*x^4)/4 + x^2*(g/2 - (3*h)/2 + (7*i)/2) + x*(f
 - 3*g + 7*h - 15*i)
```

3.79 $\int \frac{2+x}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=29

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(1+x)$$

```
[Out] -1/2*ln(1-x)+1/3*ln(2-x)+1/6*ln(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1600, 2083}

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x] + Log[2 - x]/3 + Log[1 + x]/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2083

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{4-5x^2+x^4}\,dx &= \int \frac{1}{2-x-2x^2+x^3}\,dx\\
&= \int \left(\frac{1}{3(-2+x)}-\frac{1}{2(-1+x)}+\frac{1}{6(1+x)}\right)dx\\
&= -\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$-\frac{1}{2}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{1}{6}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x] + Log[2 - x]/3 + Log[1 + x]/6
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.69

method	result	size
default	$\frac{\ln(x-2)}{3}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{6}$	20
norman	$\frac{\ln(x-2)}{3}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{6}$	20
risch	$\frac{\ln(x-2)}{3}-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{6}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(x-2)-1/2*ln(-1+x)+1/6*ln(1+x)
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.66

$$\frac{1}{6}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)+\frac{1}{3}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Fricas [A]
time = 0.51, size = 19, normalized size = 0.66

$$\frac{1}{6}\log\left(x+1\right)-\frac{1}{2}\log\left(x-1\right)+\frac{1}{3}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*log(x + 1) - 1/2*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.66

$$\frac{\log\left(x-2\right)}{3}-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/2 + log(x + 1)/6
```

Giac [A]
time = 6.26, size = 22, normalized size = 0.76

$$\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)+\frac{1}{3}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(x + 1)) - 1/2*log(abs(x - 1)) + 1/3*log(abs(x - 2))
```

Mupad [B]
time = 0.08, size = 19, normalized size = 0.66

$$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x-1\right)}{2}+\frac{\ln\left(x-2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x + 1)/6 - log(x - 1)/2 + log(x - 2)/3
```

3.80 $\int \frac{(2+x)(d+ex)}{4-5x^2+x^4} dx$

Optimal. Leaf size=42

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(1+x)$$

```
[Out] -1/2*(d+e)*ln(1-x)+1/3*(d+2*e)*ln(2-x)+1/6*(d-e)*ln(1+x)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 2099}

$$-\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/2*((d + e)*Log[1 - x]) + ((d + 2*e)*Log[2 - x])/3 + ((d - e)*Log[1 + x])
/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x)(d+ex)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex}{2-x-2x^2+x^3}\,dx\\
&= \int \left(\frac{d+2e}{3(-2+x)}+\frac{-d-e}{2(-1+x)}+\frac{d-e}{6(1+x)}\right)dx\\
&= -\frac{1}{2}(d+e)\log(1-x)+\frac{1}{3}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.93

$$\frac{1}{6}(-3(d+e)\log(1-x)+2(d+2e)\log(2-x)+(d-e)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4),x]
[Out] (-3*(d + e)*Log[1 - x] + 2*(d + 2*e)*Log[2 - x] + (d - e)*Log[1 + x])/6
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.90

method	result	size
default	$\left(\frac{d}{3}+\frac{2e}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{2}-\frac{e}{2}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)$	38
norman	$\left(\frac{d}{3}+\frac{2e}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{2}-\frac{e}{2}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln\left(1+x\right)$	38
risch	$\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] (1/3*d+2/3*e)*ln(x-2)+(-1/2*d-1/2*e)*ln(-1+x)+(1/6*d-1/6*e)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.83

$$\frac{1}{6}\left(d-e\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.42, size = 32, normalized size = 0.76

$$\frac{1}{6}\left(d-e\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2\,e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/6*(d - e)*log(x + 1) - 1/2*(d + e)*log(x - 1) + 1/3*(d + 2*e)*log(x - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(37) = 74$.
time = 1.20, size = 304, normalized size = 7.24

$$\frac{(d-e)\log\left(x+\frac{26d^3+66d^2e-9d^2(d-e)+78de^2-12de(d-e)-7d(d-e)^2+46e^3+3e^2(d-e)-8e(d-e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{6}-\frac{(d+e)\log\left(x+\frac{26d^3+66d^2e+27d^2(d+e)+78de^2+36de(d+e)-63d(d+e)^2+46e^3-9e^2(d+e)-72e(d+e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{2}+\frac{(d+2e)\log\left(x+\frac{26d^3+66d^2e-18d^2(d+2e)+78de^2-24de(d+2e)-28d(d+2e)^2+46e^3+6e^2(d+2e)-32e(d+2e)^2}{10d^3+69d^2e+102de^2+35e^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e)*log(x + (26*d**3 + 66*d**2*e - 9*d**2*(d - e) + 78*d*e**2 - 12*d*e*
(d - e) - 7*d*(d - e)**2 + 46*e**3 + 3*e**2*(d - e) - 8*e*(d - e)**2)/(10*d
**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/6 - (d + e)*log(x + (26*d**3 + 66*
d**2*e + 27*d**2*(d + e) + 78*d*e**2 + 36*d*e*(d + e) - 63*d*(d + e)**2 + 4
6*e**3 - 9*e**2*(d + e) - 72*e*(d + e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**
2 + 35*e**3))/2 + (d + 2*e)*log(x + (26*d**3 + 66*d**2*e - 18*d**2*(d + 2*e
) + 78*d*e**2 - 24*d*e*(d + 2*e) - 28*d*(d + 2*e)**2 + 46*e**3 + 6*e**2*(d
+ 2*e) - 32*e*(d + 2*e)**2)/(10*d**3 + 69*d**2*e + 102*d*e**2 + 35*e**3))/3
```

Giac [A]
time = 4.63, size = 38, normalized size = 0.90

$$\frac{1}{6}(d-e)\log(|x+1|)-\frac{1}{2}(d+e)\log(|x-1|)+\frac{1}{3}(d+2e)\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*(d - e)*log(abs(x + 1)) - 1/2*(d + e)*log(abs(x - 1)) + 1/3*(d + 2*e)*l
og(abs(x - 2))
```

Mupad [B]
time = 0.84, size = 38, normalized size = 0.90

$$\ln(x-2)\left(\frac{d}{3}+\frac{2e}{3}\right)-\ln(x-1)\left(\frac{d}{2}+\frac{e}{2}\right)+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] log(x - 2)*(d/3 + (2*e)/3) - log(x - 1)*(d/2 + e/2) + log(x + 1)*(d/6 - e/6
)
```

3.81 $\int \frac{(2+x)(d+ex+fx^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}(d+e+f)\log(1-x)+\frac{1}{3}(d+2e+4f)\log(2-x)+\frac{1}{6}(d-e+f)\log(1+x)$$

```
[Out] -1/2*(d+e+f)*ln(1-x)+1/3*(d+2*e+4*f)*ln(2-x)+1/6*(d-e+f)*ln(1+x)
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1600, 2099}

$$-\frac{1}{2}\log(1-x)(d+e+f)+\frac{1}{3}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] -1/2*((d + e + f)*Log[1 - x]) + ((d + 2*e + 4*f)*Log[2 - x])/3 + ((d - e + 
f)*Log[1 + x])/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x)\,(d+ex+fx^2)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2}{2-x-2x^2+x^3}\,dx\\
&= \int \left(\frac{d+2e+4f}{3(-2+x)}+\frac{-d-e-f}{2(-1+x)}+\frac{d-e+f}{6(1+x)}\right)dx\\
&= -\frac{1}{2}(d+e+f)\log(1-x)+\frac{1}{3}(d+2e+4f)\log(2-x)+\frac{1}{6}(d-e+f)\log(
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.94

$$\frac{1}{6}(-3(d+e+f)\log(1-x)+2(d+2e+4f)\log(2-x)+(d-e+f)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (-3*(d + e + f)*Log[1 - x] + 2*(d + 2*e + 4*f)*Log[2 - x] + (d - e + f)*Log
[1 + x])/6
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.00

method	result	size
default	$\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)$	47
norman	$\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right)\ln\left(x-2\right)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln\left(1+x\right)$	47
risch	$\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{4\ln(2-x)f}{3}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}-\frac{\ln(1-x)f}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] (1/3*d+2/3*e+4/3*f)*ln(x-2)+(-1/2*d-1/2*e-1/2*f)*ln(-1+x)+(1/6*d-1/6*e+1/6*
f)*ln(1+x)
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.85

$$\frac{1}{6}\left(d+f-e\right)\log\left(x+1\right)-\frac{1}{2}\left(d+f+e\right)\log\left(x-1\right)+\frac{1}{3}\left(d+4f+2e\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*(d + f - e)*log(x + 1) - 1/2*(d + f + e)*log(x - 1) + 1/3*(d + 4*f + 2*
e)*log(x - 2)
```

Fricas [A]
time = 0.41, size = 37, normalized size = 0.79

$$\frac{1}{6}\left(d-e+f\right)\log\left(x+1\right)-\frac{1}{2}\left(d+e+f\right)\log\left(x-1\right)+\frac{1}{3}\left(d+2e+4f\right)\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*(d - e + f)*log(x + 1) - 1/2*(d + e + f)*log(x - 1) + 1/3*(d + 2*e + 4*
f)*log(x - 2)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(49) = 98$.
time = 8.28, size = 716, normalized size = 15.23

$$\frac{(d-e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-9d^2(d-e+f)+78de^2+276def-12de(d-e+f)+222df^2+6df(d-e+f)-7d(d-e+f)^2+46e^3+204e^2f+3e^2(d-e+f)+282ef^2+36ef(d-e+f)-8e(d-e+f)^2+116f^3+51f^2(d-e+f)-13f(d-e+f)^2}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+285ef^2+154f^3}\right)}{6}-\frac{(d+e+f)\log\left(x+\frac{26d^3+66d^2e+132d^2f+27d^2(d+e+f)+78de^2+276def+36de(d+e+f)+222df^2-18df(d+e+f)-63d(d+e+f)^2+46e^3+204e^2f-9e^2(d+e+f)+282ef^2-108ef(d+e+f)-72e(d+e+f)^2+116f^3-153f^2(d+e+f)-117f(d+e+f)^2}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+285ef^2+154f^3}\right)}{2}+\frac{(d+2e+4f)\log\left(x+\frac{26d^3+66d^2e+132d^2f-18d^2(d+2e+4f)+78de^2+276def-24de(d+2e+4f)+222df^2+12df(d+2e+4f)-28d(d+2e+4f)^2+46e^3+204e^2f+6e^2(d+2e+4f)+282ef^2+72ef(d+2e+4f)-32e(d+2e+4f)^2+116f^3+102f^2(d+2e+4f)-52f(d+2e+4f)^2}{10d^3+69d^2e+102d^2f+102de^2+318def+246df^2+35e^3+174e^2f+285ef^2+154f^3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] (d - e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f - 9*d**2*(d - e + f)
+ 78*d*e**2 + 276*d*e*f - 12*d*e*(d - e + f) + 222*d*f**2 + 6*d*f*(d - e +
f) - 7*d*(d - e + f)**2 + 46*e**3 + 204*e**2*f + 3*e**2*(d - e + f) + 282*e
*f**2 + 36*e*f*(d - e + f) - 8*e*(d - e + f)**2 + 116*f**3 + 51*f**2*(d - e
 + f) - 13*f*(d - e + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2
 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/
6 - (d + e + f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 27*d**2*(d + e
+ f) + 78*d*e**2 + 276*d*e*f + 36*d*e*(d + e + f) + 222*d*f**2 - 18*d*f*(d
+ e + f) - 63*d*(d + e + f)**2 + 46*e**3 + 204*e**2*f - 9*e**2*(d + e + f)
+ 282*e*f**2 - 108*e*f*(d + e + f) - 72*e*(d + e + f)**2 + 116*f**3 - 153*f
**2*(d + e + f) - 117*f*(d + e + f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f +
 102*d*e**2 + 318*d*e*f + 246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 +
154*f**3))/2 + (d + 2*e + 4*f)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f -
18*d**2*(d + 2*e + 4*f) + 78*d*e**2 + 276*d*e*f - 24*d*e*(d + 2*e + 4*f) +
222*d*f**2 + 12*d*f*(d + 2*e + 4*f) - 28*d*(d + 2*e + 4*f)**2 + 46*e**3 + 2
04*e**2*f + 6*e**2*(d + 2*e + 4*f) + 282*e*f**2 + 72*e*f*(d + 2*e + 4*f) -
32*e*(d + 2*e + 4*f)**2 + 116*f**3 + 102*f**2*(d + 2*e + 4*f) - 52*f*(d + 2
*e + 4*f)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 102*d*e**2 + 318*d*e*f +
246*d*f**2 + 35*e**3 + 174*e**2*f + 285*e*f**2 + 154*f**3))/3
```

Giac [**A**]
time = 3.46, size = 43, normalized size = 0.91

$$\frac{1}{6}\left(d+f-e\right)\log\left(|x+1|\right)-\frac{1}{2}\left(d+f+e\right)\log\left(|x-1|\right)+\frac{1}{3}\left(d+4f+2e\right)\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*(d + f - e)*log(abs(x + 1)) - 1/2*(d + f + e)*log(abs(x - 1)) + 1/3*(d
+ 4*f + 2*e)*log(abs(x - 2))
```

Mupad [B]
time = 0.11, size = 47, normalized size = 1.00

$$\ln(x-2)\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}\right)-\ln(x-1)\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}\right)+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4),x)
[Out] log(x - 2)*(d/3 + (2*e)/3 + (4*f)/3) - log(x - 1)*(d/2 + e/2 + f/2) + log(x
 + 1)*(d/6 - e/6 + f/6)
```

3.82 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=57

$$gx-\frac{1}{2}(d+e+f+g)\log(1-x)+\frac{1}{3}(d+2e+4f+8g)\log(2-x)+\frac{1}{6}(d-e+f-g)\log(1+x)$$

```
[Out] g*x-1/2*(d+e+f+g)*ln(1-x)+1/3*(d+2*e+4*f+8*g)*ln(2-x)+1/6*(d-e+f-g)*ln(1+x)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1600, 2099}

$$-\frac{1}{2}\log(1-x)(d+e+f+g)+\frac{1}{3}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)+gx$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
```

```
[Out] g*x - ((d + e + f + g)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/3
 + ((d - e + f - g)*Log[1 + x])/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3}{2-x-2x^2+x^3}\,dx\\
&= \int \left(g+\frac{d+2e+4f+8g}{3(-2+x)}+\frac{-d-e-f-g}{2(-1+x)}+\frac{d-e+f-g}{6(1+x)}\right)dx\\
&= gx-\frac{1}{2}(d+e+f+g)\log(1-x)+\frac{1}{3}(d+2e+4f+8g)\log(2-x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.96

$$\frac{1}{6}(6gx-3(d+e+f+g)\log(1-x)+2(d+2e+4f+8g)\log(2-x)+(d-e+f-g)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4),x]
[Out] (6*g*x - 3*(d + e + f + g)*Log[1 - x] + 2*(d + 2*e + 4*f + 8*g)*Log[2 - x]
+ (d - e + f - g)*Log[1 + x])/6
```

Maple [A]
time = 0.04, size = 59, normalized size = 1.04

method	result
default	$gx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}\right)\ln(x-2)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}\right)\ln(-1+x)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)\ln(1+x)$
norman	$gx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}\right)\ln(x-2)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}\right)\ln(-1+x)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)\ln(1+x)$
risch	$gx+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(2-x)d}{3}+\frac{2\ln(2-x)e}{3}+\frac{4\ln(2-x)f}{3}+\frac{8\ln(2-x)g}{3}-\frac{\ln(1-x)d}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] g*x+(1/3*d+2/3*e+4/3*f+8/3*g)*ln(x-2)+(-1/2*d-1/2*e-1/2*f-1/2*g)*ln(-1+x)+(
1/6*d-1/6*e+1/6*f-1/6*g)*ln(1+x)
```

Maxima [A]
time = 0.31, size = 50, normalized size = 0.88

$$gx+\frac{1}{6}(d+f-g-e)\log(x+1)-\frac{1}{2}(d+f+g+e)\log(x-1)+\frac{1}{3}(d+4f+8g+2e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] g*x + 1/6*(d + f - g - e)*log(x + 1) - 1/2*(d + f + g + e)*log(x - 1) + 1/3
*(d + 4*f + 8*g + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.41, size = 47, normalized size = 0.82

$$gx+\frac{1}{6}(d-e+f-g)\log(x+1)-\frac{1}{2}(d+e+f+g)\log(x-1)+\frac{1}{3}(d+2e+4f+8g)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] g*x + 1/6*(d - e + f - g)*log(x + 1) - 1/2*(d + e + f + g)*log(x - 1) + 1/3
*(d + 2*e + 4*f + 8*g)*log(x - 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. $2(63) = 126$.
time = 59.41, size = 1389, normalized size = 24.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] g*x + (d - e + f - g)*log(x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*
g - 9*d**2*(d - e + f - g) + 78*d*e**2 + 276*d*e*f + 444*d*e*g - 12*d*e*(d
- e + f - g) + 222*d*f**2 + 636*d*f*g + 6*d*f*(d - e + f - g) + 510*d*g**2
+ 36*d*g*(d - e + f - g) - 7*d*(d - e + f - g)**2 + 46*e**3 + 204*e**2*f +
390*e**2*g + 3*e**2*(d - e + f - g) + 282*e*f**2 + 984*e*f*g + 36*e*f*(d -
e + f - g) + 930*e*g**2 + 102*e*g*(d - e + f - g) - 8*e*(d - e + f - g)**2
+ 116*f**3 + 534*f**2*g + 51*f**2*(d - e + f - g) + 924*f*g**2 + 228*f*g*(d
 - e + f - g) - 13*f*(d - e + f - g)**2 + 586*g**3 + 243*g**2*(d - e + f -
g) - 20*g*(d - e + f - g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*
g + 102*d*e**2 + 318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**
2 + 35*e**3 + 174*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2
 + 154*f**3 + 717*f**2*g + 966*f*g**2 + 323*g**3))/6 - (d + e + f + g)*log(
x + (26*d**3 + 66*d**2*e + 132*d**2*f + 174*d**2*g + 27*d**2*(d + e + f + g
) + 78*d*e**2 + 276*d*e*f + 444*d*e*g + 36*d*e*(d + e + f + g) + 222*d*f**2
 + 636*d*f*g - 18*d*f*(d + e + f + g) + 510*d*g**2 - 108*d*g*(d + e + f + g
) - 63*d*(d + e + f + g)**2 + 46*e**3 + 204*e**2*f + 390*e**2*g - 9*e**2*(d
 + e + f + g) + 282*e*f**2 + 984*e*f*g - 108*e*f*(d + e + f + g) + 930*e*g*
*2 - 306*e*g*(d + e + f + g) - 72*e*(d + e + f + g)**2 + 116*f**3 + 534*f**
2*g - 153*f**2*(d + e + f + g) + 924*f*g**2 - 684*f*g*(d + e + f + g) - 117
*f*(d + e + f + g)**2 + 586*g**3 - 729*g**2*(d + e + f + g) - 180*g*(d + e
+ f + g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g + 102*d*e**2 +
318*d*e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**2 + 35*e**3 + 174
*e**2*f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2 + 154*f**3 + 717
*f**2*g + 966*f*g**2 + 323*g**3))/2 + (d + 2*e + 4*f + 8*g)*log(x + (26*d**
3 + 66*d**2*e + 132*d**2*f + 174*d**2*g - 18*d**2*(d + 2*e + 4*f + 8*g) + 7
8*d*e**2 + 276*d*e*f + 444*d*e*g - 24*d*e*(d + 2*e + 4*f + 8*g) + 222*d*f**
2 + 636*d*f*g + 12*d*f*(d + 2*e + 4*f + 8*g) + 510*d*g**2 + 72*d*g*(d + 2*e
 + 4*f + 8*g) - 28*d*(d + 2*e + 4*f + 8*g)**2 + 46*e**3 + 204*e**2*f + 390*
e**2*g + 6*e**2*(d + 2*e + 4*f + 8*g) + 282*e*f**2 + 984*e*f*g + 72*e*f*(d
+ 2*e + 4*f + 8*g) + 930*e*g**2 + 204*e*g*(d + 2*e + 4*f + 8*g) - 32*e*(d +
 2*e + 4*f + 8*g)**2 + 116*f**3 + 534*f**2*g + 102*f**2*(d + 2*e + 4*f + 8*
g) + 924*f*g**2 + 456*f*g*(d + 2*e + 4*f + 8*g) - 52*f*(d + 2*e + 4*f + 8*g
)**2 + 586*g**3 + 486*g**2*(d + 2*e + 4*f + 8*g) - 80*g*(d + 2*e + 4*f + 8*
```

```
g)**2)/(10*d**3 + 69*d**2*e + 102*d**2*f + 213*d**2*g + 102*d*e**2 + 318*d*
e*f + 564*d*e*g + 246*d*f**2 + 894*d*f*g + 750*d*g**2 + 35*e**3 + 174*e**2*
f + 249*e**2*g + 285*e*f**2 + 852*e*f*g + 537*e*g**2 + 154*f**3 + 717*f**2*
g + 966*f*g**2 + 323*g**3))/3
```

Giac [A]
time = 4.44, size = 53, normalized size = 0.93

$$gx + \frac{1}{6}(d + f - g - e)\log(|x + 1|) - \frac{1}{2}(d + f + g + e)\log(|x - 1|) + \frac{1}{3}(d + 4f + 8g + 2e)\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] g*x + 1/6*(d + f - g - e)*log(abs(x + 1)) - 1/2*(d + f + g + e)*log(abs(x -
 1)) + 1/3*(d + 4*f + 8*g + 2*e)*log(abs(x - 2))
```

Mupad [B]
time = 0.82, size = 59, normalized size = 1.04

$$\ln(x+1)\left(\frac{d}{6} - \frac{e}{6} + \frac{f}{6} - \frac{g}{6}\right) - \ln(x-1)\left(\frac{d}{2} + \frac{e}{2} + \frac{f}{2} + \frac{g}{2}\right) + \ln(x-2)\left(\frac{d}{3} + \frac{2e}{3} + \frac{4f}{3} + \frac{8g}{3}\right) + g\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4),x)
[Out] log(x + 1)*(d/6 - e/6 + f/6 - g/6) - log(x - 1)*(d/2 + e/2 + f/2 + g/2) + l
og(x - 2)*(d/3 + (2*e)/3 + (4*f)/3 + (8*g)/3) + g*x
```

3.83 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$(g+2h)x+\frac{hx^2}{2}-\frac{1}{2}(d+e+f+g+h)\log(1-x)+\frac{1}{3}(d+2e+4f+8g+16h)\log(2-x)+\frac{1}{6}(d-e+f-g+h)\log(1+x)$$

```
[Out] (g+2*h)*x+1/2*h*x^2-1/2*(d+e+f+g+h)*ln(1-x)+1/3*(d+2*e+4*f+8*g+16*h)*ln(2-x
)+1/6*(d-e+f-g+h)*ln(1+x)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1600, 2099}

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)+x(g+2h)+\frac{hx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (g + 2*h)*x + (h*x^2)/2 - ((d + e + f + g + h)*Log[1 - x])/2 + ((d + 2*e +
4*f + 8*g + 16*h)*Log[2 - x])/3 + ((d - e + f - g + h)*Log[1 + x])/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{4-5x^2+x^4}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{2-x-2x^2+x^3}\,dx$$

$$= \int \left(g\left(1+\frac{2h}{g}\right)+\frac{d+2e+4f+8g+16h}{3(-2+x)}+\frac{-d-e-f-g}{2(-1+x)}\right.$$

$$= (g+2h)x+\frac{hx^2}{2}-\frac{1}{2}(d+e+f+g+h)\log(1-x)+\frac{1}{3}(d+2$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 0.96

$$\frac{1}{6}(6(g+2h)x+3hx^2-3(d+e+f+g+h)\log(1-x)+2(d+2(e+2f+4g+8h))\log(2-x)+(d-e+f-g+h)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4),x]
```

```
[Out] (6*(g + 2*h)*x + 3*h*x^2 - 3*(d + e + f + g + h)*Log[1 - x] + 2*(d + 2*(e +
 2*f + 4*g + 8*h))*Log[2 - x] + (d - e + f - g + h)*Log[1 + x])/6
```

Maple [A]
time = 0.04, size = 78, normalized size = 1.05

method	result
default	$\frac{hx^2}{2}+gx+2hx+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}\right)\ln(x-2)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}-\frac{h}{2}\right)\ln(-1+x)+\left(\frac{d}{6}\right.$
norman	$(g+2h)x+\frac{hx^2}{2}+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}-\frac{h}{2}\right)\ln(-1+x)+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}\right)\ln(x-2)+\left(\frac{d}{6}\right.$
risch	$\frac{hx^2}{2}+gx+2hx-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}-\frac{\ln(1-x)f}{2}-\frac{\ln(1-x)g}{2}-\frac{\ln(1-x)h}{2}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\frac{\ln(1+x)f}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*h*x^2+g*x+2*h*x+(1/3*d+2/3*e+4/3*f+8/3*g+16/3*h)*ln(x-2)+(-1/2*d-1/2*e-
1/2*f-1/2*g-1/2*h)*ln(-1+x)+(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.88

$$\frac{1}{2}hx^2+(g+2h)x+\frac{1}{6}(d+f-g+h-e)\log(x+1)-\frac{1}{2}(d+f+g+h+e)\log(x-1)+\frac{1}{3}(d+4f+8g+16h+2e)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="maxim
a")
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d + f - g + h - e)*log(x + 1) - 1/2*(d + f +
 g + h + e)*log(x - 1) + 1/3*(d + 4*f + 8*g + 16*h + 2*e)*log(x - 2)
```

Fricas [A]
time = 0.42, size = 62, normalized size = 0.84

$$\frac{1}{2}hx^2+(g+2h)x+\frac{1}{6}(d-e+f-g+h)\log(x+1)-\frac{1}{2}(d+e+f+g+h)\log(x-1)+\frac{1}{3}(d+2e+4f+8g+16h)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="frica
s")
```

```
[Out] 1/2*h*x^2 + (g + 2*h)*x + 1/6*(d - e + f - g + h)*log(x + 1) - 1/2*(d + e +
 f + g + h)*log(x - 1) + 1/3*(d + 2*e + 4*f + 8*g + 16*h)*log(x - 2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.19, size = 68, normalized size = 0.92

$$\frac{1}{2}hx^2 + gx + 2hx + \frac{1}{6}(d + f - g + h - e)\log(|x + 1|) - \frac{1}{2}(d + f + g + h + e)\log(|x - 1|) + \frac{1}{3}(d + 4f + 8g + 16h + 2e)\log(|x - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm="giac"
)
```

```
[Out] 1/2*h*x^2 + g*x + 2*h*x + 1/6*(d + f - g + h - e)*log(abs(x + 1)) - 1/2*(d
+ f + g + h + e)*log(abs(x - 1)) + 1/3*(d + 4*f + 8*g + 16*h + 2*e)*log(abs
(x - 2))
```

Mupad [**B**]
time = 0.88, size = 78, normalized size = 1.05

$$x(g + 2h) + \frac{hx^2}{2} - \ln(x - 1)\left(\frac{d}{2} + \frac{e}{2} + \frac{f}{2} + \frac{g}{2} + \frac{h}{2}\right) + \ln(x + 1)\left(\frac{d}{6} - \frac{e}{6} + \frac{f}{6} - \frac{g}{6} + \frac{h}{6}\right) + \ln(x - 2)\left(\frac{d}{3} + \frac{2e}{3} + \frac{4f}{3} + \frac{8g}{3} + \frac{16h}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4),x)
```

```
[Out] x*(g + 2*h) + (h*x^2)/2 - log(x - 1)*(d/2 + e/2 + f/2 + g/2 + h/2) + log(x
+ 1)*(d/6 - e/6 + f/6 - g/6 + h/6) + log(x - 2)*(d/3 + (2*e)/3 + (4*f)/3 +
(8*g)/3 + (16*h)/3)
```

3.84 $\int \frac{(2+x)(d+ex+fx^2+gx^3+hx^4+ix^5)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=96

$$(g+2h+5i)x+\frac{1}{2}(h+2i)x^2+\frac{ix^3}{3}-\frac{1}{2}(d+e+f+g+h+i)\log(1-x)+\frac{1}{3}(d+2e+4f+8g+16h+32i)\log(2-x)+\frac{1}{6}(d-$$

```
[Out] (g+2*h+5*i)*x+1/2*(h+2*i)*x^2+1/3*i*x^3-1/2*(d+e+f+g+h+i)*ln(1-x)+1/3*(d+2*
e+4*f+8*g+16*h+32*i)*ln(2-x)+1/6*(d-e+f-g+h-i)*ln(1+x)
```

Rubi [A]

time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {1600, 2099}

$$-\frac{1}{2}\log(1-x)(d+e+f+g+h+i)+\frac{1}{3}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)+x(g+2h+5i)+\frac{1}{2}x^2(h+2i)+\frac{ix^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4),x
]
```

```
[Out] (g + 2*h + 5*i)*x + ((h + 2*i)*x^2)/2 + (i*x^3)/3 - ((d + e + f + g + h + i
)*Log[1 - x])/2 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/3 + ((d
- e + f - g + h - i)*Log[1 + x])/6
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+84x^5\right)}{4-5x^2+x^4}\,dx &= \int \frac{d+ex+fx^2+gx^3+hx^4+84x^5}{2-x-2x^2+x^3}\,dx\\ &= \int \left(420\left(1+\frac{1}{420}(g+2h)\right)+\frac{2688+d+2e+4f+8g}{3(-2+x)}\right.\\ &= (420+g+2h)x+\frac{1}{2}(168+h)x^2+28x^3-\frac{1}{2}(84+d+e\end{aligned}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.95

$$\frac{1}{6}(6(g+2h+5i)x+3(h+2i)x^2+2ix^3-3(d+e+f+g+h+i)\log(1-x)+2(d+2e+4(f+2g+4h+8i))\log(2-x)+(d-e+f-g+h-i)\log(1+x))$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
x^4),x]
```

```
[Out] (6*(g + 2*h + 5*i)*x + 3*(h + 2*i)*x^2 + 2*i*x^3 - 3*(d + e + f + g + h + i
)*Log[1 - x] + 2*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + (d - e +
f - g + h - i)*Log[1 + x])/6
```

Maple [A]
time = 0.04, size = 102, normalized size = 1.06

method	result
norman	$\left(\frac{h}{2}+i\right)x^2+(g+2h+5i)x+\frac{i\,x^3}{3}+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\frac{g}{2}-\frac{h}{2}-\frac{i}{2}\right)\ln(-1+x)+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8}{3}\right.$
default	$\frac{i\,x^3}{3}+\frac{h\,x^2}{2}+i\,x^2+gx+2hx+5ix+\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}+\frac{32i}{3}\right)\ln(x-2)+\left(-\frac{d}{2}-\frac{e}{2}-\frac{f}{2}-\right.$
risch	$\frac{i\,x^3}{3}+\frac{h\,x^2}{2}+i\,x^2+gx+2hx+5ix-\frac{\ln(1-x)d}{2}-\frac{\ln(1-x)e}{2}-\frac{\ln(1-x)f}{2}-\frac{\ln(1-x)g}{2}-\frac{\ln(1-x)h}{2}-\frac{\ln(1-x)i}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/3*i*x^3+1/2*h*x^2+i*x^2+g*x+2*h*x+5*i*x+(1/3*d+2/3*e+4/3*f+8/3*g+16/3*h+3
2/3*i)*ln(x-2)+(-1/2*d-1/2*e-1/2*f-1/2*g-1/2*h-1/2*i)*ln(-1+x)+(1/6*d-1/6*e
+1/6*f-1/6*g+1/6*h-1/6*i)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.79

$$\frac{1}{2}(h+2i)x^2+\frac{1}{3}ix^3+(g+2h+5i)x+\frac{1}{6}(d+f-g+h-e-i)\log(x+1)-\frac{1}{2}(d+f+g+h+e+i)\log(x-1)+\frac{1}{3}(d+4f+8g+16h+2e+32i)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"maxima")
```

```
[Out] 1/2*(h + 2*I)*x^2 + 1/3*I*x^3 + (g + 2*h + 5*I)*x + 1/6*(d + f - g + h - e
- I)*log(x + 1) - 1/2*(d + f + g + h + e + I)*log(x - 1) + 1/3*(d + 4*f + 8
*g + 16*h + 2*e + 32*I)*log(x - 2)
```

Fricas [A]
time = 0.45, size = 82, normalized size = 0.85

$$\frac{1}{3}ix^3+\frac{1}{2}(h+2i)x^2+(g+2h+5i)x+\frac{1}{6}(d-e+f-g+h-i)\log(x+1)-\frac{1}{2}(d+e+f+g+h+i)\log(x-1)+\frac{1}{3}(d+2e+4f+8g+16h+32i)\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"fricas")
```

```
[Out] 1/3*i*x^3 + 1/2*(h + 2*i)*x^2 + (g + 2*h + 5*i)*x + 1/6*(d - e + f - g + h
- i)*log(x + 1) - 1/2*(d + e + f + g + h + i)*log(x - 1) + 1/3*(d + 2*e + 4
*f + 8*g + 16*h + 32*i)*log(x - 2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.56, size = 84, normalized size = 0.88

$$\frac{1}{2}hx^2+\frac{1}{3}ix^3+gx+2hx+ix^2+\frac{1}{6}(d+f-g+h-e-i)\log(|x+1|)-\frac{1}{2}(d+f+g+h+e+i)\log(|x-1|)+\frac{1}{3}(d+4f+8g+16h+2e+32i)\log(|x-2|)+5ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4),x, algorithm=
"giac")
```

```
[Out] 1/2*h*x^2 + 1/3*I*x^3 + g*x + 2*h*x + I*x^2 + 1/6*(d + f - g + h - e - I)*l
og(abs(x + 1)) - 1/2*(d + f + g + h + e + I)*log(abs(x - 1)) + 1/3*(d + 4*f
 + 8*g + 16*h + 2*e + 32*I)*log(abs(x - 2)) + 5*I*x
```

Mupad [**B**]
time = 0.88, size = 99, normalized size = 1.03

$$x(g+2h+5i)+\frac{ix^3}{3}-\ln(x-1)\left(\frac{d}{2}+\frac{e}{2}+\frac{f}{2}+\frac{g}{2}+\frac{h}{2}+\frac{i}{2}\right)+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)+\ln(x-2)\left(\frac{d}{3}+\frac{2e}{3}+\frac{4f}{3}+\frac{8g}{3}+\frac{16h}{3}+\frac{32i}{3}\right)+x^2\left(\frac{h}{2}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^2 + 4),x
)
```

```
[Out] x*(g + 2*h + 5*i) + (i*x^3)/3 - log(x - 1)*(d/2 + e/2 + f/2 + g/2 + h/2 + i
/2) + log(x + 1)*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/3 + (2
*e)/3 + (4*f)/3 + (8*g)/3 + (16*h)/3 + (32*i)/3) + x^2*(h/2 + i)
```

3.85 $\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{12(2+x)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(1+x)-\frac{19}{144}\log(2+x)$$

```
[Out] 1/12/(2+x)-1/18*ln(1-x)+1/48*ln(2-x)+1/6*ln(1+x)-19/144*ln(2+x)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1600, 2099}

$$\frac{1}{12(x+2)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{19}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(2 + x)) - Log[1 - x]/18 + Log[2 - x]/48 + Log[1 + x]/6 - (19*Log[2 +
 x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x-2x^2+x^3}{(4-5x^2+x^4)^2}\,dx &= \int \frac{1}{(2+x)^2(2-x-2x^2+x^3)}\,dx\\ &= \int \left(\frac{1}{48(-2+x)}-\frac{1}{18(-1+x)}+\frac{1}{6(1+x)}-\frac{1}{12(2+x)^2}-\frac{19}{144(2+x)}\right)dx\\ &= \frac{1}{12(2+x)}-\frac{1}{18}\log(1-x)+\frac{1}{48}\log(2-x)+\frac{1}{6}\log(1+x)-\frac{19}{144}\log(2+x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 0.91

$$\frac{1}{144}\left(\frac{12}{2+x}+24\log(-1-x)-8\log(1-x)+3\log(2-x)-19\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x - 2*x^2 + x^3)/(4 - 5*x^2 + x^4)^2,x]
[Out] (12/(2 + x) + 24*Log[-1 - x] - 8*Log[1 - x] + 3*Log[2 - x] - 19*Log[2 + x])
/144
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.72

method	result	size
default	$\frac{1}{12x+24}-\frac{19\ln(x+2)}{144}+\frac{\ln(x-2)}{48}-\frac{\ln(-1+x)}{18}+\frac{\ln(1+x)}{6}$	33
risch	$\frac{1}{12x+24}-\frac{19\ln(x+2)}{144}+\frac{\ln(x-2)}{48}-\frac{\ln(-1+x)}{18}+\frac{\ln(1+x)}{6}$	33
norman	$\frac{-\frac{1}{6}x^2-\frac{1}{12}x+\frac{1}{12}x^3+\frac{1}{6}}{x^4-5x^2+4}+\frac{\ln(x-2)}{48}-\frac{\ln(-1+x)}{18}+\frac{\ln(1+x)}{6}-\frac{19\ln(x+2)}{144}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
[Out] 1/12/(x+2)-19/144*ln(x+2)+1/48*ln(x-2)-1/18*ln(-1+x)+1/6*ln(1+x)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 0.70

$$\frac{1}{12\,(x+2)}-\frac{19}{144}\,\log\left(x+2\right)+\frac{1}{6}\,\log\left(x+1\right)-\frac{1}{18}\,\log\left(x-1\right)+\frac{1}{48}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
[Out] 1/12/(x + 2) - 19/144*log(x + 2) + 1/6*log(x + 1) - 1/18*log(x - 1) + 1/48*
log(x - 2)
```

Fricas [A]
time = 0.39, size = 45, normalized size = 0.98

$$-\frac{19\,(x+2)\log\left(x+2\right)-24\,(x+2)\log\left(x+1\right)+8\,(x+2)\log\left(x-1\right)-3\,(x+2)\log\left(x-2\right)-12}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(19*(x + 2)*log(x + 2) - 24*(x + 2)*log(x + 1) + 8*(x + 2)*log(x - 1
) - 3*(x + 2)*log(x - 2) - 12)/(x + 2)
```

Sympy [A]
time = 0.11, size = 34, normalized size = 0.74

$$\frac{\log\left(x-2\right)}{48}-\frac{\log\left(x-1\right)}{18}+\frac{\log\left(x+1\right)}{6}-\frac{19\log\left(x+2\right)}{144}+\frac{1}{12x+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] log(x - 2)/48 - log(x - 1)/18 + log(x + 1)/6 - 19*log(x + 2)/144 + 1/(12*x
+ 24)
```

Giac [A]
time = 3.03, size = 36, normalized size = 0.78

$$\frac{1}{12\left(x+2\right)}-\frac{19}{144}\log\left(|x+2|\right)+\frac{1}{6}\log\left(|x+1|\right)-\frac{1}{18}\log\left(|x-1|\right)+\frac{1}{48}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/12/(x + 2) - 19/144*log(abs(x + 2)) + 1/6*log(abs(x + 1)) - 1/18*log(abs(
x - 1)) + 1/48*log(abs(x - 2))
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.70

$$\frac{\ln\left(x+1\right)}{6}-\frac{\ln\left(x-1\right)}{18}+\frac{\ln\left(x-2\right)}{48}-\frac{19\,\ln\left(x+2\right)}{144}+\frac{1}{12\,\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2*x^2 - x^3 - 2)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x + 1)/6 - log(x - 1)/18 + log(x - 2)/48 - (19*log(x + 2))/144 + 1/(12*
(x + 2))
```

3.86 $\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=71

$$\frac{d-2e}{12(2+x)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(1+x)-\frac{1}{144}(19d-26e)\log(2+x)$$

```
[Out] 1/12*(d-2*e)/(2+x)-1/18*(d+e)*ln(1-x)+1/48*(d+2*e)*ln(2-x)+1/6*(d-e)*ln(1+x
)-1/144*(19*d-26*e)*ln(2+x)
```

Rubi [A]
time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1600, 6874}

$$\frac{d-2e}{12(x+2)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log(x+1)-\frac{1}{144}(19d-26e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e)/(12*(2 + x)) - ((d + e)*Log[1 - x])/18 + ((d + 2*e)*Log[2 - x])/4
8 + ((d - e)*Log[1 + x])/6 - ((19*d - 26*e)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx &= \int \frac{d+ex}{(2+x)^2(2-x-2x^2+x^3)}\,dx\\ &= \int \left(\frac{d+2e}{48(-2+x)}+\frac{-d-e}{18(-1+x)}+\frac{d-e}{6(1+x)}+\frac{-d+2e}{12(2+x)^2}+\frac{-19d+26e}{144(2+x)}\right.\\ &= \frac{d-2e}{12(2+x)}-\frac{1}{18}(d+e)\log(1-x)+\frac{1}{48}(d+2e)\log(2-x)+\frac{1}{6}(d-e)\log\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.93

$$\frac{1}{144}\left(\frac{12(d-2e)}{2+x}+24(d-e)\log(-1-x)-8(d+e)\log(1-x)+3(d+2e)\log(2-x)+(-19d+26e)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e))/(2 + x) + 24*(d - e)*Log[-1 - x] - 8*(d + e)*Log[1 - x] + 3
*(d + 2*e)*Log[2 - x] + (-19*d + 26*e)*Log[2 + x])/144
```

Maple [A]
time = 0.04, size = 64, normalized size = 0.90

method	result
default	$\left(-\frac{19d}{144}+\frac{13e}{72}\right)\ln\left(x+2\right)-\frac{-\frac{d}{12}+\frac{e}{6}}{x+2}+\left(\frac{d}{48}+\frac{e}{24}\right)\ln\left(x-2\right)+\left(-\frac{d}{18}-\frac{e}{18}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\frac{e}{6}\right)\ln($
risch	$\frac{d}{12x+24}-\frac{e}{6(x+2)}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)e}{24}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}-\frac{19\ln(-x-2)d}{144}+$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}\right)x+\left(\frac{d}{12}-\frac{e}{6}\right)x^3+\left(-\frac{d}{6}+\frac{e}{3}\right)x^2+\frac{d}{6}-\frac{e}{3}}{x^4-5x^2+4}+\left(-\frac{19d}{144}+\frac{13e}{72}\right)\ln\left(x+2\right)+\left(-\frac{d}{18}-\frac{e}{18}\right)\ln\left(-1+x\right)+\left(\frac{d}{6}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-19/144*d+13/72*e)*ln(x+2)-(-1/12*d+1/6*e)/(x+2)+(1/48*d+1/24*e)*ln(x-2)+(
-1/18*d-1/18*e)*ln(-1+x)+(1/6*d-1/6*e)*ln(1+x)
```

Maxima [A]
time = 0.29, size = 62, normalized size = 0.87

$$-\frac{1}{144}\left(19\,d-26\,e\right)\log\left(x+2\right)+\frac{1}{6}\left(d-e\right)\log\left(x+1\right)-\frac{1}{18}\left(d+e\right)\log\left(x-1\right)+\frac{1}{48}\left(d+2\,e\right)\log\left(x-2\right)+\frac{d-2\,e}{12\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/144*(19*d - 26*e)*log(x + 2) + 1/6*(d - e)*log(x + 1) - 1/18*(d + e)*log
(x - 1) + 1/48*(d + 2*e)*log(x - 2) + 1/12*(d - 2*e)/(x + 2)
```

Fricas [A]
time = 0.41, size = 93, normalized size = 1.31

$$-\frac{\left(\left(19\,d-26\,e\right)x+38\,d-52\,e\right)\log\left(x+2\right)-24\left(\left(d-e\right)x+2\,d-2\,e\right)\log\left(x+1\right)+8\left(\left(d+e\right)x+2\,d+2\,e\right)\log\left(x-1\right)-3\left(\left(d+2\,e\right)x+2\,d+4\,e\right)\log\left(x-2\right)-12\,d+24\,e}{144\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(((19*d - 26*e)*x + 38*d - 52*e)*log(x + 2) - 24*((d - e)*x + 2*d -
2*e)*log(x + 1) + 8*((d + e)*x + 2*d + 2*e)*log(x - 1) - 3*((d + 2*e)*x + 2
*d + 4*e)*log(x - 2) - 12*d + 24*e)/(x + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(60) = 120$.

time = 7.55, size = 1188, normalized size = 16.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (d - 2*e)/(12*x + 24) + (d - e)*log(x + (-1534775*d**6 + 8032360*d**5*e - 9
84027*d**5*(d - e) - 12991180*d**4*e**2 + 11797266*d**4*e*(d - e) + 3567168
*d**4*(d - e)**2 + 1075200*d**3*e**3 - 32721528*d**3*e**2*(d - e) - 8725248
*d**3*e*(d - e)**2 - 247104*d**3*(d - e)**3 + 16959280*d**2*e**4 + 38977296
*d**2*e**3*(d - e) - 2820096*d**2*e**2*(d - e)**2 - 10357632*d**2*e*(d - e)
**3 - 15836800*d*e**5 - 21294960*d*e**4*(d - e) + 15436800*d*e**3*(d - e)**
2 + 16277760*d*e**2*(d - e)**3 + 4283840*e**6 + 3876000*e**5*(d - e) - 6865
920*e**4*(d - e)**2 - 4078080*e**3*(d - e)**3)/(801262*d**6 - 4662251*d**5*
e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990800*d*
e**5 - 2380000*e**6))/6 - (d + e)*log(x + (-1534775*d**6 + 8032360*d**5*e +
 328009*d**5*(d + e) - 12991180*d**4*e**2 - 3932422*d**4*e*(d + e) + 396352
*d**4*(d + e)**2 + 1075200*d**3*e**3 + 10907176*d**3*e**2*(d + e) - 969472*
d**3*e*(d + e)**2 + 9152*d**3*(d + e)**3 + 16959280*d**2*e**4 - 12992432*d*
*2*e**3*(d + e) - 313344*d**2*e**2*(d + e)**2 + 383616*d**2*e*(d + e)**3 -
15836800*d*e**5 + 7098320*d*e**4*(d + e) + 1715200*d*e**3*(d + e)**2 - 6028
80*d*e**2*(d + e)**3 + 4283840*e**6 - 1292000*e**5*(d + e) - 762880*e**4*(d
 + e)**2 + 151040*e**3*(d + e)**3)/(801262*d**6 - 4662251*d**5*e + 7296938*
d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9990800*d*e**5 - 23800
00*e**6))/18 + (d + 2*e)*log(x + (-1534775*d**6 + 8032360*d**5*e - 984027*d
**5*(d + 2*e)/8 - 12991180*d**4*e**2 + 5898633*d**4*e*(d + 2*e)/4 + 55737*d
**4*(d + 2*e)**2 + 1075200*d**3*e**3 - 4090191*d**3*e**2*(d + 2*e) - 136332
*d**3*e*(d + 2*e)**2 - 3861*d**3*(d + 2*e)**3/8 + 16959280*d**2*e**4 + 4872
162*d**2*e**3*(d + 2*e) - 44064*d**2*e**2*(d + 2*e)**2 - 80919*d**2*e*(d +
2*e)**3/4 - 15836800*d*e**5 - 2661870*d*e**4*(d + 2*e) + 241200*d*e**3*(d +
 2*e)**2 + 63585*d*e**2*(d + 2*e)**3/2 + 4283840*e**6 + 484500*e**5*(d + 2*
e) - 107280*e**4*(d + 2*e)**2 - 7965*e**3*(d + 2*e)**3)/(801262*d**6 - 4662
251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9
990800*d*e**5 - 2380000*e**6))/48 - (19*d - 26*e)*log(x + (-1534775*d**6 +
8032360*d**5*e + 328009*d**5*(19*d - 26*e)/8 - 12991180*d**4*e**2 - 1966211
*d**4*e*(19*d - 26*e)/4 + 6193*d**4*(19*d - 26*e)**2 + 1075200*d**3*e**3 +
1363397*d**3*e**2*(19*d - 26*e) - 15148*d**3*e*(19*d - 26*e)**2 + 143*d**3*
```

```
(19*d - 26*e)**3/8 + 16959280*d**2*e**4 - 1624054*d**2*e**3*(19*d - 26*e) -
 4896*d**2*e**2*(19*d - 26*e)**2 + 2997*d**2*e*(19*d - 26*e)**3/4 - 1583680
0*d*e**5 + 887290*d*e**4*(19*d - 26*e) + 26800*d*e**3*(19*d - 26*e)**2 - 23
55*d*e**2*(19*d - 26*e)**3/2 + 4283840*e**6 - 161500*e**5*(19*d - 26*e) - 1
1920*e**4*(19*d - 26*e)**2 + 295*e**3*(19*d - 26*e)**3)/(801262*d**6 - 4662
251*d**5*e + 7296938*d**4*e**2 + 1388616*d**3*e**3 - 12447440*d**2*e**4 + 9
990800*d*e**5 - 2380000*e**6))/144
```

Giac [A]
time = 3.77, size = 66, normalized size = 0.93

$$-\frac{1}{144}\,(19\,d-26\,e)\log\left(|x+2|\right)+\frac{1}{6}\,(d-e)\log\left(|x+1|\right)-\frac{1}{18}\,(d+e)\log\left(|x-1|\right)+\frac{1}{48}\,(d+2\,e)\log\left(|x-2|\right)+\frac{d-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/144*(19*d - 26*e)*log(abs(x + 2)) + 1/6*(d - e)*log(abs(x + 1)) - 1/18*(
d + e)*log(abs(x - 1)) + 1/48*(d + 2*e)*log(abs(x - 2)) + 1/12*(d - 2*e)/(x
 + 2)
```

Mupad [B]
time = 0.81, size = 64, normalized size = 0.90

$$\frac{\frac{d}{12}-\frac{e}{6}}{x+2}+\ln\left(x+1\right)\,\left(\frac{d}{6}-\frac{e}{6}\right)-\ln\left(x-1\right)\,\left(\frac{d}{18}+\frac{e}{18}\right)+\ln\left(x-2\right)\,\left(\frac{d}{48}+\frac{e}{24}\right)-\ln\left(x+2\right)\,\left(\frac{19\,d}{144}-\frac{13\,e}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] (d/12 - e/6)/(x + 2) + log(x + 1)*(d/6 - e/6) - log(x - 1)*(d/18 + e/18) +
log(x - 2)*(d/48 + e/24) - log(x + 2)*((19*d)/144 - (13*e)/72)
```

3.87 $\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=82

$$\frac{d-2e+4f}{12(2+x)}-\frac{1}{18}(d+e+f)\log(1-x)+\frac{1}{48}(d+2e+4f)\log(2-x)+\frac{1}{6}(d-e+f)\log(1+x)-\frac{1}{144}(19d-26e+28f)\log$$

```
[Out] 1/12*(d-2*e+4*f)/(2+x)-1/18*(d+e+f)*ln(1-x)+1/48*(d+2*e+4*f)*ln(2-x)+1/6*(d
-e+f)*ln(1+x)-1/144*(19*d-26*e+28*f)*ln(2+x)
```

Rubi [A]

time = 0.13, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1600, 6874}

$$\frac{d-2e+4f}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f)+\frac{1}{48}\log(2-x)(d+2e+4f)+\frac{1}{6}\log(x+1)(d-e+f)-\frac{1}{144}\log(x+2)(19d-26e+28f)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f)/(12*(2 + x)) - ((d + e + f)*Log[1 - x])/18 + ((d + 2*e + 4*
f)*Log[2 - x])/48 + ((d - e + f)*Log[1 + x])/6 - ((19*d - 26*e + 28*f)*Log[
2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex+fx^2)(2-x-2x^2+x^3)}{(4-5x^2+x^4)^2}\,dx &= \int \frac{d+ex+fx^2}{(2+x)^2(2-x-2x^2+x^3)}\,dx\\ &= \int \left(\frac{d+2e+4f}{48(-2+x)}+\frac{-d-e-f}{18(-1+x)}+\frac{d-e+f}{6(1+x)}+\frac{-d+2e-4f}{12(2+x)^2}+\right.\\ &= \frac{d-2e+4f}{12(2+x)}-\frac{1}{18}(d+e+f)\log(1-x)+\frac{1}{48}(d+2e+4f)\log(2\end{aligned}$$

Mathematica [A]
time = 0.04, size = 77, normalized size = 0.94

$$\frac{1}{144}\left(\frac{12(d-2e+4f)}{2+x}+24(d-e+f)\log(-1-x)-8(d+e+f)\log(1-x)+3(d+2e+4f)\log(2-x)+(-19d+26e-28f)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x + f*x^2)*(2 - x - 2*x^2 + x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f))/(2 + x) + 24*(d - e + f)*Log[-1 - x] - 8*(d + e + f)*
Log[1 - x] + 3*(d + 2*e + 4*f)*Log[2 - x] + (-19*d + 26*e - 28*f)*Log[2 + x
])/144
```

Maple [A]
time = 0.04, size = 79, normalized size = 0.96

method	result
default	$\left(\frac{13e}{72}-\frac{7f}{36}-\frac{19d}{144}\right)\ln(x+2)-\frac{-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}}{x+2}+\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}\right)\ln(x-2)+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}\right)\ln(-1+x$
risch	$\frac{d}{12x+24}-\frac{e}{6(x+2)}+\frac{f}{3x+6}+\frac{13\ln(-x-2)e}{72}-\frac{7\ln(-x-2)f}{36}-\frac{19\ln(-x-2)d}{144}-\frac{\ln(-1+x)d}{18}-\frac{\ln(-1+x)e}{18}-\frac{\ln(-1+x)}{18}$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}\right)x^3+\left(\frac{e}{3}-\frac{2f}{3}-\frac{d}{6}\right)x^2-\frac{e}{3}+\frac{2f}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}\right)\ln(-1+x)+\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (13/72*e-7/36*f-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/3*f)/(x+2)+(1/48*d+1/24*
e+1/12*f)*ln(x-2)+(-1/18*d-1/18*e-1/18*f)*ln(-1+x)+(1/6*d-1/6*e+1/6*f)*ln(1
+x)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.89

$$-\frac{1}{144}\,(19\,d+28\,f-26\,e)\log(x+2)+\frac{1}{6}\,(d+f-e)\log(x+1)-\frac{1}{18}\,(d+f+e)\log(x-1)+\frac{1}{48}\,(d+4\,f+2\,e)\log(x-2)+\frac{d+4\,f-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxim
a")
```

```
[Out] -1/144*(19*d + 28*f - 26*e)*log(x + 2) + 1/6*(d + f - e)*log(x + 1) - 1/18*
(d + f + e)*log(x - 1) + 1/48*(d + 4*f + 2*e)*log(x - 2) + 1/12*(d + 4*f -
2*e)/(x + 2)
```

Fricas [A]
time = 0.47, size = 116, normalized size = 1.41

$$-\frac{((19\,d-26\,e+28\,f)x+38\,d-52\,e+56\,f)\log(x+2)-24\,((d-e+f)x+2\,d-2\,e+2\,f)\log(x+1)+8\,((d+e+f)x+2\,d+2\,e+2\,f)\log(x-1)-3\,((d+2\,e+4\,f)x+2\,d+4\,e+8\,f)\log(x-2)-12\,d+24\,e-48\,f}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="frica
s")
```

```
[Out] -1/144*(((19*d - 26*e + 28*f)*x + 38*d - 52*e + 56*f)*log(x + 2) - 24*((d -
 e + f)*x + 2*d - 2*e + 2*f)*log(x + 1) + 8*((d + e + f)*x + 2*d + 2*e + 2*
f)*log(x - 1) - 3*((d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) - 12*d +
 24*e - 48*f)/(x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(x**3-2*x**2-x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.61, size = 77, normalized size = 0.94

$$-\frac{1}{144}\left(19\,d+28\,f-26\,e\right)\log\left(\left|x+2\right|\right)+\frac{1}{6}\left(d+f-e\right)\log\left(\left|x+1\right|\right)-\frac{1}{18}\left(d+f+e\right)\log\left(\left|x-1\right|\right)+\frac{1}{48}\left(d+4\,f+2\,e\right)\log\left(\left|x-2\right|\right)+\frac{d+4\,f-2\,e}{12\,\left(x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(x^3-2*x^2-x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac"
)
```

```
[Out] -1/144*(19*d + 28*f - 26*e)*log(abs(x + 2)) + 1/6*(d + f - e)*log(abs(x + 1
)) - 1/18*(d + f + e)*log(abs(x - 1)) + 1/48*(d + 4*f + 2*e)*log(abs(x - 2)
) + 1/12*(d + 4*f - 2*e)/(x + 2)
```

Mupad [B]
time = 0.84, size = 79, normalized size = 0.96

$$\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}}{x+2}+\ln\left(x+1\right)\,\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}\right)-\ln\left(x-1\right)\,\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}\right)+\ln\left(x-2\right)\,\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}\right)-\ln\left(x+2\right)\,\left(\frac{19\,d}{144}-\frac{13\,e}{72}+\frac{7\,f}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x + f*x^2)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] (d/12 - e/6 + f/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f/6) - log(x - 1)*(d/1
8 + e/18 + f/18) + log(x - 2)*(d/48 + e/24 + f/12) - log(x + 2)*((19*d)/144
 - (13*e)/72 + (7*f)/36)
```

3.88 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2} dx$

Optimal. Leaf size=95

$$\frac{d-2e+4f-8g}{12(2+x)}-\frac{1}{18}(d+e+f+g)\log(1-x)+\frac{1}{48}(d+2e+4f+8g)\log(2-x)+\frac{1}{6}(d-e+f-g)\log(1+x)-\frac{1}{144}($$

```
[Out] 1/12*(d-2*e+4*f-8*g)/(2+x)-1/18*(d+e+f+g)*ln(1-x)+1/48*(d+2*e+4*f+8*g)*ln(2
-x)+1/6*(d-e+f-g)*ln(1+x)-1/144*(19*d-26*e+28*f-8*g)*ln(2+x)
```

Rubi [A]
time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {1600, 6874}

$$\frac{d-2e+4f-8g}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g)+\frac{1}{48}\log(2-x)(d+2e+4f+8g)+\frac{1}{6}\log(x+1)(d-e+f-g)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x
]
```

```
[Out] (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e + f + g)*Log[1 - x])/18 + ((d
+ 2*e + 4*f + 8*g)*Log[2 - x])/48 + ((d - e + f - g)*Log[1 + x])/6 - ((19*d
 - 26*e + 28*f - 8*g)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2} dx &= \int \frac{d+ex+fx^2+gx^3}{(2+x)^2(2-x-2x^2+x^3)} dx \\ &= \int \left(\frac{d+2e+4f+8g}{48(-2+x)}+\frac{-d-e-f-g}{18(-1+x)}+\frac{d-e+f-}{6(1+x)}\right. \\ &= \frac{d-2e+4f-8g}{12(2+x)}-\frac{1}{18}(d+e+f+g)\log(1-x)+\frac{1}{48}(\end{aligned}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 0.95

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g)}{2+x}+24(d-e+f-g)\log(-1-x)-8(d+e+f+g)\log(1-x)+3(d+2e+4f+8g)\log(2-x)+(-19d+26e-28f+8g)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^
4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g))/(2 + x) + 24*(d - e + f - g)*Log[-1 - x] - 8*(d
 + e + f + g)*Log[1 - x] + 3*(d + 2*e + 4*f + 8*g)*Log[2 - x] + (-19*d + 26
*e - 28*f + 8*g)*Log[2 + x])/144
```

Maple [A]
time = 0.05, size = 94, normalized size = 0.99

method	result
default	$\left(\frac{13e}{72}-\frac{7f}{36}+\frac{g}{18}-\frac{19d}{144}\right)\ln\left(x+2\right)-\frac{-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}}{x+2}+\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}\right)\ln\left(x-2\right)+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}\right.$
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}\right)x^3+\left(\frac{e}{3}-\frac{2f}{3}+\frac{4g}{3}-\frac{d}{6}\right)x^2-\frac{e}{3}+\frac{2f}{3}-\frac{4g}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}-\frac{g}{18}\right)\ln\left(-1+x\right)+$
risch	$\frac{d}{12x+24}-\frac{e}{6(x+2)}+\frac{f}{3x+6}-\frac{2g}{3(x+2)}+\frac{\ln(2-x)d}{48}+\frac{\ln(2-x)e}{24}+\frac{\ln(2-x)f}{12}+\frac{\ln(2-x)g}{6}+\frac{\ln(1+x)d}{6}-\frac{\ln(1+x)e}{6}+\ln($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVER
BOSE)
```

```
[Out] (13/72*e-7/36*f+1/18*g-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/3*f+2/3*g)/(x+2)+
(1/48*d+1/24*e+1/12*f+1/6*g)*ln(x-2)+(-1/18*d-1/18*e-1/18*f-1/18*g)*ln(-1+x
)+(1/6*d-1/6*e+1/6*f-1/6*g)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.91

$$-\frac{1}{144}\left(19\,d+28\,f-8\,g-26\,e\right)\log\left(x+2\right)+\frac{1}{6}\left(d+f-g-e\right)\log\left(x+1\right)-\frac{1}{18}\left(d+f+g+e\right)\log\left(x-1\right)+\frac{1}{48}\left(d+4\,f+8\,g+2\,e\right)\log\left(x-2\right)+\frac{d+4\,f-8\,g-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"maxima")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 26*e)*log(x + 2) + 1/6*(d + f - g - e)*log(x +
1) - 1/18*(d + f + g + e)*log(x - 1) + 1/48*(d + 4*f + 8*g + 2*e)*log(x - 2
) + 1/12*(d + 4*f - 8*g - 2*e)/(x + 2)
```

Fricas [A]
time = 0.83, size = 141, normalized size = 1.48

$$-\frac{((19d-26e+28f-8g)x+38d-52e+56f-16g)\log(x+2)-24((d-e+f-g)x+2d-2e+2f-2g)\log(x+1)+8((d+e+f+g)x+2d+2e+2f+2g)\log(x-1)-3((d+2e+4f+8g)x+2d+4e+8f+16g)\log(x-2)-12d+24e-48f+96g}{144(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"fricas")
```

```
[Out] -1/144*((((19*d - 26*e + 28*f - 8*g)*x + 38*d - 52*e + 56*f - 16*g)*log(x +
2) - 24*((d - e + f - g)*x + 2*d - 2*e + 2*f - 2*g)*log(x + 1) + 8*((d + e
+ f + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - 3*((d + 2*e + 4*f + 8*g)*x
 + 2*d + 4*e + 8*f + 16*g)*log(x - 2) - 12*d + 24*e - 48*f + 96*g)/(x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.70, size = 90, normalized size = 0.95

$$-\frac{1}{144}(19d+28f-8g-26e)\log(|x+2|)+\frac{1}{6}(d+f-g-e)\log(|x+1|)-\frac{1}{18}(d+f+g+e)\log(|x-1|)+\frac{1}{48}(d+4f+8g+2e)\log(|x-2|)+\frac{d+4f-8g-2e}{12(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm=
"giac")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 26*e)*log(abs(x + 2)) + 1/6*(d + f - g - e)*log
(abs(x + 1)) - 1/18*(d + f + g + e)*log(abs(x - 1)) + 1/48*(d + 4*f + 8*g +
 2*e)*log(abs(x - 2)) + 1/12*(d + 4*f - 8*g - 2*e)/(x + 2)
```

Mupad [B]
time = 0.88, size = 94, normalized size = 0.99

$$\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}}{x+2}+\ln(x+1)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}\right)-\ln(x-1)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}\right)+\ln(x-2)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}\right)-\ln(x+2)\left(\frac{19d}{144}-\frac{13e}{72}+\frac{7f}{36}-\frac{g}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((d + e*x + f*x^2 + g*x^3)*(x + 2*x^2 - x^3 - 2))/(x^4 - 5*x^2 + 4)^2,
x)
```

```
[Out] (d/12 - e/6 + f/3 - (2*g)/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f/6 - g/6) -
 log(x - 1)*(d/18 + e/18 + f/18 + g/18) + log(x - 2)*(d/48 + e/24 + f/12 +
g/6) - log(x + 2)*((19*d)/144 - (13*e)/72 + (7*f)/36 - g/18)
```

3.89 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=106

$$\frac{d-2e+4f-8g+16h}{12(2+x)}-\frac{1}{18}(d+e+f+g+h)\log(1-x)+\frac{1}{48}(d+2e+4f+8g+16h)\log(2-x)+\frac{1}{6}(d-e+f-g+h)$$

```
[Out] 1/12*(d-2*e+4*f-8*g+16*h)/(2+x)-1/18*(d+e+f+g+h)*ln(1-x)+1/48*(d+2*e+4*f+8*
g+16*h)*ln(2-x)+1/6*(d-e+f-g+h)*ln(1+x)-1/144*(19*d-26*e+28*f-8*g-80*h)*ln(
2+x)
```

Rubi [A]

time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1600, 6874}

$$\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h)+\frac{1}{6}\log(x+1)(d-e+f-g+h)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)) - ((d + e + f + g + h)*Log[1 - x]
)/18 + ((d + 2*e + 4*f + 8*g + 16*h)*Log[2 - x])/48 + ((d - e + f - g + h)*
Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80*h)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx &= \int \frac{d+ex+fx^2+gx^3+hx^4}{(2+x)^2(2-x-2x^2+x^3)}\,dx\\ &= \int \left(\frac{d+2e+4f+8g+16h}{48(-2+x)}+\frac{-d-e-f-g-h}{18(-1+x)}\right.\\ &= \frac{d-2e+4f-8g+16h}{12(2+x)}-\frac{1}{18}(d+e+f+g+h)\,\mathrm{lo}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 0.96

$$\frac{1}{144}\left(\frac{12(d-2e+4f-8g+16h)}{2+x}+24(d-e+f-g+h)\log(-1-x)-8(d+e+f+g+h)\log(1-x)+3(d+2(e+2f+4g+8h))\log(2-x)+(-19d+26e-28f+8g+80h)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*
x^2 + x^4)^2,x]
```

```
[Out] ((12*(d - 2*e + 4*f - 8*g + 16*h))/(2 + x) + 24*(d - e + f - g + h)*Log[-1
- x] - 8*(d + e + f + g + h)*Log[1 - x] + 3*(d + 2*(e + 2*f + 4*g + 8*h))*L
og[2 - x] + (-19*d + 26*e - 28*f + 8*g + 80*h)*Log[2 + x])/144
```

Maple [A]
time = 0.05, size = 109, normalized size = 1.03

method	result
default	$\left(\frac{5h}{9}+\frac{g}{18}-\frac{7f}{36}+\frac{13e}{72}-\frac{19d}{144}\right)\ln\left(x+2\right)-\frac{-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}}{x+2}+\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}\right)\ln\left(x-2\right)+($
norman	$\frac{\left(-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}\right)x+\left(\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}\right)x^3+\left(-\frac{8h}{3}+\frac{4g}{3}-\frac{2f}{3}+\frac{e}{3}-\frac{d}{6}\right)x^2+\frac{8h}{3}-\frac{4g}{3}+\frac{2f}{3}-\frac{e}{3}+\frac{d}{6}}{x^4-5x^2+4}+\left(-\frac{d}{18}-\frac{e}{18}-\frac{f}{18}-\right.$
risch	$\frac{5\ln(-x-2)h}{9}-\frac{\ln(-1+x)h}{18}+\frac{d}{12x+24}+\frac{4h}{3(x+2)}+\frac{\ln(2-x)g}{6}-\frac{\ln(1+x)g}{6}+\frac{\ln(2-x)f}{12}+\frac{\ln(1+x)f}{6}-\frac{\ln(-1+x)g}{18}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RET
URNVERBOSE)
```

```
[Out] (5/9*h+1/18*g-7/36*f+13/72*e-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/3*f+2/3*g-4
/3*h)/(x+2)+(1/48*d+1/24*e+1/12*f+1/6*g+1/3*h)*ln(x-2)+(-1/18*d-1/18*e-1/18
*f-1/18*g-1/18*h)*ln(-1+x)+(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h)*ln(1+x)
```

Maxima [A]
time = 0.30, size = 97, normalized size = 0.92

$$-\frac{1}{144}\,(19\,d+28\,f-8\,g-80\,h-26\,e)\log(x+2)+\frac{1}{6}\,(d+f-g+h-e)\log(x+1)-\frac{1}{18}\,(d+f+g+h+e)\log(x-1)+\frac{1}{48}\,(d+4\,f+8\,g+16\,h+2\,e)\log(x-2)+\frac{d+4\,f-8\,g+16\,h-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="maxima")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e)*log(x + 2) + 1/6*(d + f - g + h -
e)*log(x + 1) - 1/18*(d + f + g + h + e)*log(x - 1) + 1/48*(d + 4*f + 8*g +
 16*h + 2*e)*log(x - 2) + 1/12*(d + 4*f - 8*g + 16*h - 2*e)/(x + 2)
```

Fricas [A]
time = 3.24, size = 164, normalized size = 1.55

$$-\frac{((19\,d-26\,e+28\,f-8\,g-80\,h)x+38\,d-52\,e+56\,f-16\,g-160\,h)\log(x+2)-24\,((d-e+f-g+h)x+2\,d-2\,e+2\,f-2\,g+2\,h)\log(x+1)+8\,((d+e+f+g+h)x+2\,d+2\,e+2\,f+2\,g+2\,h)\log(x-1)-3\,((d+2\,e+4\,f+8\,g+16\,h)x+2\,d+4\,e+8\,f+16\,g+32\,h)\log(x-2)-12\,d+24\,e-48\,f+96\,g-192\,h}{144\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="fricas")
```

```
[Out] -1/144*(((19*d - 26*e + 28*f - 8*g - 80*h)*x + 38*d - 52*e + 56*f - 16*g -
160*h)*log(x + 2) - 24*((d - e + f - g + h)*x + 2*d - 2*e + 2*f - 2*g + 2*h
)*log(x + 1) + 8*((d + e + f + g + h)*x + 2*d + 2*e + 2*f + 2*g + 2*h)*log(
x - 1) - 3*((d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)
*log(x - 2) - 12*d + 24*e - 48*f + 96*g - 192*h)/(x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2
,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.32, size = 101, normalized size = 0.95

$$-\frac{1}{144}\,(19\,d+28\,f-8\,g-80\,h-26\,e)\log\left(|x+2|\right)+\frac{1}{6}\,(d+f-g+h-e)\log\left(|x+1|\right)-\frac{1}{18}\,(d+f+g+h+e)\log\left(|x-1|\right)+\frac{1}{48}\,(d+4\,f+8\,g+16\,h+2\,e)\log\left(|x-2|\right)+\frac{d+4\,f-8\,g+16\,h-2\,e}{12\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algo
rithm="giac")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e)*log(abs(x + 2)) + 1/6*(d + f - g +
 h - e)*log(abs(x + 1)) - 1/18*(d + f + g + h + e)*log(abs(x - 1)) + 1/48*(
d + 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2)) + 1/12*(d + 4*f - 8*g + 16*h -
2*e)/(x + 2)
```

Mupad [B]
time = 1.36, size = 108, normalized size = 1.02

$$\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}}{x+2}+\ln\left(x+1\right)\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}\right)-\ln\left(x-1\right)\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}+\frac{h}{18}\right)+\ln\left(x-2\right)\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}\right)+\ln\left(x+2\right)\left(\frac{13\,e}{72}-\frac{19\,d}{144}-\frac{7\,f}{36}+\frac{g}{18}+\frac{5\,h}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2
 + 4)^2,x)
```

```
[Out] (d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/3)/(x + 2) + log(x + 1)*(d/6 - e/6 + f/
6 - g/6 + h/6) - log(x - 1)*(d/18 + e/18 + f/18 + g/18 + h/18) + log(x - 2)
*(d/48 + e/24 + f/12 + g/6 + h/3) + log(x + 2)*((13*e)/72 - (19*d)/144 - (7
*f)/36 + g/18 + (5*h)/9)
```

3.90 $\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=122

$$ix+\frac{d-2e+4f-8g+16h-32i}{12(2+x)}-\frac{1}{18}(d+e+f+g+h+i)\log(1-x)+\frac{1}{48}(d+2e+4f+8g+16h+32i)\log(2-x)+$$

```
[Out] i*x+1/12*(d-2*e+4*f-8*g+16*h-32*i)/(2+x)-1/18*(d+e+f+g+h+i)*ln(1-x)+1/48*(d
+2*e+4*f+8*g+16*h+32*i)*ln(2-x)+1/6*(d-e+f-g+h-i)*ln(1+x)-1/144*(19*d-26*e+
28*f-8*g-80*h+352*i)*ln(2+x)
```

Rubi [A]
time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {1600, 6874}

$$\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{1}{18}\log(1-x)(d+e+f+g+h+i)+\frac{1}{48}\log(2-x)(d+2e+4f+8g+16h+32i)+\frac{1}{6}\log(x+1)(d-e+f-g+h-i)-\frac{1}{144}\log(x+2)(19d-26e+28f-8g-80h+352i)+ix$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4)^2,x]
```

```
[Out] i*x + (d - 2*e + 4*f - 8*g + 16*h - 32*i)/(12*(2 + x)) - ((d + e + f + g +
h + i)*Log[1 - x])/18 + ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*Log[2 - x])/48
 + ((d - e + f - g + h - i)*Log[1 + x])/6 - ((19*d - 26*e + 28*f - 8*g - 80
*h + 352*i)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2-x-2x^2+x^3)(d+ex+fx^2+gx^3+hx^4+90x^5)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+90x^5}{(2+x)^2(2-x-2x^2+x^3)}\,dx$$
$$= \int \left(90 + \frac{2880+d+2e+4f+8g+16h}{48(-2+x)} \right.$$
$$= 90x - \frac{2880-d+2e-4f+8g-16h}{12(2+x)} - $$

Mathematica [A]
time = 0.04, size = 118, normalized size = 0.97

$$\frac{1}{144}\left(144ix + \frac{12(d-2(e-2f+4g-8h+16i))}{2+x} - 8(d+e+f+g+h+i)\log(1-x) + 3(d+2e+4(f+2g+4h+8i))\log(2-x) + 24(d-e+f-g+h-i)\log(1+x) + (-19d+26e-28f+8g+80h-352i)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x - 2*x^2 + x^3)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))
/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (144*i*x + (12*(d - 2*(e - 2*f + 4*g - 8*h + 16*i)))/(2 + x) - 8*(d + e + f
 + g + h + i)*Log[1 - x] + 3*(d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x]
 + 24*(d - e + f - g + h - i)*Log[1 + x] + (-19*d + 26*e - 28*f + 8*g + 80*
h - 352*i)*Log[2 + x])/144
```

Maple [A]
time = 0.06, size = 127, normalized size = 1.04

method	result
default	$ix + \left(\frac{5h}{9} - \frac{22i}{9} + \frac{g}{18} - \frac{7f}{36} + \frac{13e}{72} - \frac{19d}{144}\right)\ln(x+2) - \frac{-\frac{d}{12}+\frac{e}{6}-\frac{f}{3}+\frac{2g}{3}-\frac{4h}{3}+\frac{8i}{3}}{x+2} + \left(\frac{d}{48} + \frac{e}{24} + \frac{f}{12} + \frac{g}{6} + \frac{h}{3} -\right.$
norman	$\frac{i\,x^5+\left(-\frac{4h}{3}+\frac{20i}{3}+\frac{2g}{3}-\frac{f}{3}+\frac{e}{6}-\frac{d}{12}\right)x+\left(\frac{4h}{3}-\frac{23i}{3}-\frac{2g}{3}+\frac{f}{3}-\frac{e}{6}+\frac{d}{12}\right)x^3+\left(-\frac{d}{6}+\frac{e}{3}-\frac{2f}{3}+\frac{4g}{3}-\frac{8h}{3}+\frac{16i}{3}\right)x^2+\frac{8h}{3}-\frac{16i}{3}+\frac{d}{6}-\frac{e}{3}+\frac{2f}{3}-\frac{4g}{3}}{x^4-5x^2+4} +$
risch	$\frac{5\ln(-x-2)h}{9} - \frac{\ln(-1+x)h}{18} + \frac{d}{12x+24} - \frac{\ln(-1+x)i}{18} - \frac{\ln(1+x)i}{6} - \frac{22\ln(-x-2)i}{9} + \frac{4h}{3(x+2)} + ix + \frac{\ln(2-x)g}{6} - \frac{\ln(1+}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,metho
d=_RETURNVERBOSE)
```

```
[Out] i*x+(5/9*h-22/9*i+1/18*g-7/36*f+13/72*e-19/144*d)*ln(x+2)-(-1/12*d+1/6*e-1/
3*f+2/3*g-4/3*h+8/3*i)/(x+2)+(1/48*d+1/24*e+1/12*f+1/6*g+1/3*h+2/3*i)*ln(x-
2)+(-1/18*d-1/18*e-1/18*f-1/18*g-1/18*h-1/18*i)*ln(-1+x)+(1/6*d-1/6*e+1/6*f
-1/6*g+1/6*h-1/6*i)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.86

$$-\frac{1}{144} (19 d + 28 f - 8 g - 80 h - 26 e + 352 i) \log (x + 2) + \frac{1}{6} (d + f - g + h - e - i) \log (x + 1) - \frac{1}{18} (d + f + g + h + e + i) \log (x - 1) + \frac{1}{48} (d + 4 f + 8 g + 16 h + 2 e + 32 i) \log (x - 2) + i x + \frac{d + 4 f - 8 g + 16 h - 2 e - 32 i}{12 (x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="maxima")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e + 352*I)*log(x + 2) + 1/6*(d + f -
g + h - e - I)*log(x + 1) - 1/18*(d + f + g + h + e + I)*log(x - 1) + 1/48*
(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(x - 2) + I*x + 1/12*(d + 4*f - 8*g
+ 16*h - 2*e - 32*I)/(x + 2)
```

Fricas [A]
time = 19.30, size = 200, normalized size = 1.64

$$\frac{144 i x^2 + 288 i x - ((19 d - 26 e + 28 f - 8 g - 80 h + 352 i) x + 38 d - 52 e + 56 f - 16 g - 160 h + 704 i) \log (x + 2) + 24 ((d - e + f - g + h - i) x + 2 d - 2 e + 2 f - 2 g + 2 h - 2 i) \log (x + 1) - 8 ((d + e + f + g + h + i) x + 2 d + 2 e + 2 f + 2 g + 2 h + 2 i) \log (x - 1) + 3 ((d + 2 e + 4 f + 8 g + 16 h + 32 i) x + 2 d + 4 e + 8 f + 16 g + 32 h + 64 i) \log (x - 2) + 12 d - 24 e + 48 f - 96 g + 192 h - 384 i}{144 (x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="fricas")
```

```
[Out] 1/144*(144*i*x^2 + 288*i*x - ((19*d - 26*e + 28*f - 8*g - 80*h + 352*i)*x +
 38*d - 52*e + 56*f - 16*g - 160*h + 704*i)*log(x + 2) + 24*((d - e + f - g
 + h - i)*x + 2*d - 2*e + 2*f - 2*g + 2*h - 2*i)*log(x + 1) - 8*((d + e + f
 + g + h + i)*x + 2*d + 2*e + 2*f + 2*g + 2*h + 2*i)*log(x - 1) + 3*((d + 2
*e + 4*f + 8*g + 16*h + 32*i)*x + 2*d + 4*e + 8*f + 16*g + 32*h + 64*i)*log
(x - 2) + 12*d - 24*e + 48*f - 96*g + 192*h - 384*i)/(x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2-x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**
2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.78, size = 109, normalized size = 0.89

$$-\frac{1}{144} (19 d + 28 f - 8 g - 80 h - 26 e + 352 i) \log (|x + 2|) + \frac{1}{6} (d + f - g + h - e - i) \log (|x + 1|) - \frac{1}{18} (d + f + g + h + e + i) \log (|x - 1|) + \frac{1}{48} (d + 4 f + 8 g + 16 h + 2 e + 32 i) \log (|x - 2|) + i x + \frac{d + 4 f - 8 g + 16 h - 2 e - 32 i}{12 (x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2-x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x
, algorithm="giac")
```

```
[Out] -1/144*(19*d + 28*f - 8*g - 80*h - 26*e + 352*I)*log(abs(x + 2)) + 1/6*(d +
 f - g + h - e - I)*log(abs(x + 1)) - 1/18*(d + f + g + h + e + I)*log(abs(
x - 1)) + 1/48*(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(abs(x - 2)) + I*x +
1/12*(d + 4*f - 8*g + 16*h - 2*e - 32*I)/(x + 2)
```

Mupad [B]
time = 1.67, size = 127, normalized size = 1.04

$$i\,x+\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2\,g}{3}+\frac{4\,h}{3}-\frac{8\,i}{3}}{x+2}+\ln\left(x+1\right)\,\left(\frac{d}{6}-\frac{e}{6}+\frac{f}{6}-\frac{g}{6}+\frac{h}{6}-\frac{i}{6}\right)+\ln\left(x-2\right)\,\left(\frac{d}{48}+\frac{e}{24}+\frac{f}{12}+\frac{g}{6}+\frac{h}{3}+\frac{2\,i}{3}\right)-\ln\left(x-1\right)\,\left(\frac{d}{18}+\frac{e}{18}+\frac{f}{18}+\frac{g}{18}+\frac{h}{18}+\frac{i}{18}\right)-\ln\left(x+2\right)\,\left(\frac{19\,d}{144}-\frac{13\,e}{72}+\frac{7\,f}{36}-\frac{g}{18}-\frac{5\,h}{9}+\frac{22\,i}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 2*x^2 - x^3 - 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4
 - 5*x^2 + 4)^2,x)
```

```
[Out] i*x + (d/12 - e/6 + f/3 - (2*g)/3 + (4*h)/3 - (8*i)/3)/(x + 2) + log(x + 1)
*(d/6 - e/6 + f/6 - g/6 + h/6 - i/6) + log(x - 2)*(d/48 + e/24 + f/12 + g/6
 + h/3 + (2*i)/3) - log(x - 1)*(d/18 + e/18 + f/18 + g/18 + h/18 + i/18) -
log(x + 2)*((19*d)/144 - (13*e)/72 + (7*f)/36 - g/18 - (5*h)/9 + (22*i)/9)
```

3.91 $\int \frac{2-3x+x^2}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{5+3x}{12\left(2+3x+x^2\right)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(1+x)+\frac{31}{144}\log(2+x)$$

```
[Out] 1/12*(-5-3*x)/(x^2+3*x+2)-1/36*ln(1-x)+1/144*ln(2-x)-7/36*ln(1+x)+31/144*ln
(2+x)
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1600, 988, 1086, 646, 31}

$$-\frac{3x+5}{12\left(x^2+3x+2\right)}-\frac{1}{36}\log(1-x)+\frac{1}{144}\log(2-x)-\frac{7}{36}\log(x+1)+\frac{31}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -1/12*(5 + 3*x)/(2 + 3*x + x^2) - Log[1 - x]/36 + Log[2 - x]/144 - (7*Log[1
 + x])/36 + (31*Log[2 + x])/144
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
```

```
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-3x+x^2}{(4-5x^2+x^4)^2}\,dx &= \int \frac{1}{(2-3x+x^2)\,(2+3x+x^2)^2}\,dx \\
&= -\frac{5+3x}{12\,(2+3x+x^2)} + \frac{1}{72}\int \frac{-18+48x-18x^2}{(2-3x+x^2)\,(2+3x+x^2)}\,dx \\
&= -\frac{5+3x}{12\,(2+3x+x^2)} + \frac{\int \frac{252-108x}{2-3x+x^2}\,dx}{5184} + \frac{\int \frac{-900+108x}{2+3x+x^2}\,dx}{5184} \\
&= -\frac{5+3x}{12\,(2+3x+x^2)} + \frac{1}{144}\int \frac{1}{-2+x}\,dx - \frac{1}{36}\int \frac{1}{-1+x}\,dx - \frac{7}{36}\int \frac{1}{1+x}\,dx + \frac{3}{14} \\
&= -\frac{5+3x}{12\,(2+3x+x^2)} - \frac{1}{36}\log(1-x) + \frac{1}{144}\log(2-x) - \frac{7}{36}\log(1+x) + \frac{31}{144}\log(2
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 48, normalized size = 0.86

$$\frac{1}{144}\left(-\frac{12(5+3x)}{2+3x+x^2}-4\log(1-x)+\log(2-x)-28\log(1+x)+31\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + x^2)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(5 + 3*x))/(2 + 3*x + x^2) - 4*Log[1 - x] + Log[2 - x] - 28*Log[1 + x
] + 31*Log[2 + x])/144
```

Maple [A]
time = 0.04, size = 40, normalized size = 0.71

method	result	size
default	$-\frac{1}{12(x+2)}+\frac{31\ln(x+2)}{144}+\frac{\ln(x-2)}{144}-\frac{\ln(-1+x)}{36}-\frac{1}{6(1+x)}-\frac{7\ln(1+x)}{36}$	40
risch	$\frac{-\frac{x}{4}-\frac{5}{12}}{x^2+3x+2}+\frac{\ln(x-2)}{144}-\frac{\ln(-1+x)}{36}-\frac{7\ln(1+x)}{36}+\frac{31\ln(x+2)}{144}$	42
norman	$\frac{\frac{1}{3}x^2+\frac{3}{4}x-\frac{1}{4}x^3-\frac{5}{6}}{x^4-5x^2+4}+\frac{\ln(x-2)}{144}-\frac{\ln(-1+x)}{36}-\frac{7\ln(1+x)}{36}+\frac{31\ln(x+2)}{144}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/(x+2)+31/144*ln(x+2)+1/144*ln(x-2)-1/36*ln(-1+x)-1/6/(1+x)-7/36*ln(1+
x)
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.75

$$-\frac{3\,x+5}{12\,(x^2+3\,x+2)}+\frac{31}{144}\,\log\left(x+2\right)-\frac{7}{36}\,\log\left(x+1\right)-\frac{1}{36}\,\log\left(x-1\right)+\frac{1}{144}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*x + 5)/(x^2 + 3*x + 2) + 31/144*log(x + 2) - 7/36*log(x + 1) - 1/3
6*log(x - 1) + 1/144*log(x - 2)
```

Fricas [A]
time = 0.40, size = 72, normalized size = 1.29

$$\frac{31\,(x^2+3\,x+2)\log\left(x+2\right)-28\,(x^2+3\,x+2)\log\left(x+1\right)-4\,(x^2+3\,x+2)\log\left(x-1\right)+(x^2+3\,x+2)\log\left(x-2\right)-36\,x-60}{144\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(31*(x^2 + 3*x + 2)*log(x + 2) - 28*(x^2 + 3*x + 2)*log(x + 1) - 4*(x
^2 + 3*x + 2)*log(x - 1) + (x^2 + 3*x + 2)*log(x - 2) - 36*x - 60)/(x^2 + 3
*x + 2)
```

Sympy [A]
time = 0.16, size = 46, normalized size = 0.82

$$\frac{-3x-5}{12x^2+36x+24}+\frac{\log(x-2)}{144}-\frac{\log(x-1)}{36}-\frac{7\log(x+1)}{36}+\frac{31\log(x+2)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (-3*x - 5)/(12*x**2 + 36*x + 24) + log(x - 2)/144 - log(x - 1)/36 - 7*log(x
 + 1)/36 + 31*log(x + 2)/144
```

Giac [A]
time = 4.21, size = 46, normalized size = 0.82

$$-\frac{3x+5}{12(x+2)(x+1)}+\frac{31}{144}\log(|x+2|)-\frac{7}{36}\log(|x+1|)-\frac{1}{36}\log(|x-1|)+\frac{1}{144}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*x + 5)/((x + 2)*(x + 1)) + 31/144*log(abs(x + 2)) - 7/36*log(abs(x
 + 1)) - 1/36*log(abs(x - 1)) + 1/144*log(abs(x - 2))
```

Mupad [B]
time = 0.05, size = 42, normalized size = 0.75

$$\frac{\ln(x-2)}{144}-\frac{7\ln(x+1)}{36}-\frac{\ln(x-1)}{36}+\frac{31\ln(x+2)}{144}-\frac{\frac{x}{4}+\frac{5}{12}}{x^2+3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 3*x + 2)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 2)/144 - (7*log(x + 1))/36 - log(x - 1)/36 + (31*log(x + 2))/144 -
(x/4 + 5/12)/(3*x + x^2 + 2)
```

3.92 $\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{5d-6e+(3d-4e)x}{12\,(2+3x+x^2)}-\frac{1}{36}(d+e)\log(1-x)+\frac{1}{144}(d+2e)\log(2-x)-\frac{1}{36}(7d-13e)\log(1+x)+\frac{1}{144}(31d-50e)\log(2+x)$$

```
[Out] 1/12*(-5*d+6*e-(3*d-4*e)*x)/(x^2+3*x+2)-1/36*(d+e)*ln(1-x)+1/144*(d+2*e)*ln
(2-x)-1/36*(7*d-13*e)*ln(1+x)+1/144*(31*d-50*e)*ln(2+x)
```

Rubi [A]
time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1600, 1030, 1086, 646, 31}

$$-\frac{x(3d-4e)+5d-6e}{12\,(x^2+3x+2)}-\frac{1}{36}(d+e)\log(1-x)+\frac{1}{144}(d+2e)\log(2-x)-\frac{1}{36}(7d-13e)\log(x+1)+\frac{1}{144}(31d-50e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -1/12*(5*d - 6*e + (3*d - 4*e)*x)/(2 + 3*x + x^2) - ((d + e)*Log[1 - x])/36
 + ((d + 2*e)*Log[2 - x])/144 - ((7*d - 13*e)*Log[1 + x])/36 + ((31*d - 50*
e)*Log[2 + x])/144
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{(d+ex)(2-3x+x^2)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= -\frac{5d-6e+(3d-4e)x}{12(2+3x+x^2)} - \frac{1}{72}\int \frac{6(3d-10e)-24(2d-3e)x+6(3d-4e)x^2}{(2-3x+x^2)(2+3x+x^2)}$$
$$= -\frac{5d-6e+(3d-4e)x}{12(2+3x+x^2)} - \frac{\int \frac{108(3d-10e)-288(2d-3e)+(-36(3d-10e)+72(3d-4e))x}{2-3x+x^2}\,dx}{5184} -$$
$$= -\frac{5d-6e+(3d-4e)x}{12(2+3x+x^2)} - \frac{1}{36}(7d-13e)\int \frac{1}{1+x}\,dx - \frac{1}{144}(-d-2e)\int \frac{1}{-2}$$
$$= -\frac{5d-6e+(3d-4e)x}{12(2+3x+x^2)} - \frac{1}{36}(d+e)\log(1-x) + \frac{1}{144}(d+2e)\log(2-x) -$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.90

$$\frac{1}{144}\left(\frac{12(-5d+6e-3dx+4ex)}{2+3x+x^2} - 4(d+e)\log(1-x) + (d+2e)\log(2-x) + 4(-7d+13e)\log(1+x) + (31d-50e)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 - 3*x + x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-5*d + 6*e - 3*d*x + 4*e*x))/(2 + 3*x + x^2) - 4*(d + e)*Log[1 - x] +
 (d + 2*e)*Log[2 - x] + 4*(-7*d + 13*e)*Log[1 + x] + (31*d - 50*e)*Log[2 +
x])/144
```

Maple [A]
time = 0.04, size = 78, normalized size = 0.88

method	result
default	$-\frac{\frac{d}{12}-\frac{e}{6}}{x+2} + \left(\frac{31d}{144}-\frac{25e}{72}\right)\ln(x+2) + \left(\frac{d}{144}+\frac{e}{72}\right)\ln(x-2) + \left(-\frac{d}{36}-\frac{e}{36}\right)\ln(-1+x) + \left(-\frac{7d}{36}+\frac{13e}{36}\right)\ln$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}\right)x-\frac{5d}{12}+\frac{e}{2}}{x^2+3x+2} - \frac{\ln(-1+x)d}{36} - \frac{\ln(-1+x)e}{36} - \frac{7\ln(-1-x)d}{36} + \frac{13\ln(-1-x)e}{36} + \frac{\ln(2-x)d}{144} + \frac{\ln(2-x)e}{72} + \frac{31\ln(x+2)d}{144} -$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}\right)x+\left(\frac{d}{3}-\frac{e}{2}\right)x^2-\frac{5d}{6}+e}{x^4-5x^2+4} + \left(-\frac{7d}{36}+\frac{13e}{36}\right)\ln(1+x) + \left(-\frac{d}{36}-\frac{e}{36}\right)\ln(-1+x) + \left(\frac{d}{144}+\frac{e}{72}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/12*d-1/6*e)/(x+2)+(31/144*d-25/72*e)*ln(x+2)+(1/144*d+1/72*e)*ln(x-2)+(
-1/36*d-1/36*e)*ln(-1+x)+(-7/36*d+13/36*e)*ln(1+x)-(1/6*d-1/6*e)/(1+x)
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.91

$$\frac{1}{144}\,(31\,d-50\,e)\log\left(x+2\right)-\frac{1}{36}\,(7\,d-13\,e)\log\left(x+1\right)-\frac{1}{36}\,(d+e)\log\left(x-1\right)+\frac{1}{144}\,(d+2\,e)\log\left(x-2\right)-\frac{(3\,d-4\,e)x+5\,d-6\,e}{12\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(31*d - 50*e)*log(x + 2) - 1/36*(7*d - 13*e)*log(x + 1) - 1/36*(d + e
)*log(x - 1) + 1/144*(d + 2*e)*log(x - 2) - 1/12*((3*d - 4*e)*x + 5*d - 6*e
)/(x^2 + 3*x + 2)
```

Fricas [A]
time = 0.41, size = 153, normalized size = 1.72

$$-\frac{12\,(3\,d-4\,e)x-((31\,d-50\,e)x^2+3\,(31\,d-50\,e)x+62\,d-100\,e)\log\left(x+2\right)+4\,((7\,d-13\,e)x^2+3\,(7\,d-13\,e)x+14\,d-26\,e)\log\left(x+1\right)+4\,((d+e)x^2+3\,(d+e)x+2\,d+2\,e)\log\left(x-1\right)-((d+2\,e)x^2+3\,(d+2\,e)x+2\,d+4\,e)\log\left(x-2\right)+60\,d-72\,e}{144\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e)*x - ((31*d - 50*e)*x^2 + 3*(31*d - 50*e)*x + 62*d -
100*e)*log(x + 2) + 4*((7*d - 13*e)*x^2 + 3*(7*d - 13*e)*x + 14*d - 26*e)*l
og(x + 1) + 4*((d + e)*x^2 + 3*(d + e)*x + 2*d + 2*e)*log(x - 1) - ((d + 2*
e)*x^2 + 3*(d + 2*e)*x + 2*d + 4*e)*log(x - 2) + 60*d - 72*e)/(x^2 + 3*x +
2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(80) = 160$.
time = 7.61, size = 1255, normalized size = 14.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x**2-3*x+2)/(x**4-5*x**2+4)**2,x)
```

```
[Out] -(d + e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 10439775*d**5*(d + e)
 - 511591980*d**4*e**2 - 94132290*d**4*e*(d + e) + 667200*d**4*(d + e)**2 +
 469491120*d**3*e**3 + 333672552*d**3*e**2*(d + e) - 2703328*d**3*e*(d + e)
**2 - 198000*d**3*(d + e)**3 + 322778400*d**2*e**4 - 582497712*d**2*e**3*(d
 + e) + 1752768*d**2*e**2*(d + e)**2 + 1107552*d**2*e*(d + e)**3 - 86349385
6*d*e**5 + 500776560*d*e**4*(d + e) + 4226944*d*e**3*(d + e)**2 - 1880640*d
*e**2*(d + e)**3 + 429000000*e**6 - 169242912*e**5*(d + e) - 4538112*e**4*(
d + e)**2 + 964224*e**3*(d + e)**3)/(13474125*d**6 - 102860175*d**5*e + 274
190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*
e**5 - 256183200*e**6))/36 + (d + 2*e)*log(x + (-24383100*d**6 + 187408066*
```

```
d**5*e - 10439775*d**5*(d + 2*e)/4 - 511591980*d**4*e**2 + 47066145*d**4*e*
(d + 2*e)/2 + 41700*d**4*(d + 2*e)**2 + 469491120*d**3*e**3 - 83418138*d**3
*e**2*(d + 2*e) - 168958*d**3*e*(d + 2*e)**2 + 12375*d**3*(d + 2*e)**3/4 +
322778400*d**2*e**4 + 145624428*d**2*e**3*(d + 2*e) + 109548*d**2*e**2*(d +
 2*e)**2 - 34611*d**2*e*(d + 2*e)**3/2 - 863493856*d*e**5 - 125194140*d*e**
4*(d + 2*e) + 264184*d*e**3*(d + 2*e)**2 + 29385*d*e**2*(d + 2*e)**3 + 4290
00000*e**6 + 42310728*e**5*(d + 2*e) - 283632*e**4*(d + 2*e)**2 - 15066*e**
3*(d + 2*e)**3)/(13474125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2 - 2
24142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*e**
6))/144 - (7*d - 13*e)*log(x + (-24383100*d**6 + 187408066*d**5*e + 1043977
5*d**5*(7*d - 13*e) - 511591980*d**4*e**2 - 94132290*d**4*e*(7*d - 13*e) +
667200*d**4*(7*d - 13*e)**2 + 469491120*d**3*e**3 + 333672552*d**3*e**2*(7*
d - 13*e) - 2703328*d**3*e*(7*d - 13*e)**2 - 198000*d**3*(7*d - 13*e)**3 +
322778400*d**2*e**4 - 582497712*d**2*e**3*(7*d - 13*e) + 1752768*d**2*e**2*
(7*d - 13*e)**2 + 1107552*d**2*e*(7*d - 13*e)**3 - 863493856*d*e**5 + 50077
6560*d*e**4*(7*d - 13*e) + 4226944*d*e**3*(7*d - 13*e)**2 - 1880640*d*e**2*
(7*d - 13*e)**3 + 429000000*e**6 - 169242912*e**5*(7*d - 13*e) - 4538112*e*
*4*(7*d - 13*e)**2 + 964224*e**3*(7*d - 13*e)**3)/(13474125*d**6 - 10286017
5*d**5*e + 274190390*d**4*e**2 - 224142072*d**3*e**3 - 245084096*d**2*e**4
+ 535797456*d*e**5 - 256183200*e**6))/36 + (31*d - 50*e)*log(x + (-24383100
*d**6 + 187408066*d**5*e - 10439775*d**5*(31*d - 50*e)/4 - 511591980*d**4*e
**2 + 47066145*d**4*e*(31*d - 50*e)/2 + 41700*d**4*(31*d - 50*e)**2 + 46949
1120*d**3*e**3 - 83418138*d**3*e**2*(31*d - 50*e) - 168958*d**3*e*(31*d - 5
0*e)**2 + 12375*d**3*(31*d - 50*e)**3/4 + 322778400*d**2*e**4 + 145624428*d
**2*e**3*(31*d - 50*e) + 109548*d**2*e**2*(31*d - 50*e)**2 - 34611*d**2*e*(
31*d - 50*e)**3/2 - 863493856*d*e**5 - 125194140*d*e**4*(31*d - 50*e) + 264
184*d*e**3*(31*d - 50*e)**2 + 29385*d*e**2*(31*d - 50*e)**3 + 429000000*e**
6 + 42310728*e**5*(31*d - 50*e) - 283632*e**4*(31*d - 50*e)**2 - 15066*e**3
*(31*d - 50*e)**3)/(13474125*d**6 - 102860175*d**5*e + 274190390*d**4*e**2
- 224142072*d**3*e**3 - 245084096*d**2*e**4 + 535797456*d*e**5 - 256183200*
e**6))/144 + (-5*d + 6*e + x*(-3*d + 4*e))/(12*x**2 + 36*x + 24)
```

Giac [A]

time = 3.96, size = 85, normalized size = 0.96

$$\frac{1}{144}\,(31\,d-50\,e)\log\left(|x+2|\right)-\frac{1}{36}\,(7\,d-13\,e)\log\left(|x+1|\right)-\frac{1}{36}\,(d+e)\log\left(|x-1|\right)+\frac{1}{144}\,(d+2\,e)\log\left(|x-2|\right)-\frac{(3\,d-4\,e)x+5\,d-6\,e}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(x^2-3*x+2)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(31*d - 50*e)*log(abs(x + 2)) - 1/36*(7*d - 13*e)*log(abs(x + 1)) - 1
/36*(d + e)*log(abs(x - 1)) + 1/144*(d + 2*e)*log(abs(x - 2)) - 1/12*((3*d
- 4*e)*x + 5*d - 6*e)/((x + 2)*(x + 1))
```

Mupad [B]

time = 0.10, size = 79, normalized size = 0.89

$$\ln\left(x-2\right)\left(\frac{d}{144}+\frac{e}{72}\right)-\ln\left(x-1\right)\left(\frac{d}{36}+\frac{e}{36}\right)-\ln\left(x+1\right)\left(\frac{7\,d}{36}-\frac{13\,e}{36}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+x\left(\frac{d}{4}-\frac{e}{3}\right)}{x^2+3\,x+2}+\ln\left(x+2\right)\left(\frac{31\,d}{144}-\frac{25\,e}{72}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x^2 - 3*x + 2))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 2)*(d/144 + e/72) - log(x - 1)*(d/36 + e/36) - log(x + 1)*((7*d)/36
 - (13*e)/36) - ((5*d)/12 - e/2 + x*(d/4 - e/3))/(3*x + x^2 + 2) + log(x +
2)*((31*d)/144 - (25*e)/72)
```

3.93 $\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=105

$$-\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)}-\frac{1}{36}(d+e+f)\log(1-x)+\frac{1}{144}(d+2e+4f)\log(2-x)-\frac{1}{36}(7d-13e+19f)\log$$

```
[Out] 1/12*(-5*d+6*e-8*f-(3*d-4*e+6*f)*x)/(x^2+3*x+2)-1/36*(d+e+f)*ln(1-x)+1/144*
(d+2*e+4*f)*ln(2-x)-1/36*(7*d-13*e+19*f)*ln(1+x)+1/144*(31*d-50*e+76*f)*ln(
2+x)
```

Rubi [A]

time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1600, 1074, 1086, 646, 31}

$$-\frac{x(3d-4e+6f)+5d-6e+8f}{12(x^2+3x+2)}-\frac{1}{36}\log(1-x)(d+e+f)+\frac{1}{144}\log(2-x)(d+2e+4f)-\frac{1}{36}\log(x+1)(7d-13e+19f)+\frac{1}{144}\log(x+2)(31d-50e+76f)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -1/12*(5*d - 6*e + 8*f + (3*d - 4*e + 6*f)*x)/(2 + 3*x + x^2) - ((d + e + f
)*Log[1 - x])/36 + ((d + 2*e + 4*f)*Log[2 - x])/144 - ((7*d - 13*e + 19*f)*
Log[1 + x])/36 + ((31*d - 50*e + 76*f)*Log[2 + x])/144
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
```

```
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{72}\int \frac{6(3d-10e+12f)-24(2d\ldots}{(2-3x+\ldots}$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{\int \frac{-288(2d-3e+5f)+108(3d-10e+12f)+(\ldots}{2-3x+\ldots}}{518\ldots}$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{144}(-31d+50e-76f)\int \frac{1}{2+\ldots}$$
$$= -\frac{5d-6e+8f+(3d-4e+6f)x}{12(2+3x+x^2)} - \frac{1}{36}(d+e+f)\log(1-x) + \frac{1}{144}(\ldots$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.92

$$\frac{1}{144}\left(-\frac{12(-6e+8f-4ex+6fx+d(5+3x))}{2+3x+x^2} - 4(d+e+f)\log(1-x) + (d+2e+4f)\log(2-x) - 4(7d-13e+19f)\log(1+x) + (31d-50e+76f)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((-12*(-6*e + 8*f - 4*e*x + 6*f*x + d*(5 + 3*x)))/(2 + 3*x + x^2) - 4*(d +
e + f)*Log[1 - x] + (d + 2*e + 4*f)*Log[2 - x] - 4*(7*d - 13*e + 19*f)*Log[
1 + x] + (31*d - 50*e + 76*f)*Log[2 + x])/144
```

Maple [A]
time = 0.04, size = 96, normalized size = 0.91

method	result
default	$-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}}{x+2}+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}\right)\ln(x+2)+\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}\right)\ln(x-2)+\left(-\frac{d}{36}-\frac{e}{36}-\frac{f}{36}\right)\ln(-1+\ldots$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}\right)x^2-\frac{5d}{6}+e-\frac{4f}{3}}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}\right)\ln(1+x)+\left(-\frac{d}{36}-\frac{e}{36}-\frac{f}{36}\right)\ln\ldots$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}}{x^2+3x+2}-\frac{7\ln(-1-x)d}{36}+\frac{13\ln(-1-x)e}{36}-\frac{19\ln(-1-x)f}{36}+\frac{31\ln(x+2)d}{144}-\frac{25\ln(x+2)e}{72}+\frac{19\ln(x+2)f}{36}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(1/12*d-1/6*e+1/3*f)/(x+2)+(31/144*d-25/72*e+19/36*f)*ln(x+2)+(1/144*d+1/7
2*e+1/36*f)*ln(x-2)+(-1/36*d-1/36*e-1/36*f)*ln(-1+x)+(-7/36*d+13/36*e-19/36
*f)*ln(1+x)-(1/6*d-1/6*e+1/6*f)/(1+x)
```

Maxima [A]
time = 0.29, size = 97, normalized size = 0.92

$$\frac{1}{144}\,(31\,d+76\,f-50\,e)\log\left(x+2\right)-\frac{1}{36}\,(7\,d+19\,f-13\,e)\log\left(x+1\right)-\frac{1}{36}\,(d+f+e)\log\left(x-1\right)+\frac{1}{144}\,(d+4\,f+2\,e)\log\left(x-2\right)-\frac{(3\,d+6\,f-4\,e)x+5\,d+8\,f-6\,e}{12\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(31*d + 76*f - 50*e)*log(x + 2) - 1/36*(7*d + 19*f - 13*e)*log(x + 1)
 - 1/36*(d + f + e)*log(x - 1) + 1/144*(d + 4*f + 2*e)*log(x - 2) - 1/12*((
3*d + 6*f - 4*e)*x + 5*d + 8*f - 6*e)/(x^2 + 3*x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(95) = 190.
time = 0.47, size = 191, normalized size = 1.82

$$-\frac{12\,(3\,d-4\,e+6\,f)x-((31\,d-50\,e+76\,f)x^2+3\,(31\,d-50\,e+76\,f)x+62\,d-100\,e+152\,f)\log\left(x+2\right)+4\,((7\,d-13\,e+19\,f)x^2+3\,(7\,d-13\,e+19\,f)x+14\,d-26\,e+38\,f)\log\left(x+1\right)+4\,((d+e+f)x^2+3\,(d+e+f)x+2\,d+2\,e+2\,f)\log\left(x-1\right)-((d+2\,e+4\,f)x^2+3\,(d+2\,e+4\,f)x+2\,d+4\,e+8\,f)\log\left(x-2\right)+60\,d-72\,e+96\,f}{144\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f)*x - ((31*d - 50*e + 76*f)*x^2 + 3*(31*d - 50*e
 + 76*f)*x + 62*d - 100*e + 152*f)*log(x + 2) + 4*((7*d - 13*e + 19*f)*x^2
+ 3*(7*d - 13*e + 19*f)*x + 14*d - 26*e + 38*f)*log(x + 1) + 4*((d + e + f)
*x^2 + 3*(d + e + f)*x + 2*d + 2*e + 2*f)*log(x - 1) - ((d + 2*e + 4*f)*x^2
 + 3*(d + 2*e + 4*f)*x + 2*d + 4*e + 8*f)*log(x - 2) + 60*d - 72*e + 96*f)/
(x^2 + 3*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.34, size = 101, normalized size = 0.96

$$\frac{1}{144}\,(31\,d+76\,f-50\,e)\log\left(|x+2|\right)-\frac{1}{36}\,(7\,d+19\,f-13\,e)\log\left(|x+1|\right)-\frac{1}{36}\,(d+f+e)\log\left(|x-1|\right)+\frac{1}{144}\,(d+4\,f+2\,e)\log\left(|x-2|\right)-\frac{(3\,d+6\,f-4\,e)x+5\,d+8\,f-6\,e}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(31*d + 76*f - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19*f - 13*e)*log(a
bs(x + 1)) - 1/36*(d + f + e)*log(abs(x - 1)) + 1/144*(d + 4*f + 2*e)*log(a
bs(x - 2)) - 1/12*((3*d + 6*f - 4*e)*x + 5*d + 8*f - 6*e)/((x + 2)*(x + 1))
```

Mupad [B]
time = 0.83, size = 97, normalized size = 0.92

$$\ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}\right)-\ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}\right)-\ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}\right)+\ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}\right)}{x^2+3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 2)*(d/144 + e/72 + f/36) - log(x + 1)*((7*d)/36 - (13*e)/36 + (19*f
)/36) - log(x - 1)*(d/36 + e/36 + f/36) + log(x + 2)*((31*d)/144 - (25*e)/7
2 + (19*f)/36) - ((5*d)/12 - e/2 + (2*f)/3 + x*(d/4 - e/3 + f/2))/(3*x + x^
2 + 2)
```

3.94 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{d-e+f-g}{6(1+x)}-\frac{d-2e+4f-8g}{12(2+x)}-\frac{1}{36}(d+e+f+g)\log(1-x)+\frac{1}{144}(d+2e+4f+8g)\log(2-x)-\frac{1}{36}(7d-13e$$

```
[Out] 1/6*(-d+e-f+g)/(1+x)+1/12*(-d+2*e-4*f+8*g)/(2+x)-1/36*(d+e+f+g)*ln(1-x)+1/1
44*(d+2*e+4*f+8*g)*ln(2-x)-1/36*(7*d-13*e+19*f-25*g)*ln(1+x)+1/144*(31*d-50
*e+76*f-104*g)*ln(2+x)
```

Rubi [A]
time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1600, 6860}

$$-\frac{d-2e+4f-8g}{12(x+2)}-\frac{d-e+f-g}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g)+\frac{1}{144}\log(2-x)(d+2e+4f+8g)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] -1/6*(d - e + f - g)/(1 + x) - (d - 2*e + 4*f - 8*g)/(12*(2 + x)) - ((d + e
 + f + g)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g)*Log[2 - x])/144 - ((7*d -
 13*e + 19*f - 25*g)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g)*Log[2 +
 x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g}{144(-2+x)} + \frac{-d-e-f-g}{36(-1+x)} + \frac{d-e+f-g}{6(1+x)^2} + \frac{-\right.$$
$$= -\frac{d-e+f-g}{6(1+x)} - \frac{d-2e+4f-8g}{12(2+x)} - \frac{1}{36}(d+e+f+g)\log(1$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 0.97

$$\frac{1}{144}\left(\frac{12(-5d+6e-8f+12g-3dx+4ex-6fx+10gx)}{2+3x+x^2} - 4(d+e+f+g)\log(1-x) + (d+2e+4f+8g)\log(2-x) + 4(-7d+13e-19f+25g)\log(1+x) + (31d-50e+76f-104g)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x
]
```

```
[Out] ((12*(-5*d + 6*e - 8*f + 12*g - 3*d*x + 4*e*x - 6*f*x + 10*g*x))/(2 + 3*x +
 x^2) - 4*(d + e + f + g)*Log[1 - x] + (d + 2*e + 4*f + 8*g)*Log[2 - x] + 4
*(-7*d + 13*e - 19*f + 25*g)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g)*Log[
2 + x])/144
```

Maple [A]
time = 0.05, size = 114, normalized size = 0.97

method	result
default	$-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}}{x+2} + \left(\frac{31d}{144} - \frac{25e}{72} + \frac{19f}{36} - \frac{13g}{18}\right)\ln(x+2) + \left(\frac{d}{144} + \frac{e}{72} + \frac{f}{36} + \frac{g}{18}\right)\ln(x-2) + \left(-\frac{d}{36} - \frac{e}{36} -\right.$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}\right)x^2-\frac{5d}{6}+e+2g-\frac{4f}{3}}{x^4-5x^2+4} + \left(-\frac{7d}{36} + \frac{13e}{36} - \frac{19f}{36} + \frac{25g}{36}\right)\ln(1+x) +$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}+g}{x^2+3x+2} + \frac{\ln(2-x)d}{144} + \frac{\ln(2-x)e}{72} + \frac{\ln(2-x)f}{36} + \frac{\ln(2-x)g}{18} + \frac{31\ln(x+2)d}{144} - \frac{25\ln(x+2)e}{72} + \frac{19\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] -(1/12*d-1/6*e+1/3*f-2/3*g)/(x+2)+(31/144*d-25/72*e+19/36*f-13/18*g)*ln(x+2
)+(1/144*d+1/72*e+1/36*f+1/18*g)*ln(x-2)+(-1/36*d-1/36*e-1/36*f-1/36*g)*ln(
-1+x)+(-7/36*d+13/36*e-19/36*f+25/36*g)*ln(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g)/(
1+x)
```

Maxima [A]
time = 0.29, size = 113, normalized size = 0.97

$$\frac{1}{144}\,(31\,d+76\,f-104\,g-50\,e)\log\left(x+2\right)-\frac{1}{36}\,(7\,d+19\,f-25\,g-13\,e)\log\left(x+1\right)-\frac{1}{36}\,(d+f+g+e)\log\left(x-1\right)+\frac{1}{144}\,(d+4\,f+8\,g+2\,e)\log\left(x-2\right)-\frac{(3\,d+6\,f-10\,g-4\,e)x+5\,d+8\,f-12\,g-6\,e}{12\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/144*(31*d + 76*f - 104*g - 50*e)*log(x + 2) - 1/36*(7*d + 19*f - 25*g - 1
3*e)*log(x + 1) - 1/36*(d + f + g + e)*log(x - 1) + 1/144*(d + 4*f + 8*g +
2*e)*log(x - 2) - 1/12*((3*d + 6*f - 10*g - 4*e)*x + 5*d + 8*f - 12*g - 6*e
)/(x^2 + 3*x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(105) = 210.
time = 0.86, size = 229, normalized size = 1.96

$$-\frac{12\,(3\,d-4\,e+6\,f-10\,g)x-\left((31\,d-50\,e+76\,f-104\,g)x^2+3\,(31\,d-50\,e+76\,f-104\,g)x+62\,d-100\,e+152\,f-208\,g\right)\log\left(x+2\right)+4\left((7\,d-13\,e+19\,f-25\,g)x^2+3\,(7\,d-13\,e+19\,f-25\,g)x+14\,d-26\,e+38\,f-50\,g\right)\log\left(x+1\right)+4\left((d+e+f+g)x^2+3\,(d+e+f+g)x+2\,d+2\,e+2\,f+2\,g\right)\log\left(x-1\right)-\left((d+2\,e+4\,f+8\,g)x^2+3\,(d+2\,e+4\,f+8\,g)x+2\,d+4\,e+8\,f+16\,g\right)\log\left(x-2\right)+60\,d-72\,e+96\,f-144\,g}{144\,(x^2+3\,x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g)*x - ((31*d - 50*e + 76*f - 104*g)*x^2 +
 3*(31*d - 50*e + 76*f - 104*g)*x + 62*d - 100*e + 152*f - 208*g)*log(x + 2
) + 4*((7*d - 13*e + 19*f - 25*g)*x^2 + 3*(7*d - 13*e + 19*f - 25*g)*x + 14
*d - 26*e + 38*f - 50*g)*log(x + 1) + 4*((d + e + f + g)*x^2 + 3*(d + e + f
 + g)*x + 2*d + 2*e + 2*f + 2*g)*log(x - 1) - ((d + 2*e + 4*f + 8*g)*x^2 +
3*(d + 2*e + 4*f + 8*g)*x + 2*d + 4*e + 8*f + 16*g)*log(x - 2) + 60*d - 72*
e + 96*f - 144*g)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.01, size = 117, normalized size = 1.00

$$\frac{1}{144}\,(31\,d+76\,f-104\,g-50\,e)\log\left(|x+2|\right)-\frac{1}{36}\,(7\,d+19\,f-25\,g-13\,e)\log\left(|x+1|\right)-\frac{1}{36}\,(d+f+g+e)\log\left(|x-1|\right)+\frac{1}{144}\,(d+4\,f+8\,g+2\,e)\log\left(|x-2|\right)-\frac{(3\,d+6\,f-10\,g-4\,e)x+5\,d+8\,f-12\,g-6\,e}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/144*(31*d + 76*f - 104*g - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19*f - 25*
g - 13*e)*log(abs(x + 1)) - 1/36*(d + f + g + e)*log(abs(x - 1)) + 1/144*(d
 + 4*f + 8*g + 2*e)*log(abs(x - 2)) - 1/12*((3*d + 6*f - 10*g - 4*e)*x + 5*
d + 8*f - 12*g - 6*e)/((x + 2)*(x + 1))
```

Mupad [B]
time = 0.91, size = 115, normalized size = 0.98

$$\ln(x-2)\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}\right)-\ln(x+1)\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}-\frac{25\,g}{36}\right)-\ln(x-1)\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}\right)+\ln(x+2)\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}-\frac{13\,g}{18}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}-g+x\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5\,g}{6}\right)}{x^2+3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 2)*(d/144 + e/72 + f/36 + g/18) - log(x + 1)*((7*d)/36 - (13*e)/36
+ (19*f)/36 - (25*g)/36) - log(x - 1)*(d/36 + e/36 + f/36 + g/36) + log(x +
 2)*((31*d)/144 - (25*e)/72 + (19*f)/36 - (13*g)/18) - ((5*d)/12 - e/2 + (2
*f)/3 - g + x*(d/4 - e/3 + f/2 - (5*g)/6))/(3*x + x^2 + 2)
```

3.95 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=131

$$-\frac{d-e+f-g+h}{6(1+x)}-\frac{d-2e+4f-8g+16h}{12(2+x)}-\frac{1}{36}(d+e+f+g+h)\log(1-x)+\frac{1}{144}(d+2e+4f+8g+16h)\log$$

```
[Out] 1/6*(-d+e-f+g-h)/(1+x)+1/12*(-d+2*e-4*f+8*g-16*h)/(2+x)-1/36*(d+e+f+g+h)*ln
(1-x)+1/144*(d+2*e+4*f+8*g+16*h)*ln(2-x)-1/36*(7*d-13*e+19*f-25*g+31*h)*ln(
1+x)+1/144*(31*d-50*e+76*f-104*g+112*h)*ln(2+x)
```

Rubi [A]
time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {1600, 6860}

$$-\frac{d-e+f-g+h}{6(x+1)}-\frac{d-2e+4f-8g+16h}{12(x+2)}-\frac{1}{36}\log(1-x)(d+e+f+g+h)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2
,x]
```

```
[Out] -1/6*(d - e + f - g + h)/(1 + x) - (d - 2*e + 4*f - 8*g + 16*h)/(12*(2 + x)
) - ((d + e + f + g + h)*Log[1 - x])/36 + ((d + 2*e + 4*f + 8*g + 16*h)*Log
[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x])/36 + ((31*d -
 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{(2-3x+x^2)(2+3x+x^2)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g+16h}{144(-2+x)} + \frac{-d-e-f-g-h}{36(-1+x)} + \frac{d}{\cdots}\right.$$
$$= -\frac{d-e+f-g+h}{6(1+x)} - \frac{d-2e+4f-8g+16h}{12(2+x)} - \frac{1}{36}(d+\cdots$$

Mathematica [A]
time = 0.04, size = 136, normalized size = 1.04

$$\frac{1}{144}\left(-\frac{12(d(5+3x)+2(4f-6g+10h+3fx-5gx+9hx-e(3+2x)))}{2+3x+x^2} - 4(d+e+f+g+h)\log(1-x) + (d+2(e+2f+4g+8h))\log(2-x) - 4(7d-13e+19f-25g+31h)\log(1+x) + (31d-50e+76f-104g+112h)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] ((-12*(d*(5 + 3*x) + 2*(4*f - 6*g + 10*h + 3*f*x - 5*g*x + 9*h*x - e*(3 + 2
*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h)*Log[1 - x] + (d + 2*(e + 2*f
 + 4*g + 8*h))*Log[2 - x] - 4*(7*d - 13*e + 19*f - 25*g + 31*h)*Log[1 + x]
+ (31*d - 50*e + 76*f - 104*g + 112*h)*Log[2 + x])/144
```

Maple [A]
time = 0.06, size = 132, normalized size = 1.01

method	result
default	$-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}}{x+2}+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}-\frac{13g}{18}+\frac{7h}{9}\right)\ln\left(x+2\right)+\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}\right)\ln\left(x-2\right)+$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}-\frac{3h}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}+2h\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}+\frac{17h}{6}\right)x^2-\frac{5d}{6}+e+2g-\frac{10h}{3}-\frac{4f}{3}}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\frac{13e}{36}-\frac{19f}{36}+\right.$
risch	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}-\frac{3h}{2}\right)x-\frac{5d}{12}+\frac{e}{2}-\frac{2f}{3}+g-\frac{5h}{3}}{x^2+3x+2}-\frac{\ln(-1+x)d}{36}-\frac{\ln(-1+x)e}{36}-\frac{\ln(-1+x)f}{36}-\frac{\ln(-1+x)g}{36}-\frac{\ln(-1+x)h}{36}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] -(1/12*d-1/6*e+1/3*f-2/3*g+4/3*h)/(x+2)+(31/144*d-25/72*e+19/36*f-13/18*g+7
/9*h)*ln(x+2)+(1/144*d+1/72*e+1/36*f+1/18*g+1/9*h)*ln(x-2)+(-1/36*d-1/36*e-
1/36*f-1/36*g-1/36*h)*ln(-1+x)+(-7/36*d+13/36*e-19/36*f+25/36*g-31/36*h)*ln
(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h)/(1+x)
```

Maxima [A]
time = 0.29, size = 129, normalized size = 0.98

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-50\,e\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-13\,e\right)\log\left(x+1\right)-\frac{1}{36}\left(d+f+g+h+e\right)\log\left(x-1\right)+\frac{1}{144}\left(d+4\,f+8\,g+16\,h+2\,e\right)\log\left(x-2\right)-\frac{\left(3\,d+6\,f-10\,g+18\,h-4\,e\right)x+5\,d+8\,f-12\,g+20\,h-6\,e}{12\left(x^{2}+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="maxima")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e)*log(x + 2) - 1/36*(7*d + 19*f -
25*g + 31*h - 13*e)*log(x + 1) - 1/36*(d + f + g + h + e)*log(x - 1) + 1/14
4*(d + 4*f + 8*g + 16*h + 2*e)*log(x - 2) - 1/12*((3*d + 6*f - 10*g + 18*h
- 4*e)*x + 5*d + 8*f - 12*g + 20*h - 6*e)/(x^2 + 3*x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(119) = 238.
time = 5.11, size = 267, normalized size = 2.04

$$-\frac{12\left(3\,d-4\,e+6\,f-10\,g+18\,h\right)x-\left(\left(31\,d-50\,e+76\,f-104\,g+112\,h\right)x^{2}+3\left(31\,d-50\,e+76\,f-104\,g+112\,h\right)x+62\,d-100\,e+152\,f-208\,g+224\,h\right)\log\left(x+2\right)+4\left(\left(7\,d-13\,e+19\,f-25\,g+31\,h\right)x^{2}+3\left(7\,d-13\,e+19\,f-25\,g+31\,h\right)x+14\,d-26\,e+38\,f-50\,g+62\,h\right)\log\left(x+1\right)+4\left(\left(d+e+f+g+h\right)x^{2}+3\left(d+e+f+g+h\right)x+2\,d+2\,e+2\,f+2\,g+2\,h\right)\log\left(x-1\right)-\left(\left(d+2\,e+4\,f+8\,g+16\,h\right)x^{2}+3\left(d+2\,e+4\,f+8\,g+16\,h\right)x+2\,d+4\,e+8\,f+16\,g+32\,h\right)\log\left(x-2\right)+60\,d-72\,e+96\,f-144\,g+240\,h}{144\left(x^{2}+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h)*x - ((31*d - 50*e + 76*f - 104*g
 + 112*h)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h)*x + 62*d - 100*e + 1
52*f - 208*g + 224*h)*log(x + 2) + 4*((7*d - 13*e + 19*f - 25*g + 31*h)*x^2
 + 3*(7*d - 13*e + 19*f - 25*g + 31*h)*x + 14*d - 26*e + 38*f - 50*g + 62*h
)*log(x + 1) + 4*((d + e + f + g + h)*x^2 + 3*(d + e + f + g + h)*x + 2*d +
 2*e + 2*f + 2*g + 2*h)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h)*x^2 + 3*
(d + 2*e + 4*f + 8*g + 16*h)*x + 2*d + 4*e + 8*f + 16*g + 32*h)*log(x - 2)
+ 60*d - 72*e + 96*f - 144*g + 240*h)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.65, size = 133, normalized size = 1.02

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-50\,e\right)\log\left(\left|x+2\right|\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-13\,e\right)\log\left(\left|x+1\right|\right)-\frac{1}{36}\left(d+f+g+h+e\right)\log\left(\left|x-1\right|\right)+\frac{1}{144}\left(d+4\,f+8\,g+16\,h+2\,e\right)\log\left(\left|x-2\right|\right)-\frac{\left(3\,d+6\,f-10\,g+18\,h-4\,e\right)x+5\,d+8\,f-12\,g+20\,h-6\,e}{12\left(x+2\right)\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="giac")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e)*log(abs(x + 2)) - 1/36*(7*d + 19
*f - 25*g + 31*h - 13*e)*log(abs(x + 1)) - 1/36*(d + f + g + h + e)*log(abs
(x - 1)) + 1/144*(d + 4*f + 8*g + 16*h + 2*e)*log(abs(x - 2)) - 1/12*((3*d
+ 6*f - 10*g + 18*h - 4*e)*x + 5*d + 8*f - 12*g + 20*h - 6*e)/((x + 2)*(x +
 1))
```

Mupad [B]
time = 1.33, size = 133, normalized size = 1.02

$$\ln (x-2)\,\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}\right)-\ln (x-1)\,\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}+\frac{h}{36}\right)-\ln (x+1)\,\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}-\frac{25\,g}{36}+\frac{31\,h}{36}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}-g+\frac{5\,h}{3}+x\,\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5\,g}{6}+\frac{3\,h}{2}\right)}{x^2+3\,x+2}+\ln (x+2)\,\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}-\frac{13\,g}{18}+\frac{7\,h}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4)^2
,x)
```

```
[Out] log(x - 2)*(d/144 + e/72 + f/36 + g/18 + h/9) - log(x - 1)*(d/36 + e/36 + f
/36 + g/36 + h/36) - log(x + 1)*((7*d)/36 - (13*e)/36 + (19*f)/36 - (25*g)/
36 + (31*h)/36) - ((5*d)/12 - e/2 + (2*f)/3 - g + (5*h)/3 + x*(d/4 - e/3 +
f/2 - (5*g)/6 + (3*h)/2))/(3*x + x^2 + 2) + log(x + 2)*((31*d)/144 - (25*e)
/72 + (19*f)/36 - (13*g)/18 + (7*h)/9)
```

3.96 $\int \frac{(2-3x+x^2)(d+ex+fx^2+gx^3+hx^4+ix^5)}{(4-5x^2+x^4)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{d-e+f-g+h-i}{6(1+x)}-\frac{d-2e+4f-8g+16h-32i}{12(2+x)}-\frac{1}{36}(d+e+f+g+h+i)\log(1-x)+\frac{1}{144}(d+2e+4f+$$

```
[Out] 1/6*(-d+e-f+g-h+i)/(1+x)+1/12*(-d+2*e-4*f+8*g-16*h+32*i)/(2+x)-1/36*(d+e+f+
g+h+i)*ln(1-x)+1/144*(d+2*e+4*f+8*g+16*h+32*i)*ln(2-x)-1/36*(7*d-13*e+19*f-
25*g+31*h-37*i)*ln(1+x)+1/144*(31*d-50*e+76*f-104*g+112*h-32*i)*ln(2+x)
```

Rubi [A]

time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1600, 6860}

$$-\frac{d-2e+4f-8g+16h-32i}{12(x+2)}-\frac{d-e+f-g+h-i}{6(x+1)}-\frac{1}{36}\log(1-x)(d+e+f+g+h+i)+\frac{1}{144}\log(2-x)(d+2e+4f+8g+16h+32i)-\frac{1}{36}\log(x+1)(7d-13e+19f-25g+31h-37i)+\frac{1}{144}\log(x+2)(31d-50e+76f-104g+112h-32i)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2
+ x^4)^2,x]
```

```
[Out] -1/6*(d - e + f - g + h - i)/(1 + x) - (d - 2*e + 4*f - 8*g + 16*h - 32*i)/
(12*(2 + x)) - ((d + e + f + g + h + i)*Log[1 - x])/36 + ((d + 2*e + 4*f +
8*g + 16*h + 32*i)*Log[2 - x])/144 - ((7*d - 13*e + 19*f - 25*g + 31*h - 37
*i)*Log[1 + x])/36 + ((31*d - 50*e + 76*f - 104*g + 112*h - 32*i)*Log[2 + x
])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(2-3x+x^2)\,(d+ex+fx^2+gx^3+hx^4+96x^5)}{(4-5x^2+x^4)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+96x^5}{(2-3x+x^2)\,(2+3x+x^2)^2}\,dx$$
$$= \int \left(\frac{3072+d+2e+4f+8g+16h}{144(-2+x)} + \frac{-96-d-}{36(}\right.$$
$$= \frac{96-d+e-f+g-h}{6(1+x)} + \frac{3072-d+2e-4f+8}{12(2+x)}$$

Mathematica [A]
time = 0.05, size = 153, normalized size = 1.04

$$\frac{1}{144}\left(\frac{12(-d(5+3x)+2(-4f+6g-10h+18i-3fx+5gx-9hx+17ix+e(3+2x)))}{2+3x+x^2} - 4(d+e+f+g+h+i)\log(1-x) + (d+2e+4(f+2g+4h+8i))\log(2-x) + 4(-7d+13e-19f+25g-31h+37i)\log(1+x) + (31d-50e+76f-104g+112h-32i)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - 3*x + x^2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 -
5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(-(d*(5 + 3*x)) + 2*(-4*f + 6*g - 10*h + 18*i - 3*f*x + 5*g*x - 9*h*x
+ 17*i*x + e*(3 + 2*x))))/(2 + 3*x + x^2) - 4*(d + e + f + g + h + i)*Log[1
 - x] + (d + 2*e + 4*(f + 2*g + 4*h + 8*i))*Log[2 - x] + 4*(-7*d + 13*e - 1
9*f + 25*g - 31*h + 37*i)*Log[1 + x] + (31*d - 50*e + 76*f - 104*g + 112*h
- 32*i)*Log[2 + x])/144
```

Maple [A]
time = 0.07, size = 150, normalized size = 1.02

method	result
default	$-\frac{\frac{d}{12}-\frac{e}{6}+\frac{f}{3}-\frac{2g}{3}+\frac{4h}{3}-\frac{8i}{3}}{x+2}+\left(\frac{31d}{144}-\frac{25e}{72}+\frac{19f}{36}-\frac{13g}{18}-\frac{2i}{9}+\frac{7h}{9}\right)\ln(x+2)+\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}+\frac{2i}{9}\right.$
norman	$\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}+\frac{17i}{6}-\frac{3h}{2}\right)x^3+\left(\frac{3d}{4}-\frac{5e}{6}+f-\frac{4g}{3}+2h-\frac{10i}{3}\right)x+\left(\frac{d}{3}-\frac{e}{2}+\frac{5f}{6}-\frac{3g}{2}+\frac{17h}{6}-\frac{11i}{2}\right)x^2+6i-\frac{5d}{6}+e+2g-\frac{10h}{3}-\frac{4f}{3}}{x^4-5x^2+4}+\left(-\frac{7d}{36}+\right.$
risch	$-\frac{7\ln(-1-x)d}{36}+\frac{13\ln(-1-x)e}{36}-\frac{\ln(-1+x)i}{36}-\frac{2\ln(x+2)i}{9}-\frac{\ln(-1+x)h}{36}-\frac{13\ln(x+2)g}{18}+\frac{\ln(2-x)g}{18}+\frac{\left(-\frac{d}{4}+\frac{e}{3}-\frac{f}{2}+\frac{5g}{6}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_R
ETURNVERBOSE)
```

```
[Out] -(1/12*d-1/6*e+1/3*f-2/3*g+4/3*h-8/3*i)/(x+2)+(31/144*d-25/72*e+19/36*f-13/
18*g-2/9*i+7/9*h)*ln(x+2)+(1/144*d+1/72*e+1/36*f+1/18*g+1/9*h+2/9*i)*ln(x-2
)+(-1/36*d-1/36*e-1/36*f-1/36*g-1/36*h-1/36*i)*ln(-1+x)+(-7/36*d+13/36*e-19
/36*f+25/36*g-31/36*h+37/36*i)*ln(1+x)-(1/6*d-1/6*e+1/6*f-1/6*g+1/6*h-1/6*i
)/(1+x)
```

Maxima [A]
time = 0.28, size = 135, normalized size = 0.92

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-50\,e-32i\right)\log\left(x+2\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-13\,e-37i\right)\log\left(x+1\right)-\frac{1}{36}\left(d+f+g+h+e+i\right)\log\left(x-1\right)+\frac{1}{144}\left(d+4\,f+8\,g+16\,h+2\,e+32i\right)\log\left(x-2\right)-\frac{\left(3\,d+6\,f-10\,g+18\,h-4\,e-34i\right)x+5\,d+8\,f-12\,g+20\,h-6\,e-36i}{12\left(x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="maxima")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e - 32*I)*log(x + 2) - 1/36*(7*d +
19*f - 25*g + 31*h - 13*e - 37*I)*log(x + 1) - 1/36*(d + f + g + h + e + I)
*log(x - 1) + 1/144*(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(x - 2) - 1/12*(
(3*d + 6*f - 10*g + 18*h - 4*e - 34*I)*x + 5*d + 8*f - 12*g + 20*h - 6*e -
36*I)/(x^2 + 3*x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(135) = 270$.
time = 20.44, size = 305, normalized size = 2.07

$$-\frac{12\left(3\,d-4\,e+6\,f-10\,g+18\,h-34\,i\right)x-\left(\left(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i\right)x^2+3\left(31\,d-50\,e+76\,f-104\,g+112\,h-32\,i\right)x+62\,d-100\,e+152\,f-208\,g+224\,h-64\,i\right)\log\left(x+2\right)+4\left(\left(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i\right)x^2+3\left(7\,d-13\,e+19\,f-25\,g+31\,h-37\,i\right)x+14\,d-26\,e+38\,f-50\,g+62\,h-74\,i\right)\log\left(x+1\right)+4\left(\left(d+e+f+g+h+i\right)x^2+3\left(d+e+f+g+h+i\right)x+2\,d+2\,e+2\,f+2\,g+2\,h+2\,i\right)\log\left(x-1\right)-\left(\left(d+2\,e+4\,f+8\,g+16\,h+32\,i\right)x^2+3\left(d+2\,e+4\,f+8\,g+16\,h+32\,i\right)x+2\,d+4\,e+8\,f+16\,g+32\,h+64\,i\right)\log\left(x-2\right)+60\,d-72\,e+96\,f-144\,g+240\,h-432\,i}{144\left(x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="fricas")
```

```
[Out] -1/144*(12*(3*d - 4*e + 6*f - 10*g + 18*h - 34*i)*x - ((31*d - 50*e + 76*f
- 104*g + 112*h - 32*i)*x^2 + 3*(31*d - 50*e + 76*f - 104*g + 112*h - 32*i)
*x + 62*d - 100*e + 152*f - 208*g + 224*h - 64*i)*log(x + 2) + 4*((7*d - 13
*e + 19*f - 25*g + 31*h - 37*i)*x^2 + 3*(7*d - 13*e + 19*f - 25*g + 31*h -
37*i)*x + 14*d - 26*e + 38*f - 50*g + 62*h - 74*i)*log(x + 1) + 4*((d + e +
 f + g + h + i)*x^2 + 3*(d + e + f + g + h + i)*x + 2*d + 2*e + 2*f + 2*g +
 2*h + 2*i)*log(x - 1) - ((d + 2*e + 4*f + 8*g + 16*h + 32*i)*x^2 + 3*(d +
2*e + 4*f + 8*g + 16*h + 32*i)*x + 2*d + 4*e + 8*f + 16*g + 32*h + 64*i)*lo
g(x - 2) + 60*d - 72*e + 96*f - 144*g + 240*h - 432*i)/(x^2 + 3*x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x+2)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)*
*2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.49, size = 139, normalized size = 0.95

$$\frac{1}{144}\left(31\,d+76\,f-104\,g+112\,h-50\,e-32i\right)\log\left(|x+2|\right)-\frac{1}{36}\left(7\,d+19\,f-25\,g+31\,h-13\,e-37i\right)\log\left(|x+1|\right)-\frac{1}{36}\left(d+f+g+h+e+i\right)\log\left(|x-1|\right)+\frac{1}{144}\left(d+4\,f+8\,g+16\,h+2\,e+32i\right)\log\left(|x-2|\right)-\frac{(3\,d+6\,f-10\,g+18\,h-4\,e-34i)x+5\,d+8\,f-12\,g+20\,h-6\,e-36i}{12\,(x+2)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, al
gorithm="giac")
```

```
[Out] 1/144*(31*d + 76*f - 104*g + 112*h - 50*e - 32*I)*log(abs(x + 2)) - 1/36*(7
*d + 19*f - 25*g + 31*h - 13*e - 37*I)*log(abs(x + 1)) - 1/36*(d + f + g +
h + e + I)*log(abs(x - 1)) + 1/144*(d + 4*f + 8*g + 16*h + 2*e + 32*I)*log(
abs(x - 2)) - 1/12*((3*d + 6*f - 10*g + 18*h - 4*e - 34*I)*x + 5*d + 8*f -
12*g + 20*h - 6*e - 36*I)/((x + 2)*(x + 1))
```

Mupad [B]
time = 1.68, size = 151, normalized size = 1.03

$$\ln\left(x-2\right)\,\left(\frac{d}{144}+\frac{e}{72}+\frac{f}{36}+\frac{g}{18}+\frac{h}{9}+\frac{2\,i}{9}\right)-\ln\left(x-1\right)\,\left(\frac{d}{36}+\frac{e}{36}+\frac{f}{36}+\frac{g}{36}+\frac{h}{36}+\frac{i}{36}\right)-\ln\left(x+1\right)\,\left(\frac{7\,d}{36}-\frac{13\,e}{36}+\frac{19\,f}{36}-\frac{25\,g}{36}+\frac{31\,h}{36}-\frac{37\,i}{36}\right)+\ln\left(x+2\right)\,\left(\frac{31\,d}{144}-\frac{25\,e}{72}+\frac{19\,f}{36}-\frac{13\,g}{18}+\frac{7\,h}{9}-\frac{2\,i}{9}\right)-\frac{\frac{5\,d}{12}-\frac{e}{2}+\frac{2\,f}{3}-g+\frac{5\,h}{3}-3\,i+x\,\left(\frac{d}{4}-\frac{e}{3}+\frac{f}{2}-\frac{5\,g}{6}+\frac{3\,h}{2}-\frac{17\,i}{6}\right)}{x^2+3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 3*x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^
2 + 4)^2,x)
```

```
[Out] log(x - 2)*(d/144 + e/72 + f/36 + g/18 + h/9 + (2*i)/9) - log(x - 1)*(d/36
+ e/36 + f/36 + g/36 + h/36 + i/36) - log(x + 1)*((7*d)/36 - (13*e)/36 + (1
9*f)/36 - (25*g)/36 + (31*h)/36 - (37*i)/36) + log(x + 2)*((31*d)/144 - (25
*e)/72 + (19*f)/36 - (13*g)/18 + (7*h)/9 - (2*i)/9) - ((5*d)/12 - e/2 + (2*
f)/3 - g + (5*h)/3 - 3*i + x*(d/4 - e/3 + f/2 - (5*g)/6 + (3*h)/2 - (17*i)/
6))/(3*x + x^2 + 2)
```

3.97 $\int \frac{2+x}{(4-5x^2+x^4)^2} \, dx$

Optimal. Leaf size=68

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(1+x)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(1+x)+\frac{1}{144}\log(2+x)$$

```
[Out] 1/12/(1-x)+1/36/(2-x)-1/36/(1+x)+1/18*ln(1-x)-35/432*ln(2-x)+1/54*ln(1+x)+1
/144*ln(2+x)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1600, 2099}

$$\frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(x+1)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(x+1)+\frac{1}{144}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] 1/(12*(1 - x)) + 1/(36*(2 - x)) - 1/(36*(1 + x)) + Log[1 - x]/18 - (35*Log[
2 - x])/432 + Log[1 + x]/54 + Log[2 + x]/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{(4-5x^2+x^4)^2} \, dx &= \int \frac{1}{(2+x)(2-x-2x^2+x^3)^2} \, dx \\
&= \int \left(\frac{1}{36(-2+x)^2}-\frac{35}{432(-2+x)}+\frac{1}{12(-1+x)^2}+\frac{1}{18(-1+x)}+\frac{1}{36(1+x)^2}+ \right. \\
&= \frac{1}{12(1-x)}+\frac{1}{36(2-x)}-\frac{1}{36(1+x)}+\frac{1}{18}\log(1-x)-\frac{35}{432}\log(2-x)+\frac{1}{54}\log(1
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.88

$$\frac{1}{432}\left(\frac{12(5+6x-5x^2)}{2-x-2x^2+x^3}+24\log(1-x)-35\log(2-x)+8\log(1+x)+3\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(5 + 6*x - 5*x^2))/(2 - x - 2*x^2 + x^3) + 24*Log[1 - x] - 35*Log[2 -
x] + 8*Log[1 + x] + 3*Log[2 + x])/432
```

Maple [A]
time = 0.03, size = 47, normalized size = 0.69

method	result	size
default	$\frac{\ln(x+2)}{144}-\frac{1}{36(x-2)}-\frac{35\ln(x-2)}{432}-\frac{1}{12(-1+x)}+\frac{\ln(-1+x)}{18}-\frac{1}{36(1+x)}+\frac{\ln(1+x)}{54}$	47
risch	$\frac{-\frac{5}{36}x^2+\frac{1}{6}x+\frac{5}{36}}{x^3-2x^2-x+2}-\frac{35\ln(x-2)}{432}+\frac{\ln(-1+x)}{18}+\frac{\ln(1+x)}{54}+\frac{\ln(x+2)}{144}$	52
norman	$\frac{-\frac{1}{9}x^2+\frac{17}{36}x-\frac{5}{36}x^3+\frac{5}{18}}{x^4-5x^2+4}-\frac{35\ln(x-2)}{432}+\frac{\ln(-1+x)}{18}+\frac{\ln(1+x)}{54}+\frac{\ln(x+2)}{144}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/144*ln(x+2)-1/36/(x-2)-35/432*ln(x-2)-1/12/(-1+x)+1/18*ln(-1+x)-1/36/(1+x
)+1/54*ln(1+x)
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.76

$$-\frac{5\,x^2-6\,x-5}{36\,(x^3-2\,x^2-x+2)}+\frac{1}{144}\,\log\left(x+2\right)+\frac{1}{54}\,\log\left(x+1\right)+\frac{1}{18}\,\log\left(x-1\right)-\frac{35}{432}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/(x^3 - 2*x^2 - x + 2) + 1/144*log(x + 2) + 1/54*log
(x + 1) + 1/18*log(x - 1) - 35/432*log(x - 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(50) = 100.
time = 0.39, size = 103, normalized size = 1.51

$$-\frac{60\,x^2-3\,(x^3-2\,x^2-x+2)\log\left(x+2\right)-8\,(x^3-2\,x^2-x+2)\log\left(x+1\right)-24\,(x^3-2\,x^2-x+2)\log\left(x-1\right)+35\,(x^3-2\,x^2-x+2)\log\left(x-2\right)-72\,x-60}{432\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(60*x^2 - 3*(x^3 - 2*x^2 - x + 2)*log(x + 2) - 8*(x^3 - 2*x^2 - x +
2)*log(x + 1) - 24*(x^3 - 2*x^2 - x + 2)*log(x - 1) + 35*(x^3 - 2*x^2 - x +
 2)*log(x - 2) - 72*x - 60)/(x^3 - 2*x^2 - x + 2)
```

Sympy [A]
time = 0.14, size = 53, normalized size = 0.78

$$\frac{-5x^2+6x+5}{36x^3-72x^2-36x+72}-\frac{35\log{(x-2)}}{432}+\frac{\log{(x-1)}}{18}+\frac{\log{(x+1)}}{54}+\frac{\log{(x+2)}}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**4-5*x**2+4)**2,x)
```

```
[Out] (-5*x**2 + 6*x + 5)/(36*x**3 - 72*x**2 - 36*x + 72) - 35*log(x - 2)/432 + l
og(x - 1)/18 + log(x + 1)/54 + log(x + 2)/144
```

Giac [A]
time = 3.99, size = 56, normalized size = 0.82

$$-\frac{5\,x^2-6\,x-5}{36\,(x+1)(x-1)(x-2)}+\frac{1}{144}\,\log\left(|x+2|\right)+\frac{1}{54}\,\log\left(|x+1|\right)+\frac{1}{18}\,\log\left(|x-1|\right)-\frac{35}{432}\,\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] -1/36*(5*x^2 - 6*x - 5)/((x + 1)*(x - 1)*(x - 2)) + 1/144*log(abs(x + 2)) +
 1/54*log(abs(x + 1)) + 1/18*log(abs(x - 1)) - 35/432*log(abs(x - 2))
```

Mupad [B]
time = 0.05, size = 52, normalized size = 0.76

$$\frac{\ln\left(x-1\right)}{18}+\frac{\ln\left(x+1\right)}{54}-\frac{35\,\ln\left(x-2\right)}{432}+\frac{\ln\left(x+2\right)}{144}-\frac{-\frac{5\,x^2}{36}+\frac{x}{6}+\frac{5}{36}}{-x^3+2\,x^2+x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)/18 + log(x + 1)/54 - (35*log(x - 2))/432 + log(x + 2)/144 - (x/6
 - (5*x^2)/36 + 5/36)/(x + 2*x^2 - x^3 - 2)
```

3.98 $\int \frac{(2+x)(d+ex)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=105

$$\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}-\frac{d-e}{36(1+x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(1+x)+\frac{1}{1}$$

```
[Out] 1/12*(d+e)/(1-x)+1/36*(d+2*e)/(2-x)+1/36*(-d+e)/(1+x)+1/36*(2*d+5*e)*ln(1-x
)-1/432*(35*d+58*e)*ln(2-x)+1/108*(2*d+e)*ln(1+x)+1/144*(d-2*e)*ln(2+x)
```

Rubi [A]
time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1600, 6874}

$$-\frac{d-e}{36(x+1)}+\frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log(2-x)+\frac{1}{108}(2d+e)\log(x+1)+\frac{1}{144}(d-2e)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e)/(12*(1 - x)) + (d + 2*e)/(36*(2 - x)) - (d - e)/(36*(1 + x)) + ((2*
d + 5*e)*Log[1 - x])/36 - ((35*d + 58*e)*Log[2 - x])/432 + ((2*d + e)*Log[1
 + x])/108 + ((d - 2*e)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x)(d+ex)}{\left(4-5x^2+x^4\right)^2}\,dx &= \int \frac{d+ex}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx\\ &= \int \left(\frac{d+2e}{36(-2+x)^2}+\frac{-35d-58e}{432(-2+x)}+\frac{d+e}{12(-1+x)^2}+\frac{2d+5e}{36(-1+x)}+\frac{d-e}{36(1+x)^2}+\frac{}{10}\right.\\ &= \frac{d+e}{12(1-x)}+\frac{d+2e}{36(2-x)}-\frac{d-e}{36(1+x)}+\frac{1}{36}(2d+5e)\log(1-x)-\frac{1}{432}(35d+58e)\log\end{aligned}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.92

$$\frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2e(5-2x^2))}{2-x-2x^2+x^3}+12(2d+5e)\log(1-x)-(35d+58e)\log(2-x)+4(2d+e)\log(1+x)+3(d-2e)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*e*(5 - 2*x^2)))/(2 - x - 2*x^2 + x^3) + 12*(2
*d + 5*e)*Log[1 - x] - (35*d + 58*e)*Log[2 - x] + 4*(2*d + e)*Log[1 + x] +
3*(d - 2*e)*Log[2 + x])/432
```

Maple [A]
time = 0.04, size = 92, normalized size = 0.88

method	result
default	$\left(\frac{d}{144}-\frac{e}{72}\right)\ln\left(x+2\right)+\left(-\frac{35d}{432}-\frac{29e}{216}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}}{x-2}-\frac{\frac{d}{12}+\frac{e}{12}}{-1+x}+\left(\frac{d}{18}+\frac{5e}{36}\right)\ln\left(-1+x\right)-\frac{\frac{d}{36}-\frac{e}{3}}{1+x}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}\right)\ln\left(x-2\right)+\left(\frac{d}{18}+\frac{5e}{36}\right)\ln\left(-1+x\right)+\left(\frac{d}{54}\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}\right)x^2+\frac{dx}{6}+\frac{5d}{36}+\frac{5e}{18}}{x^3-2x^2-x+2}-\frac{35\ln(2-x)d}{432}-\frac{29\ln(2-x)e}{216}+\frac{\ln(x+2)d}{144}-\frac{\ln(x+2)e}{72}+\frac{\ln(-1-x)d}{54}+\frac{\ln(-1-x)e}{108}+\frac{\ln(-1+}{18}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/144*d-1/72*e)*ln(x+2)+(-35/432*d-29/216*e)*ln(x-2)-(1/36*d+1/18*e)/(x-2)
-(1/12*d+1/12*e)/(-1+x)+(1/18*d+5/36*e)*ln(-1+x)-(1/36*d-1/36*e)/(1+x)+(1/5
4*d+1/108*e)*ln(1+x)
```

Maxima [A]
time = 0.29, size = 94, normalized size = 0.90

$$\frac{1}{144}\left(d-2\,e\right)\log\left(x+2\right)+\frac{1}{108}\left(2\,d+e\right)\log\left(x+1\right)+\frac{1}{36}\left(2\,d+5\,e\right)\log\left(x-1\right)-\frac{1}{432}\left(35\,d+58\,e\right)\log\left(x-2\right)-\frac{(5\,d+4\,e)x^2-6\,dx-5\,d-10\,e}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d - 2*e)*log(x + 2) + 1/108*(2*d + e)*log(x + 1) + 1/36*(2*d + 5*e)*
log(x - 1) - 1/432*(35*d + 58*e)*log(x - 2) - 1/36*((5*d + 4*e)*x^2 - 6*d*x
 - 5*d - 10*e)/(x^3 - 2*x^2 - x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(87) = 174.
time = 0.42, size = 211, normalized size = 2.01

$$-\frac{12\,(5\,d+4\,e)x^2-72\,dx-3\,((d-2\,e)x^3-2\,(d-2\,e)x^2-(d-2\,e)x+2\,d-4\,e)\log\left(x+2\right)-4\,((2\,d+e)x^3-2\,(2\,d+e)x^2-(2\,d+e)x+4\,d+2\,e)\log\left(x+1\right)-12\,((2\,d+5\,e)x^3-2\,(2\,d+5\,e)x^2-(2\,d+5\,e)x+4\,d+10\,e)\log\left(x-1\right)+((35\,d+58\,e)x^3-2\,(35\,d+58\,e)x^2-(35\,d+58\,e)x+70\,d+116\,e)\log\left(x-2\right)-60\,d-120\,e}{432\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")

[Out] -1/432*(12*(5*d + 4*e)*x^2 - 72*d*x - 3*((d - 2*e)*x^3 - 2*(d - 2*e)*x^2 - (d - 2*e)*x + 2*d - 4*e)*log(x + 2) - 4*((2*d + e)*x^3 - 2*(2*d + e)*x^2 - (2*d + e)*x + 4*d + 2*e)*log(x + 1) - 12*((2*d + 5*e)*x^3 - 2*(2*d + 5*e)*x^2 - (2*d + 5*e)*x + 4*d + 10*e)*log(x - 1) + ((35*d + 58*e)*x^3 - 2*(35*d + 58*e)*x^2 - (35*d + 58*e)*x + 70*d + 116*e)*log(x - 2) - 60*d - 120*e)/(x^3 - 2*x^2 - x + 2)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(82) = 164.
time = 6.03, size = 1034, normalized size = 9.85

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)*(e*x+d)/(x**4-5*x**2+4)**2,x)

[Out] (d - 2*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*(d - 2*e)/4 + 364910432*d**3*e**2 - 18128055*d**3*e*(d - 2*e) - 83772*d**3*(d - 2*e)**2 + 686697536*d**2*e**3 - 60296868*d**2*e**2*(d - 2*e) - 597816*d**2*e*(d - 2*e)**2 + 65907*d**2*(d - 2*e)**3/4 + 614357568*d*e**4 - 85949220*d*e**3*(d - 2*e) - 1500048*d*e**2*(d - 2*e)**2 + 105840*d*e*(d - 2*e)**3 + 208470400*e**5 - 45136356*e**4*(d - 2*e) - 1196064*e**3*(d - 2*e)**2 + 128277*e**2*(d - 2*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/144 + (2*d + e)*log(x + (8710660*d**5 + 91884504*d**4*e - 2526593*d**4*(2*d + e) + 364910432*d**3*e**2 - 24170740*d**3*e*(2*d + e) - 148928*d**3*(2*d + e)**2 + 686697536*d**2*e**3 - 80395824*d**2*e**2*(2*d + e) - 1062784*d**2*e*(2*d + e)**2 + 39056*d**2*(2*d + e)**3 + 614357568*d*e**4 - 114598960*d*e**3*(2*d + e) - 2666752*d*e**2*(2*d + e)**2 + 250880*d*e*(2*d + e)**3 + 208470400*e**5 - 60181808*e**4*(2*d + e) - 2126336*e**3*(2*d + e)**2 + 304064*e**2*(2*d + e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/108 + (2*d + 5*e)*log(x + (8710660*d**5 + 91884504*d**4*e - 7579779*d**4*(2*d + 5*e) + 364910432*d**3*e**2 - 72512220*d**3*e*(2*d + 5*e) - 1340352*d**3*(2*d + 5*e)**2 + 686697536*d**2*e**3 - 241187472*d**2*e**2*(2*d + 5*e) - 9565056*d**2*e*(2*d + 5*e)**2 + 1054512*d**2*(2*d + 5*e)**3 + 614357568*d*e**4 - 343796880*d*e**3*(2*d + 5*e) - 24000768*d*e**2*(2*d + 5*e)**2 + 6773760*d*e*(2*d + 5*e)**3 + 208470400*e**5 - 180545424*e**4*(2*d + 5*e) - 19137024*e**3*(2*d + 5*e)**2 + 8209728*e**2*(2*d + 5*e)**3)/(3374210*d**5 + 38645295*d**4*e + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320*e**5))/36 - (35*d + 58*e)*log(x + (8710660*d**5 + 91884504*d**4*e + 2526593*d**4*(35*d + 58*e)/4 + 364910432*d**3*e**2 + 6042685*d**3*e*(35*d + 58*e) - 9308*d**3*(35*d + 58*e)

```
**2 + 686697536*d**2*e**3 + 20098956*d**2*e**2*(35*d + 58*e) - 66424*d**2*e
*(35*d + 58*e)**2 - 2441*d**2*(35*d + 58*e)**3/4 + 614357568*d*e**4 + 28649
740*d*e**3*(35*d + 58*e) - 166672*d*e**2*(35*d + 58*e)**2 - 3920*d*e*(35*d
+ 58*e)**3 + 208470400*e**5 + 15045452*e**4*(35*d + 58*e) - 132896*e**3*(35
*d + 58*e)**2 - 4751*e**2*(35*d + 58*e)**3)/(3374210*d**5 + 38645295*d**4*e
 + 170558380*d**3*e**2 + 362061760*d**2*e**3 + 370298160*d*e**4 + 146466320
*e**5))/432 + (6*d*x + 5*d + 10*e + x**2*(-5*d - 4*e))/(36*x**3 - 72*x**2 -
 36*x + 72)
```

Giac [A]
time = 3.39, size = 98, normalized size = 0.93

$$\frac{1}{144}\,(d-2\,e)\log\left(|x+2|\right)+\frac{1}{108}\,(2\,d+e)\log\left(|x+1|\right)+\frac{1}{36}\,(2\,d+5\,e)\log\left(|x-1|\right)-\frac{1}{432}\,(35\,d+58\,e)\log\left(|x-2|\right)-\frac{(5\,d+4\,e)x^2-6\,dx-5\,d-10\,e}{36\,(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d - 2*e)*log(abs(x + 2)) + 1/108*(2*d + e)*log(abs(x + 1)) + 1/36*(2
*d + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 58*e)*log(abs(x - 2)) - 1/36*((5*
d + 4*e)*x^2 - 6*d*x - 5*d - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B]
time = 0.09, size = 90, normalized size = 0.86

$$\ln\left(x-1\right)\,\left(\frac{d}{18}+\frac{5\,e}{36}\right)-\frac{\left(-\frac{5\,d}{36}-\frac{e}{9}\right)\,x^2+\frac{d\,x}{6}+\frac{5\,d}{36}+\frac{5\,e}{18}}{-x^3+2\,x^2+x-2}+\ln\left(x+1\right)\,\left(\frac{d}{54}+\frac{e}{108}\right)+\ln\left(x+2\right)\,\left(\frac{d}{144}-\frac{e}{72}\right)-\ln\left(x-2\right)\,\left(\frac{35\,d}{432}+\frac{29\,e}{216}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/18 + (5*e)/36) - ((5*d)/36 + (5*e)/18 - x^2*((5*d)/36 + e/9)
+ (d*x)/6)/(x + 2*x^2 - x^3 - 2) + log(x + 1)*(d/54 + e/108) + log(x + 2)*(
d/144 - e/72) - log(x - 2)*((35*d)/432 + (29*e)/216)
```

3.99 $\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=122

$$\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}-\frac{d-e+f}{36(1+x)}+\frac{1}{36}(2d+5e+8f)\log(1-x)-\frac{1}{432}(35d+58e+92f)\log(2-x)+\frac{1}{108}(2d+e-$$

```
[Out] 1/12*(d+e+f)/(1-x)+1/36*(d+2*e+4*f)/(2-x)+1/36*(-d+e-f)/(1+x)+1/36*(2*d+5*e
+8*f)*ln(1-x)-1/432*(35*d+58*e+92*f)*ln(2-x)+1/108*(2*d+e-4*f)*ln(1+x)+1/14
4*(d-2*e+4*f)*ln(2+x)
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1600, 6874}

$$-\frac{d-e+f}{36(x+1)}+\frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f)-\frac{1}{432}\log(2-x)(35d+58e+92f)+\frac{1}{108}\log(x+1)(2d+e-4f)+\frac{1}{144}\log(x+2)(d-2e+4f)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f)/(12*(1 - x)) + (d + 2*e + 4*f)/(36*(2 - x)) - (d - e + f)/(36*(
1 + x)) + ((2*d + 5*e + 8*f)*Log[1 - x])/36 - ((35*d + 58*e + 92*f)*Log[2 -
 x])/432 + ((2*d + e - 4*f)*Log[1 + x])/108 + ((d - 2*e + 4*f)*Log[2 + x])/
144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f}{36(-2+x)^2}+\frac{-35d-58e-92f}{432(-2+x)}+\frac{d+e+f}{12(-1+x)^2}+\frac{2d+5e+8f}{36(-1+x)}+\frac{d}{36}\right.$$
$$= \frac{d+e+f}{12(1-x)}+\frac{d+2e+4f}{36(2-x)}-\frac{d-e+f}{36(1+x)}+\frac{1}{36}(2d+5e+8f)\log(1-x)-\frac{1}{432}$$

Mathematica [A]
time = 0.04, size = 121, normalized size = 0.99

$$\frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+e(10-4x^2)+2f(4+3x-4x^2))}{2-x-2x^2+x^3}+12(2d+5e+8f)\log(1-x)-(35d+58e+92f)\log(2-x)+4(2d+e-4f)\log(1+x)+3(d-2e+4f)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + e*(10 - 4*x^2) + 2*f*(4 + 3*x - 4*x^2)))/(2 - x
 - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f)*Log[1 - x] - (35*d + 58*e + 92*f)*Lo
g[2 - x] + 4*(2*d + e - 4*f)*Log[1 + x] + 3*(d - 2*e + 4*f)*Log[2 + x])/432
```

Maple [A]
time = 0.04, size = 113, normalized size = 0.93

method	result
default	$\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}\right)\ln\left(x+2\right)+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}}{x-2}-\frac{\frac{d}{12}+\frac{e}{12}+\frac{f}{12}}{-1+x}+\left(\frac{d}{18}+\frac{5e}{36}+$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{4f}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}\right)\ln\left(x-2\right)+\left(\frac{d}{18}+\frac{5e}{36}\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}}{x^3-2x^2-x+2}+\frac{\ln(-1-x)d}{54}+\frac{\ln(-1-x)e}{108}-\frac{\ln(-1-x)f}{27}+\frac{\ln(x+2)d}{144}-\frac{\ln(x+2)e}{72}+\frac{\ln(x+2)}{36}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/144*d-1/72*e+1/36*f)*ln(x+2)+(-35/432*d-29/216*e-23/108*f)*ln(x-2)-(1/36
*d+1/18*e+1/9*f)/(x-2)-(1/12*d+1/12*e+1/12*f)/(-1+x)+(1/18*d+5/36*e+2/9*f)*
ln(-1+x)-(1/36*d-1/36*e+1/36*f)/(1+x)+(1/54*d+1/108*e-1/27*f)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 114, normalized size = 0.93

$$\frac{1}{144}\left(d+4f-2e\right)\log\left(x+2\right)+\frac{1}{108}\left(2d-4f+e\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+8f+5e\right)\log\left(x-1\right)-\frac{1}{432}\left(35d+92f+58e\right)\log\left(x-2\right)-\frac{\left(5d+8f+4e\right)x^2-6\left(d+f\right)x-5d-8f-10e}{36\left(x^3-2x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d + 4*f - 2*e)*log(x + 2) + 1/108*(2*d - 4*f + e)*log(x + 1) + 1/36*
(2*d + 8*f + 5*e)*log(x - 1) - 1/432*(35*d + 92*f + 58*e)*log(x - 2) - 1/36
*((5*d + 8*f + 4*e)*x^2 - 6*(d + f)*x - 5*d - 8*f - 10*e)/(x^3 - 2*x^2 - x
+ 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(104) = 208$.
time = 0.48, size = 267, normalized size = 2.19

$$-\frac{12\left(5d+4e+8f\right)x^2-72\left(d+f\right)x-3\left(\left(d-2e+4f\right)x^3-2\left(d-2e+4f\right)x^2-\left(d-2e+4f\right)x+2d-4e+8f\right)\log\left(x+2\right)-4\left(\left(2d+e-4f\right)x^3-2\left(2d+e-4f\right)x^2-\left(2d+e-4f\right)x+4d+2e-8f\right)\log\left(x+1\right)-12\left(\left(2d+5e+8f\right)x^3-2\left(2d+5e+8f\right)x^2-\left(2d+5e+8f\right)x+4d+10e+16f\right)\log\left(x-1\right)+\left(\left(35d+58e+92f\right)x^3-2\left(35d+58e+92f\right)x^2-\left(35d+58e+92f\right)x+70d+116e+184f\right)\log\left(x-2\right)-60d-120e-96f}{432\left(x^3-2x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f)*x^3 -
2*(d - 2*e + 4*f)*x^2 - (d - 2*e + 4*f)*x + 2*d - 4*e + 8*f)*log(x + 2) - 4
*((2*d + e - 4*f)*x^3 - 2*(2*d + e - 4*f)*x^2 - (2*d + e - 4*f)*x + 4*d + 2
*e - 8*f)*log(x + 1) - 12*((2*d + 5*e + 8*f)*x^3 - 2*(2*d + 5*e + 8*f)*x^2
- (2*d + 5*e + 8*f)*x + 4*d + 10*e + 16*f)*log(x - 1) + ((35*d + 58*e + 92*
f)*x^3 - 2*(35*d + 58*e + 92*f)*x^2 - (35*d + 58*e + 92*f)*x + 70*d + 116*e
 + 184*f)*log(x - 2) - 60*d - 120*e - 96*f)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.57, size = 118, normalized size = 0.97

$$\frac{1}{144}\left(d+4f-2e\right)\log\left(|x+2|\right)+\frac{1}{108}\left(2d-4f+e\right)\log\left(|x+1|\right)+\frac{1}{36}\left(2d+8f+5e\right)\log\left(|x-1|\right)-\frac{1}{432}\left(35d+92f+58e\right)\log\left(|x-2|\right)-\frac{(5d+8f+4e)x^2-6(d+f)x-5d-8f-10e}{36(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d + 4*f - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + e)*log(abs(x + 1
)) + 1/36*(2*d + 8*f + 5*e)*log(abs(x - 1)) - 1/432*(35*d + 92*f + 58*e)*lo
g(abs(x - 2)) - 1/36*((5*d + 8*f + 4*e)*x^2 - 6*(d + f)*x - 5*d - 8*f - 10*
e)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B]
time = 0.13, size = 113, normalized size = 0.93

$$\ln\left(x-1\right)\left(\frac{d}{18}+\frac{5e}{36}+\frac{2f}{9}\right)+\ln\left(x+1\right)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}\right)+\ln\left(x+2\right)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}\right)-\ln\left(x-2\right)\left(\frac{35d}{432}+\frac{29e}{216}+\frac{23f}{108}\right)-\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}}{-x^3+2x^2+x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9) + log(x + 1)*(d/54 + e/108 - f/27) +
 log(x + 2)*(d/144 - e/72 + f/36) - log(x - 2)*((35*d)/432 + (29*e)/216 + (
23*f)/108) - ((5*d)/36 + (5*e)/18 + (2*f)/9 + x*(d/6 + f/6) - x^2*((5*d)/36
 + e/9 + (2*f)/9))/(x + 2*x^2 - x^3 - 2)
```

3.100 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=141

$$\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}-\frac{d-e+f-g}{36(1+x)}+\frac{1}{36}(2d+5e+8f+11g)\log(1-x)-\frac{1}{432}(35d+58e+92f+13$$

```
[Out] 1/12*(d+e+f+g)/(1-x)+1/36*(d+2*e+4*f+8*g)/(2-x)+1/36*(-d+e-f+g)/(1+x)+1/36*
(2*d+5*e+8*f+11*g)*ln(1-x)-1/432*(35*d+58*e+92*f+136*g)*ln(2-x)+1/108*(2*d+
e-4*f+7*g)*ln(1+x)+1/144*(d-2*e+4*f-8*g)*ln(2+x)
```

Rubi [A]
time = 0.17, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1600, 6874}

$$-\frac{d-e+f-g}{36(x+1)}+\frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g)+\frac{1}{108}\log(x+1)(2d+e-4f+7g)+\frac{1}{144}\log(x+2)(d-2e+4f-8g)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f + g)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g)/(36*(2 - x)) - (d - e
+ f - g)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g)*Log[1 - x])/36 - ((35*d +
 58*e + 92*f + 136*g)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g)*Log[1 + x])/
108 + ((d - 2*e + 4*f - 8*g)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+x)\left(d+ex+fx^2+gx^3\right)}{\left(4-5x^2+x^4\right)^2}\,dx &= \int \frac{d+ex+fx^2+gx^3}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx\\ &= \int \left(\frac{d+2e+4f+8g}{36(-2+x)^2}+\frac{-35d-58e-92f-136g}{432(-2+x)}+\frac{d+e+f+g}{12(-1+x)^2}\right.\\ &= \frac{d+e+f+g}{12(1-x)}+\frac{d+2e+4f+8g}{36(2-x)}-\frac{d-e+f-g}{36(1+x)}+\frac{1}{36}(2d+5e+\end{aligned}$$

Mathematica [A]
time = 0.05, size = 144, normalized size = 1.02

$$\frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2(g(8-5x^2)+f(4+3x-4x^2)+e(5-2x^2)))}{2-x-2x^2+x^3}+12(2d+5e+8f+11g)\log(1-x)-(35d+58e+92f+136g)\log(2-x)+4(2d+e-4f+7g)\log(1+x)+3(d-2e+4f-8g)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(g*(8 - 5*x^2) + f*(4 + 3*x - 4*x^2) + e*(5 -
 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e + 8*f + 11*g)*Log[1 - x] -
 (35*d + 58*e + 92*f + 136*g)*Log[2 - x] + 4*(2*d + e - 4*f + 7*g)*Log[1 +
x] + 3*(d - 2*e + 4*f - 8*g)*Log[2 + x])/432
```

Maple [A]
time = 0.05, size = 134, normalized size = 0.95

method	result
default	$\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}\right)\ln\left(x+2\right)+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}+\frac{2g}{9}}{x-2}-\frac{\frac{d}{12}+\frac{e}{12}+\frac{f}{12}+}{-1+x}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{4f}{9}+\frac{8g}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}\right)\ln($
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}}{x^3-2x^2-x+2}-\frac{35\ln(2-x)d}{432}-\frac{29\ln(2-x)e}{216}-\frac{23\ln(2-x)f}{108}-\frac{17\ln(2-x)g}{54}+\frac{\ln(-1-x)}{54}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/144*d-1/72*e+1/36*f-1/18*g)*ln(x+2)+(-35/432*d-29/216*e-23/108*f-17/54*g
)*ln(x-2)-(1/36*d+1/18*e+1/9*f+2/9*g)/(x-2)-(1/12*d+1/12*e+1/12*f+1/12*g)/(
-1+x)+(1/18*d+5/36*e+2/9*f+11/36*g)*ln(-1+x)-(1/36*d-1/36*e+1/36*f-1/36*g)/
(1+x)+(1/54*d+1/108*e-1/27*f+7/108*g)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 132, normalized size = 0.94

$$\frac{1}{144}\left(d+4f-8g-2e\right)\log\left(x+2\right)+\frac{1}{108}\left(2d-4f+7g+e\right)\log\left(x+1\right)+\frac{1}{36}\left(2d+8f+11g+5e\right)\log\left(x-1\right)-\frac{1}{432}\left(35d+92f+136g+58e\right)\log\left(x-2\right)-\frac{(5d+8f+10g+4e)x^2-6\left(d+f\right)x-5d-8f-16g-10e}{36\left(x^3-2x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="maxima")
```

```
[Out] 1/144*(d + 4*f - 8*g - 2*e)*log(x + 2) + 1/108*(2*d - 4*f + 7*g + e)*log(x
+ 1) + 1/36*(2*d + 8*f + 11*g + 5*e)*log(x - 1) - 1/432*(35*d + 92*f + 136*
g + 58*e)*log(x - 2) - 1/36*((5*d + 8*f + 10*g + 4*e)*x^2 - 6*(d + f)*x - 5
*d - 8*f - 16*g - 10*e)/(x^3 - 2*x^2 - x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(123) = 246$.
time = 0.88, size = 321, normalized size = 2.28

$$-\frac{12\,(5\,d+4\,e+8\,f+10\,g)x^2-72\,(d+f)x-3\,((d-2\,e+4\,f-8\,g)x^3-2\,(d-2\,e+4\,f-8\,g)x^2-(d-2\,e+4\,f-8\,g)x+2\,d-4\,e+8\,f-16\,g)\log(x+2)-4\,((2\,d+e-4\,f+7\,g)x^3-2\,(2\,d+e-4\,f+7\,g)x^2-(2\,d+e-4\,f+7\,g)x+4\,d+2\,e-8\,f+14\,g)\log(x+1)-12\,((2\,d+5\,e+8\,f+11\,g)x^3-2\,(2\,d+5\,e+8\,f+11\,g)x^2-(2\,d+5\,e+8\,f+11\,g)x+4\,d+10\,e+16\,f+22\,g)\log(x-1)+((35\,d+58\,e+92\,f+136\,g)x^3-2\,(35\,d+58\,e+92\,f+136\,g)x^2-(35\,d+58\,e+92\,f+136\,g)x+70\,d+116\,e+184\,f+272\,g)\log(x-2)-60\,d-120\,e-96\,f-192\,g}{432\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g)*x^2 - 72*(d + f)*x - 3*((d - 2*e + 4*f
- 8*g)*x^3 - 2*(d - 2*e + 4*f - 8*g)*x^2 - (d - 2*e + 4*f - 8*g)*x + 2*d -
4*e + 8*f - 16*g)*log(x + 2) - 4*((2*d + e - 4*f + 7*g)*x^3 - 2*(2*d + e -
4*f + 7*g)*x^2 - (2*d + e - 4*f + 7*g)*x + 4*d + 2*e - 8*f + 14*g)*log(x +
1) - 12*((2*d + 5*e + 8*f + 11*g)*x^3 - 2*(2*d + 5*e + 8*f + 11*g)*x^2 - (2
*d + 5*e + 8*f + 11*g)*x + 4*d + 10*e + 16*f + 22*g)*log(x - 1) + ((35*d +
58*e + 92*f + 136*g)*x^3 - 2*(35*d + 58*e + 92*f + 136*g)*x^2 - (35*d + 58*
e + 92*f + 136*g)*x + 70*d + 116*e + 184*f + 272*g)*log(x - 2) - 60*d - 120
*e - 96*f - 192*g)/(x^3 - 2*x^2 - x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.53, size = 136, normalized size = 0.96

$$\frac{1}{144}\,(d+4\,f-8\,g-2\,e)\log(|x+2|)+\frac{1}{108}\,(2\,d-4\,f+7\,g+e)\log(|x+1|)+\frac{1}{36}\,(2\,d+8\,f+11\,g+5\,e)\log(|x-1|)-\frac{1}{432}\,(35\,d+92\,f+136\,g+58\,e)\log(|x-2|)-\frac{(5\,d+8\,f+10\,g+4\,e)x^2-6\,(d+f)x-5\,d-8\,f-16\,g-10\,e}{36\,(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="giac")
```

```
[Out] 1/144*(d + 4*f - 8*g - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + 7*g + e)*l
og(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 5*e)*log(abs(x - 1)) - 1/432*(35*
d + 92*f + 136*g + 58*e)*log(abs(x - 2)) - 1/36*((5*d + 8*f + 10*g + 4*e)*x
^2 - 6*(d + f)*x - 5*d - 8*f - 16*g - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B]
time = 0.88, size = 131, normalized size = 0.93

$$\ln(x-1)\left(\frac{d}{18}+\frac{5\,e}{36}+\frac{2\,f}{9}+\frac{11\,g}{36}\right)+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}\right)+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7\,g}{108}\right)-\ln(x-2)\left(\frac{35\,d}{432}+\frac{29\,e}{216}+\frac{23\,f}{108}+\frac{17\,g}{54}\right)-\frac{\left(-\frac{5\,d}{36}-\frac{e}{9}-\frac{2\,f}{9}-\frac{5\,g}{18}\right)\,x^2+\left(\frac{d}{6}+\frac{f}{6}\right)\,x+\frac{5\,d}{36}+\frac{5\,e}{18}+\frac{2\,f}{9}+\frac{4\,g}{9}}{-x^3+2\,x^2+x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36) + log(x + 2)*(d/144 - e/
72 + f/36 - g/18) + log(x + 1)*(d/54 + e/108 - f/27 + (7*g)/108) - log(x -
2)*((35*d)/432 + (29*e)/216 + (23*f)/108 + (17*g)/54) - ((5*d)/36 + (5*e)/1
8 + (2*f)/9 + (4*g)/9 - x^2*((5*d)/36 + e/9 + (2*f)/9 + (5*g)/18) + x*(d/6
+ f/6))/(x + 2*x^2 - x^3 - 2)
```

3.101 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=158

$$\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}-\frac{d-e+f-g+h}{36(1+x)}+\frac{1}{36}(2d+5e+8f+11g+14h)\log(1-x)-\frac{1}{43}$$

```
[Out] 1/12*(d+e+f+g+h)/(1-x)+1/36*(d+2*e+4*f+8*g+16*h)/(2-x)+1/36*(-d+e-f+g-h)/(1
+x)+1/36*(2*d+5*e+8*f+11*g+14*h)*ln(1-x)-1/432*(35*d+58*e+92*f+136*g+176*h)
*ln(2-x)+1/108*(2*d+e-4*f+7*g-10*h)*ln(1+x)+1/144*(d-2*e+4*f-8*g+16*h)*ln(2
+x)
```

Rubi [A]
time = 0.19, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1600, 6874}

$$-\frac{d-e+f-g+h}{36(x+1)}+\frac{d+e+f+g+h}{12(1-x)}+\frac{d+2e+4f+8g+16h}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h)+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x]
```

```
[Out] (d + e + f + g + h)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h)/(36*(2 - x)
) - (d - e + f - g + h)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 11*g + 14*h)*Log
[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h)*Log[2 - x])/432 + ((2*d
 + e - 4*f + 7*g - 10*h)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h)*Lo
g[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{d+2e+4f+8g+16h}{36(-2+x)^2} + \frac{-35d-58e-92f-136g-17}{432(-2+x)}\right.$$
$$= \frac{d+e+f+g+h}{12(1-x)} + \frac{d+2e+4f+8g+16h}{36(2-x)} - \frac{d-e+f-g}{36(1+x)}$$

Mathematica [A]
time = 0.06, size = 169, normalized size = 1.07

$$\frac{1}{432}\left(\frac{12(d(5+6x-5x^2)+2(8g+10h+3hx-5gx^2-10hx^2+f(4+3x-4x^2)+e(5-2x^2)))}{2-x-2x^2+x^3}+12(2d+5e+8f+11g+14h)\log(1-x)-(35d+58e+92f+136g+176h)\log(2-x)+4(2d+e-4f+7g-10h)\log(1+x)+3(d-2e+4f-8g+16h)\log(2+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(4 - 5*x^2 + x^4)^2,x
]
```

```
[Out] ((12*(d*(5 + 6*x - 5*x^2) + 2*(8*g + 10*h + 3*h*x - 5*g*x^2 - 10*h*x^2 + f*
(4 + 3*x - 4*x^2) + e*(5 - 2*x^2))))/(2 - x - 2*x^2 + x^3) + 12*(2*d + 5*e
+ 8*f + 11*g + 14*h)*Log[1 - x] - (35*d + 58*e + 92*f + 136*g + 176*h)*Log[
2 - x] + 4*(2*d + e - 4*f + 7*g - 10*h)*Log[1 + x] + 3*(d - 2*e + 4*f - 8*g
 + 16*h)*Log[2 + x])/432
```

Maple [A]
time = 0.06, size = 155, normalized size = 0.98

method	result
default	$\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}\right)\ln\left(x+2\right)+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}-\frac{11h}{27}\right)\ln\left(x-2\right)-\frac{\frac{d}{36}+\frac{e}{18}+\frac{f}{9}+\frac{2g}{9}+\frac{4h}{9}}{x-2}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}+\frac{8h}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}-\frac{17h}{18}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{10h}{9}+\frac{4f}{9}+\frac{8g}{9}}{x^4-5x^2+4}+\left(-\frac{35d}{432}-\frac{29e}{216}-\right.$
risch	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}\right)x^2+\left(\frac{h}{6}+\frac{f}{6}+\frac{d}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}+\frac{5h}{9}}{x^3-2x^2-x+2}+\frac{\ln(-1-x)d}{54}+\frac{\ln(-1-x)e}{108}-\frac{\ln(-1-x)f}{27}+\frac{7\ln(-1-x)g}{108}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNVERBOSE
)
```

```
[Out] (1/144*d-1/72*e+1/36*f-1/18*g+1/9*h)*ln(x+2)+(-35/432*d-29/216*e-23/108*f-1
7/54*g-11/27*h)*ln(x-2)-(1/36*d+1/18*e+1/9*f+2/9*g+4/9*h)/(x-2)-(1/12*d+1/1
2*e+1/12*f+1/12*g+1/12*h)/(-1+x)+(1/18*d+5/36*e+2/9*f+11/36*g+7/18*h)*ln(-1
+x)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h)/(1+x)+(1/54*d+1/108*e-1/27*f+7/108
*g-5/54*h)*ln(1+x)
```

Maxima [**A**]
time = 0.28, size = 151, normalized size = 0.96

$$\frac{1}{144}\,(d+4\,f-8\,g+16\,h-2\,e)\log\left(x+2\right)+\frac{1}{108}\,(2\,d-4\,f+7\,g-10\,h+e)\log\left(x+1\right)+\frac{1}{36}\,(2\,d+8\,f+11\,g+14\,h+5\,e)\log\left(x-1\right)-\frac{1}{432}\,(35\,d+92\,f+136\,g+176\,h+58\,e)\log\left(x-2\right)-\frac{(5\,d+8\,f+10\,g+20\,h+4\,e)x^2-6\,(d+f+h)x-5\,d-8\,f-16\,g-20\,h-10\,e}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="max
ima")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e)*log(x + 2) + 1/108*(2*d - 4*f + 7*g - 10
*h + e)*log(x + 1) + 1/36*(2*d + 8*f + 11*g + 14*h + 5*e)*log(x - 1) - 1/43
2*(35*d + 92*f + 136*g + 176*h + 58*e)*log(x - 2) - 1/36*((5*d + 8*f + 10*g
 + 20*h + 4*e)*x^2 - 6*(d + f + h)*x - 5*d - 8*f - 16*g - 20*h - 10*e)/(x^3
 - 2*x^2 - x + 2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(140) = 280.
time = 3.60, size = 376, normalized size = 2.38

$$-\frac{12\,(5\,d+4\,e+8\,f+10\,g+20\,h)x^2-72\,(d+f+h)x-3\,((d-2\,e+4\,f-8\,g+16\,h)x^3-2\,(d-2\,e+4\,f-8\,g+16\,h)x^2-(d-2\,e+4\,f-8\,g+16\,h)x+2\,d-4\,e+8\,f-16\,g+32\,h)\log\left(x+2\right)-4\,((2\,d+e-4\,f+7\,g-10\,h)x^3-2\,(2\,d+e-4\,f+7\,g-10\,h)x^2-(2\,d+e-4\,f+7\,g-10\,h)x+4\,d+2\,e-8\,f+14\,g-20\,h)\log\left(x+1\right)-12\,((2\,d+5\,e+8\,f+11\,g+14\,h)x^3-2\,(2\,d+5\,e+8\,f+11\,g+14\,h)x^2-(2\,d+5\,e+8\,f+11\,g+14\,h)x+4\,d+10\,e+16\,f+22\,g+28\,h)\log\left(x-1\right)+((35\,d+58\,e+92\,f+136\,g+176\,h)x^3-2\,(35\,d+58\,e+92\,f+136\,g+176\,h)x^2-(35\,d+58\,e+92\,f+136\,g+176\,h)x+70\,d+116\,e+184\,f+272\,g+352\,h)\log\left(x-2\right)-60\,d-120\,e-96\,f-192\,g-240\,h}{432\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="fri
cas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h)*x^2 - 72*(d + f + h)*x - 3*((d -
 2*e + 4*f - 8*g + 16*h)*x^3 - 2*(d - 2*e + 4*f - 8*g + 16*h)*x^2 - (d - 2*
e + 4*f - 8*g + 16*h)*x + 2*d - 4*e + 8*f - 16*g + 32*h)*log(x + 2) - 4*((2
*d + e - 4*f + 7*g - 10*h)*x^3 - 2*(2*d + e - 4*f + 7*g - 10*h)*x^2 - (2*d
+ e - 4*f + 7*g - 10*h)*x + 4*d + 2*e - 8*f + 14*g - 20*h)*log(x + 1) - 12*
((2*d + 5*e + 8*f + 11*g + 14*h)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h)*x^
2 - (2*d + 5*e + 8*f + 11*g + 14*h)*x + 4*d + 10*e + 16*f + 22*g + 28*h)*lo
g(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h)*x^3 - 2*(35*d + 58*e + 92*
f + 136*g + 176*h)*x^2 - (35*d + 58*e + 92*f + 136*g + 176*h)*x + 70*d + 11
6*e + 184*f + 272*g + 352*h)*log(x - 2) - 60*d - 120*e - 96*f - 192*g - 240
*h)/(x^3 - 2*x^2 - x + 2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.46, size = 155, normalized size = 0.98

$$\frac{1}{144} \left(d+4 f-8 g+16 h-2 e\right) \log \left(|x+2|\right)+\frac{1}{108} \left(2 d-4 f+7 g-10 h+e\right) \log \left(|x+1|\right)+\frac{1}{36} \left(2 d+8 f+11 g+14 h+5 e\right) \log \left(|x-1|\right)-\frac{1}{432} \left(35 d+92 f+136 g+176 h+58 e\right) \log \left(|x-2|\right)-\frac{(5 d+8 f+10 g+20 h+4 e)x^{2}-6 \left(d+f+h\right)x-5 d-8 f-16 g-20 h-10 e}{36 \left(x+1\right)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorithm="gia
c")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e)*log(abs(x + 2)) + 1/108*(2*d - 4*f + 7*g
 - 10*h + e)*log(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 14*h + 5*e)*log(abs
(x - 1)) - 1/432*(35*d + 92*f + 136*g + 176*h + 58*e)*log(abs(x - 2)) - 1/3
6*((5*d + 8*f + 10*g + 20*h + 4*e)*x^2 - 6*(d + f + h)*x - 5*d - 8*f - 16*g
 - 20*h - 10*e)/((x + 1)*(x - 1)*(x - 2))
```

Mupad [B]
time = 1.39, size = 152, normalized size = 0.96

$$\ln \left(x-1\right) \left(\frac{d}{18}+\frac{5 e}{36}+\frac{2 f}{9}+\frac{11 g}{36}+\frac{7 h}{18}\right)-\frac{\left(-\frac{5 d}{36}-\frac{e}{9}-\frac{2 f}{9}-\frac{5 g}{18}-\frac{5 h}{9}\right) x^{2}+\left(\frac{d}{6}+\frac{f}{6}+\frac{h}{6}\right) x+\frac{5 d}{36}+\frac{5 e}{18}+\frac{2 f}{9}+\frac{4 g}{9}+\frac{5 h}{9}}{-x^{3}+2 x^{2}+x-2}+\ln \left(x+2\right) \left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}\right)+\ln \left(x+1\right) \left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7 g}{108}-\frac{5 h}{54}\right)-\ln \left(x-2\right) \left(\frac{35 d}{432}+\frac{29 e}{216}+\frac{23 f}{108}+\frac{17 g}{54}+\frac{11 h}{27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4))/(x^4 - 5*x^2 + 4)^2,x)
```

```
[Out] log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36 + (7*h)/18) - ((5*d)/36 +
 (5*e)/18 + (2*f)/9 + (4*g)/9 + (5*h)/9 - x^2*((5*d)/36 + e/9 + (2*f)/9 + (
5*g)/18 + (5*h)/9) + x*(d/6 + f/6 + h/6))/(x + 2*x^2 - x^3 - 2) + log(x + 2
)*(d/144 - e/72 + f/36 - g/18 + h/9) + log(x + 1)*(d/54 + e/108 - f/27 + (7
*g)/108 - (5*h)/54) - log(x - 2)*((35*d)/432 + (29*e)/216 + (23*f)/108 + (1
7*g)/54 + (11*h)/27)
```

3.102 $\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+ix^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=177

$$\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}-\frac{d-e+f-g+h-i}{36(1+x)}+\frac{1}{36}(2d+5e+8f+11g+14h+$$

```
[Out] 1/12*(d+e+f+g+h+i)/(1-x)+1/36*(d+2*e+4*f+8*g+16*h+32*i)/(2-x)+1/36*(-d+e-f+
g-h+i)/(1+x)+1/36*(2*d+5*e+8*f+11*g+14*h+17*i)*ln(1-x)-1/432*(35*d+58*e+92*
f+136*g+176*h+160*i)*ln(2-x)+1/108*(2*d+e-4*f+7*g-10*h+13*i)*ln(1+x)+1/144*
(d-2*e+4*f-8*g+16*h-32*i)*ln(2+x)
```

Rubi [A]
time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {1600, 6874}

$$-\frac{d-e+f-g+h-i}{36(x+1)}+\frac{d+e+f+g+h+i}{12(1-x)}+\frac{d+2e+4f+8g+16h+32i}{36(2-x)}+\frac{1}{36}\log(1-x)(2d+5e+8f+11g+14h+17i)-\frac{1}{432}\log(2-x)(35d+58e+92f+136g+176h+160i)+\frac{1}{108}\log(x+1)(2d+e-4f+7g-10h+13i)+\frac{1}{144}\log(x+2)(d-2e+4f-8g+16h-32i)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 + x^4)^2
,x]
```

```
[Out] (d + e + f + g + h + i)/(12*(1 - x)) + (d + 2*e + 4*f + 8*g + 16*h + 32*i)/
(36*(2 - x)) - (d - e + f - g + h - i)/(36*(1 + x)) + ((2*d + 5*e + 8*f + 1
1*g + 14*h + 17*i)*Log[1 - x])/36 - ((35*d + 58*e + 92*f + 136*g + 176*h +
160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g - 10*h + 13*i)*Log[1 + x])/1
08 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2 + x])/144
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(2+x)\left(d+ex+fx^2+gx^3+hx^4+102x^5\right)}{\left(4-5x^2+x^4\right)^2}\,dx = \int \frac{d+ex+fx^2+gx^3+hx^4+102x^5}{(2+x)\left(2-x-2x^2+x^3\right)^2}\,dx$$
$$= \int \left(\frac{3264+d+2e+4f+8g+16h}{36(-2+x)^2} + \frac{-16320-35d\ \cdots}{\cdots}\right.$$
$$= \frac{102+d+e+f+g+h}{12(1-x)} + \frac{3264+d+2e+4f+8g+\cdots}{36(2-x)}$$

Mathematica [A]
time = 0.07, size = 195, normalized size = 1.10

$$\frac{5d+10e+8f+16g+20h+40i+6dx+6fx+6hx-5dx^2-4ex^2-8fx^2-10gx^2-20hx^2-34ix^2}{36\left(2-x-2x^2+x^3\right)} + \frac{1}{36}(2d+5e+8f+11g+14h+17i)\log(1-x) + \frac{1}{432}(-35d-58e-92f-136g-176h-160i)\log(2-x) + \frac{1}{108}(2d+e-4f+7g-10h+13i)\log(1+x) + \frac{1}{144}(d-2e+4f-8g+16h-32i)\log(2+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(4 - 5*x^2 +
x^4)^2,x]
```

```
[Out] (5*d + 10*e + 8*f + 16*g + 20*h + 40*i + 6*d*x + 6*f*x + 6*h*x - 5*d*x^2 -
4*e*x^2 - 8*f*x^2 - 10*g*x^2 - 20*h*x^2 - 34*i*x^2)/(36*(2 - x - 2*x^2 + x^
3)) + ((2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*Log[1 - x])/36 + ((-35*d - 58
*e - 92*f - 136*g - 176*h - 160*i)*Log[2 - x])/432 + ((2*d + e - 4*f + 7*g
- 10*h + 13*i)*Log[1 + x])/108 + ((d - 2*e + 4*f - 8*g + 16*h - 32*i)*Log[2
 + x])/144
```

Maple [A]
time = 0.08, size = 176, normalized size = 0.99

method	result
default	$\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}-\frac{2i}{9}\right)\ln(x+2)+\left(-\frac{35d}{432}-\frac{29e}{216}-\frac{23f}{108}-\frac{17g}{54}-\frac{11h}{27}-\frac{10i}{27}\right)\ln(x-2)-\frac{\frac{d}{36}+\cdots}{\cdots}$
norman	$\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}-\frac{17i}{18}\right)x^3+\left(\frac{17d}{36}+\frac{5e}{18}+\frac{5f}{9}+\frac{4g}{9}+\frac{8h}{9}+\frac{10i}{9}\right)x+\left(-\frac{d}{9}-\frac{2e}{9}-\frac{5f}{18}-\frac{5g}{9}-\frac{17h}{18}-\frac{17i}{9}\right)x^2+\frac{5d}{18}+\frac{5e}{9}+\frac{10h}{9}+\frac{20i}{9}+\frac{4f}{9}+\frac{8g}{9}}{x^4-5x^2+4}+\cdots$
risch	$\frac{\ln(-1-x)d}{54}+\frac{\ln(-1-x)e}{108}+\frac{17\ln(-1+x)i}{36}-\frac{2\ln(x+2)i}{9}+\frac{7\ln(-1+x)h}{18}+\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}-\frac{17i}{18}\right)x^2+\left(\frac{h}{6}+\frac{f}{6}+\frac{d}{6}\right)x+\cdots}{x^3-2x^2-x+2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x,method=_RETURNV
ERBOSE)
```

```
[Out] (1/144*d-1/72*e+1/36*f-1/18*g+1/9*h-2/9*i)*ln(x+2)+(-35/432*d-29/216*e-23/1
08*f-17/54*g-11/27*h-10/27*i)*ln(x-2)-(1/36*d+1/18*e+1/9*f+2/9*g+4/9*h+8/9*
i)/(x-2)-(1/12*d+1/12*e+1/12*f+1/12*g+1/12*h+1/12*i)/(-1+x)+(1/18*d+5/36*e+
```

```
2/9*f+11/36*g+7/18*h+17/36*i)*ln(-1+x)-(1/36*d-1/36*e+1/36*f-1/36*g+1/36*h-
1/36*i)/(1+x)+(1/54*d+1/108*e-1/27*f+7/108*g-5/54*h+13/108*i)*ln(1+x)
```

Maxima [A]
time = 0.28, size = 157, normalized size = 0.89

$$\frac{1}{144}\,(d+4\,f-8\,g+16\,h-2\,e-32i)\log\left(x+2\right)+\frac{1}{108}\,(2\,d-4\,f+7\,g-10\,h+e+13i)\log\left(x+1\right)+\frac{1}{36}\,(2\,d+8\,f+11\,g+14\,h+5\,e+17i)\log\left(x-1\right)-\frac{1}{432}\,(35\,d+92\,f+136\,g+176\,h+58\,e+160i)\log\left(x-2\right)-\frac{(5\,d+8\,f+10\,g+20\,h+4\,e+34i)x^2-6\,(d+f+h)x-5\,d-8\,f-16\,g-20\,h-10\,e-40i}{36\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="maxima")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(x + 2) + 1/108*(2*d - 4*f + 7
*g - 10*h + e + 13*I)*log(x + 1) + 1/36*(2*d + 8*f + 11*g + 14*h + 5*e + 17
*I)*log(x - 1) - 1/432*(35*d + 92*f + 136*g + 176*h + 58*e + 160*I)*log(x -
 2) - 1/36*((5*d + 8*f + 10*g + 20*h + 4*e + 34*I)*x^2 - 6*(d + f + h)*x -
5*d - 8*f - 16*g - 20*h - 10*e - 40*I)/(x^3 - 2*x^2 - x + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(159) = 318$.
time = 21.48, size = 430, normalized size = 2.43

$$-\frac{12\,(5\,d+4\,e+8\,f+10\,g+20\,h+34\,i)x^2-72\,(d+f+h)x-3\,((d-2\,e+4\,f-8\,g+16\,h-32\,i)x^3-2\,(d-2\,e+4\,f-8\,g+16\,h-32\,i)x^2-(d-2\,e+4\,f-8\,g+16\,h-32\,i)x+2\,d-4\,e+8\,f-16\,g+32\,h-64\,i)\log\left(x+2\right)-4\,((2\,d+e-4\,f+7\,g-10\,h+13\,i)x^3-2\,(2\,d+e-4\,f+7\,g-10\,h+13\,i)x^2-(2\,d+e-4\,f+7\,g-10\,h+13\,i)x+4\,d+2\,e-8\,f+14\,g-20\,h+26\,i)\log\left(x+1\right)-12\,((2\,d+5\,e+8\,f+11\,g+14\,h+17\,i)x^3-2\,(2\,d+5\,e+8\,f+11\,g+14\,h+17\,i)x^2-(2\,d+5\,e+8\,f+11\,g+14\,h+17\,i)x+4\,d+10\,e+16\,f+22\,g+28\,h+34\,i)\log\left(x-1\right)+((35\,d+58\,e+92\,f+136\,g+176\,h+160\,i)x^3-2\,(35\,d+58\,e+92\,f+136\,g+176\,h+160\,i)x^2-(35\,d+58\,e+92\,f+136\,g+176\,h+160\,i)x+70\,d+116\,e+184\,f+272\,g+352\,h+320\,i)\log\left(x-2\right)-60\,d-120\,e-96\,f-192\,g-240\,h-480\,i}{432\,(x^3-2\,x^2-x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="fricas")
```

```
[Out] -1/432*(12*(5*d + 4*e + 8*f + 10*g + 20*h + 34*i)*x^2 - 72*(d + f + h)*x -
3*((d - 2*e + 4*f - 8*g + 16*h - 32*i)*x^3 - 2*(d - 2*e + 4*f - 8*g + 16*h
- 32*i)*x^2 - (d - 2*e + 4*f - 8*g + 16*h - 32*i)*x + 2*d - 4*e + 8*f - 16*
g + 32*h - 64*i)*log(x + 2) - 4*((2*d + e - 4*f + 7*g - 10*h + 13*i)*x^3 -
2*(2*d + e - 4*f + 7*g - 10*h + 13*i)*x^2 - (2*d + e - 4*f + 7*g - 10*h + 1
3*i)*x + 4*d + 2*e - 8*f + 14*g - 20*h + 26*i)*log(x + 1) - 12*((2*d + 5*e
+ 8*f + 11*g + 14*h + 17*i)*x^3 - 2*(2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*
x^2 - (2*d + 5*e + 8*f + 11*g + 14*h + 17*i)*x + 4*d + 10*e + 16*f + 22*g +
 28*h + 34*i)*log(x - 1) + ((35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*x^
3 - 2*(35*d + 58*e + 92*f + 136*g + 176*h + 160*i)*x^2 - (35*d + 58*e + 92*
f + 136*g + 176*h + 160*i)*x + 70*d + 116*e + 184*f + 272*g + 352*h + 320*i
)*log(x - 2) - 60*d - 120*e - 96*f - 192*g - 240*h - 480*i)/(x^3 - 2*x^2 -
x + 2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(x**4-5*x**2+4)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.79, size = 161, normalized size = 0.91

$$\frac{1}{144}(d+4f-8g+16h-2e-32i)\log(|x+2|)+\frac{1}{108}(2d-4f+7g-10h+e+13i)\log(|x+1|)+\frac{1}{36}(2d+8f+11g+14h+5e+17i)\log(|x-1|)-\frac{1}{432}(35d+92f+136g+176h+58e+160i)\log(|x-2|)-\frac{(5d+8f+10g+20h+4e+34i)x^2-6(d+f+h)x-5d-8f-16g-20h-10e-40i}{36(x+1)(x-1)(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)*(i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(x^4-5*x^2+4)^2,x, algorith
m="giac")
```

```
[Out] 1/144*(d + 4*f - 8*g + 16*h - 2*e - 32*I)*log(abs(x + 2)) + 1/108*(2*d - 4*
f + 7*g - 10*h + e + 13*I)*log(abs(x + 1)) + 1/36*(2*d + 8*f + 11*g + 14*h
+ 5*e + 17*I)*log(abs(x - 1)) - 1/432*(35*d + 92*f + 136*g + 176*h + 58*e +
 160*I)*log(abs(x - 2)) - 1/36*((5*d + 8*f + 10*g + 20*h + 4*e + 34*I)*x^2
- 6*(d + f + h)*x - 5*d - 8*f - 16*g - 20*h - 10*e - 40*I)/((x + 1)*(x - 1)
*(x - 2))
```

Mupad [B]
time = 1.75, size = 170, normalized size = 0.96

$$\ln(x-1)\left(\frac{d}{18}+\frac{5e}{36}+\frac{2f}{9}+\frac{11g}{36}+\frac{7h}{18}+\frac{17i}{36}\right)+\ln(x+2)\left(\frac{d}{144}-\frac{e}{72}+\frac{f}{36}-\frac{g}{18}+\frac{h}{9}-\frac{2i}{9}\right)+\ln(x+1)\left(\frac{d}{54}+\frac{e}{108}-\frac{f}{27}+\frac{7g}{108}-\frac{5h}{54}+\frac{13i}{108}\right)-\ln(x-2)\left(\frac{35d}{432}+\frac{29e}{216}+\frac{23f}{108}+\frac{17g}{54}+\frac{11h}{27}+\frac{10i}{27}\right)-\frac{\left(-\frac{5d}{36}-\frac{e}{9}-\frac{2f}{9}-\frac{5g}{18}-\frac{5h}{9}-\frac{17i}{18}\right)x^2+\left(\frac{d}{6}+\frac{f}{6}+\frac{h}{6}\right)x+\frac{5d}{36}+\frac{5e}{18}+\frac{2f}{9}+\frac{4g}{9}+\frac{5h}{9}+\frac{10i}{9}}{-x^3+2x^2+x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 2)*(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5))/(x^4 - 5*x^2 + 4)^2
,x)
```

```
[Out] log(x - 1)*(d/18 + (5*e)/36 + (2*f)/9 + (11*g)/36 + (7*h)/18 + (17*i)/36) +
 log(x + 2)*(d/144 - e/72 + f/36 - g/18 + h/9 - (2*i)/9) + log(x + 1)*(d/54
 + e/108 - f/27 + (7*g)/108 - (5*h)/54 + (13*i)/108) - log(x - 2)*((35*d)/4
32 + (29*e)/216 + (23*f)/108 + (17*g)/54 + (11*h)/27 + (10*i)/27) - ((5*d)/
36 + (5*e)/18 + (2*f)/9 + (4*g)/9 + (5*h)/9 + (10*i)/9 - x^2*((5*d)/36 + e/
9 + (2*f)/9 + (5*g)/18 + (5*h)/9 + (17*i)/18) + x*(d/6 + f/6 + h/6))/(x + 2
*x^2 - x^3 - 2)
```

3.103 $\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=717

$$-\frac{(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3(b^2-4ac)\,(2ce-bg)\,(b+2cx^2)\,\sqrt{}}{256c^3}$$

```
[Out] 1/32*(-b*g+2*c*e)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c^2+1/63*x*(7*c*f*x^2+3
*b*f+9*c*d)*(c*x^4+b*x^2+a)^(3/2)/c+1/10*g*(c*x^4+b*x^2+a)^(5/2)/c+3/512*(-
4*a*c+b^2)^2*(-b*g+2*c*e)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(
1/2))/c^(7/2)-3/256*(-4*a*c+b^2)*(-b*g+2*c*e)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(
1/2)/c^3+1/315*x*(9*b^2*c*d+90*a*c^2*d-4*b^3*f+9*a*b*c*f+3*c*(14*a*c*f-4*b^
2*f+9*b*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2-1/315*(-84*a^2*c^2*f+57*a*b^2*c
*f-144*a*b*c^2*d-8*b^4*f+18*b^3*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/
2)+x^2*c^(1/2))+1/315*a^(1/4)*(-84*a^2*c^2*f+57*a*b^2*c*f-144*a*b*c^2*d-8*b
^4*f+18*b^3*c*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/
2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/630*a^(1/4)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*(18*b^3*c*d-144*a*b*c^2*d-8*b^4*f+57*a*b^2*c*f-84*a^2*c^2*f+(24*a*b
*c*f-180*a*c^2*d-4*b^3*f+9*b^2*c*d)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1687, 1190, 1211, 1117, 1209, 1261, 654, 626, 635, 212}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/315*((18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f
)*x*Sqrt[a + b*x^2 + c*x^4])/(c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*(b^2 -
4*a*c)*(2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^3) + (x*
(9*b^2*c*d + 90*a*c^2*d - 4*b^3*f + 9*a*b*c*f + 3*c*(9*b*c*d - 4*b^2*f + 14
*a*c*f)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + ((2*c*e - b*g)*(b + 2*c*x
^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (x*(3*(3*c*d + b*f) + 7*c*f*x^2)*
(a + b*x^2 + c*x^4)^(3/2))/(63*c) + (g*(a + b*x^2 + c*x^4)^(5/2))/(10*c) +
(3*(b^2 - 4*a*c)^2*(2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
```

```
b*x^2 + c*x^4])])/(512*c^(7/2)) + (a^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*
b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) -
(a^(1/4)*(18*b^3*c*d - 144*a*b*c^2*d - 8*b^4*f + 57*a*b^2*c*f - 84*a^2*c^2*
f + Sqrt[a]*Sqrt[c]*(9*b^2*c*d - 180*a*c^2*d - 4*b^3*f + 24*a*b*c*f))*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(
11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
```

```
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \left(d+ex+fx^2+gx^3\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(d+fx^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx + \int x\left(e+gx^2\right)\left(a+b\right.$$

$$= \frac{x(3(3cd+bf)+7cfx^2)\left(a+bx^2+cx^4\right)^{3/2}}{63c} + \frac{1}{2}\text{Subst}\left(\int (e\right.$$

$$= \frac{x(9b^2cd+90ac^2d-4b^3f+9abcf+3c(9bcd-4b^2f+14ac}{315c^2}$$

$$= \frac{x(9b^2cd+90ac^2d-4b^3f+9abcf+3c(9bcd-4b^2f+14ac}{315c^2}$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

$$= -\frac{\left(18b^3cd-144abc^2d-8b^4f+57ab^2cf-84a^2c^2f\right)x\sqrt{a+}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.64, size = 2588, normalized size = 3.61

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-945*b^4*g
 + 2*b^3*c*(945*e + x*(512*f + 315*g*x)) - 12*b^2*c*(-525*a*g + c*x*(192*d
+ 105*e*x + 64*f*x^2 + 42*g*x^3)) - 8*b*c^2*(3*a*(525*e + 256*f*x + 147*g*x
^2) + 2*c*x^3*(1152*d + 945*e*x + 800*f*x^2 + 693*g*x^3)) - 16*c^2*(504*a^2
*g + 2*c^2*x^5*(360*d + 7*x*(45*e + 40*f*x + 36*g*x^2)) + a*c*x*(2160*d + 7
*x*(225*e + 16*x*(11*f + 9*g*x))))) + (2304*I)*Sqrt[2]*b^3*c^(3/2)*(b - Sqr
t[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
```

]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (18432*I)*Sqrt[2]
*a*b*c^(5/2)*(-b + Sqrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
]) + (7296*I)*Sqrt[2]*a*b^2*c^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])]) + (1024*I)*Sqrt[2]*b^4*Sqrt[c]*(-b + Sqrt[b^2 - 4*
a*c])*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (10752*I)*Sqrt[2]*a^2*c^(5/
2)*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I
*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (2304
*I)*Sqrt[2]*a*b^2*c^(5/2)*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] - (46080*I)*Sqrt[2]*a^2*c^(7/2)*d*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (1024*I)*Sqrt[2]*a*
b^3*c^(3/2)*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] + (6144*I)*Sqrt[2]*a^2*b*c^(5/2)*f*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
)]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 1890*b^4*c*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b
*x^2 + c*x^4]] + 15120*a*b^2*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*Sqrt[a +
 b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] - 3024
0*a^2*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*Sqrt[a + b*x^2 + c*x^4]*Log[b +
 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] + 945*b^5*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*g*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]] - 7560*a*b^3*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*g*Sqrt[a +
b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]] + 15120
*a^2*b*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*g*Sqrt[a + b*x^2 + c*x^4]*Log[b

```
+ 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(161280*c^(7/2)*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(681) = 1362$.
time = 0.08, size = 1580, normalized size = 2.20

method	result	size
elliptic	Expression too large to display	1376
default	Expression too large to display	1580
risch	Expression too large to display	2729

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128*b^3/c^2*x^2*(c*x^4+b*x^2+a)^
(1/2)-3/32*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-5/
64*a*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)+3/64*a*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))+1/10*c*x^8*(c*x^4+b*x^2+a)^(1/2)+11/80*b*x^6*(c*
x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b*x
^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2
))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2))+
f*(1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(1
1/9*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a*b-4/5*(11/9*a*c+
1/21*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(76/63*a*b-4/5*(11/9*a*c+1/21
*b^2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b^2)/
c*a-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*
(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e*(1/64*b^2*
x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32*a*b^2/c^(
3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/8*c*x^6*(c*x^4+b*x^2
+a)^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+5/16*a*x^2*(c*x^4+b*x^2+a)^(1/2)
-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^(5/2)*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))/c^(1/2))+d*(1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x
^2+a)^(1/2)+1/3*(9/7*a*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(
9/7*a*c+3/35*b^2)/c*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
```

```
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a
*c+3/35*b^2)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(g*x^3 + f*x^2 + x*e + d), x)
```

Fricas [A]
time = 0.31, size = 911, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/322560*(512*sqrt(1/2)*((18*(b^3*c^2 - 8*a*b*c^3)*d - (8*b^4*c - 57*a*b^2
*c^2 + 84*a^2*c^3)*f)*x*sqrt((b^2 - 4*a*c)/c^2) - (18*(b^4*c - 8*a*b^2*c^2)
*d - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/
c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 5
12*sqrt(1/2)*((9*(2*b^3*c^2 + 20*a*c^4 - (16*a*b + b^2)*c^3)*d - (8*b^4*c +
 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)*f)*x*sqrt((b^2 - 4*a*c)/c
^2) - (9*(2*b^4*c - 20*a*b*c^3 - (16*a*b^2 - b^3)*c^2)*d - (8*b^5 + 12*(7*a
^2*b - 2*a*b^2)*c^2 - (57*a*b^3 - 4*b^4)*c)*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*
c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))
+ 945*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e - (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*g)*sqrt(c)*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) - 4*(8064*c^5*g*x^9 + 8960*c^5*f*x^8 + 1
008*(10*c^5*e + 11*b*c^4*g)*x^7 + 1280*(9*c^5*d + 10*b*c^4*f)*x^6 + 504*(30
*b*c^4*e + (b^2*c^3 + 32*a*c^4)*g)*x^5 + 256*(72*b*c^4*d + (3*b^2*c^3 + 77*
a*c^4)*f)*x^4 + 126*(10*(b^2*c^3 + 20*a*c^4)*e - (5*b^3*c^2 - 28*a*b*c^3)*g
)*x^3 + 256*(9*(b^2*c^3 + 15*a*c^4)*d - 4*(b^3*c^2 - 6*a*b*c^3)*f)*x^2 - 46
08*(b^3*c^2 - 8*a*b*c^3)*d + 256*(8*b^4*c - 57*a*b^2*c^2 + 84*a^2*c^3)*f -
```

```
63*(10*(3*b^3*c^2 - 20*a*b*c^3)*e - (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3
)*g)*x)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(d+ex+fx^2+gx^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)*(d + e*x + f*x**2 + g*x**3), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,x^4+b\,x^2+a\right)}^{3/2}\,\left(g\,x^3+f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)*(d + e*x + f*x^2 + g*x^3),x)
[Out] int((a + b*x^2 + c*x^4)^(3/2)*(d + e*x + f*x^2 + g*x^3), x)
```

3.104 $\int (d+ex+fx^2+gx^3)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=505

$$\frac{(5bcd-2b^2f+6acf)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{(2ce-bg)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{x(5cd+bf+3cfx^2)\sqrt{a}}{15c}$$

```
[Out] 1/6*g*(c*x^4+b*x^2+a)^(3/2)/c-1/32*(-4*a*c+b^2)*(-b*g+2*c*e)*arctanh(1/2*(2
*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)+1/16*(-b*g+2*c*e)*(2*c*x^2
+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/15*x*(3*c*f*x^2+b*f+5*c*d)*(c*x^4+b*x^2+a)^
(1/2)/c+1/15*(6*a*c*f-2*b^2*f+5*b*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(
1/2)+x^2*c^(1/2))-1/15*a^(1/4)*(6*a*c*f-2*b^2*f+5*b*c*d)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+
a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2))*(5*c*d-
2*b*f+3*f*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1687, 1190, 1211, 1117, 1209, 1261, 654, 626, 635, 212}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,(6acf-2b^2f+5bcd)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,(3\sqrt{a}\sqrt{c}f-2bf+5cd)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}\,(6acf-2b^2f+5bcd)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{(b^2-4ac)(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}+\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}\,(2ce-bg)}{16c^2}+\frac{x\sqrt{a+bx^2+cx^4}\,(bf+5cd+3cfx^2)}{15c}+\frac{g(a+bx^2+cx^4)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b*c*d - 2*b^2*f + 6*a*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt
[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])
/(16*c^2) + (x*(5*c*d + b*f + 3*c*f*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) +
(g*(a + b*x^2 + c*x^4)^(3/2))/(6*c) - ((b^2 - 4*a*c)*(2*c*e - b*g)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)) - (a^(1/4)
*(5*b*c*d - 2*b^2*f + 6*a*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)
*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d - 2*b*f + 3*Sqrt[a]*Sqrt[c]*f)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqr
t[a + b*x^2 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \left(d+ex+fx^2+gx^3\right) \sqrt{a+bx^2+cx^4}\, dx = \int \left(d+fx^2\right) \sqrt{a+bx^2+cx^4}\, dx + \int x\left(e+gx^2\right) \sqrt{a+bx^2-$$

$$= \frac{x(5cd+bf+3cfx^2)\sqrt{a+bx^2+cx^4}}{15c} + \frac{1}{2}\text{Subst}\bigg(\int (e+gx)$$

$$= \frac{x(5cd+bf+3cfx^2)\sqrt{a+bx^2+cx^4}}{15c} + \frac{g(a+bx^2+cx^4)^{3/2}}{6c}$$

$$= \frac{(5bcd-2b^2f+6acf)\, x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{(2ce-bg)\,(b+2c}{1}$$

$$= \frac{(5bcd-2b^2f+6acf)\, x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{(2ce-bg)\,(b+2c}{1}$$

$$= \frac{(5bcd-2b^2f+6acf)\, x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{(2ce-bg)\,(b+2c}{1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.55, size = 661, normalized size = 1.31

$$\frac{2\sqrt{c}\left(a+bx^2+cx^4\right)\left(-15b^2g+2bc(15e+x(8f+5gx))+4c(10ag+cx(20d+x(15e+2x(6f+5gx))))\right)+\frac{-8i\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)(-5bcd+2b^2f-6acf)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+8i\sqrt{2}\sqrt{c}\left(-2b^3f+bc\left(-5\sqrt{b^2-4ac}\,d+8af\right)+b^2\left(5cd+2\sqrt{b^2-4ac}\,f\right)-2ac\left(10cd+3\sqrt{b^2-4ac}\,f\right)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-15(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(-2ce+bg)\sqrt{a+bx^2+cx^4}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{480c^{5/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(a + b*x^2 + c*x^4)*(-15*b^2*g + 2*b*c*(15*e + x*(8*f + 5*g*x))
+ 4*c*(10*a*g + c*x*(20*d + x*(15*e + 2*x*(6*f + 5*g*x))))) + ((-8*I)*Sqrt[
2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*(-5*b*c*d + 2*b^2*f - 6*a*c*f)*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] + (8*I)*Sqrt[2]*Sqrt[c]*(-2*b^3*f + b*c*(-5*Sqrt[b^2 - 4*a*c]*d + 8
*a*f) + b^2*(5*c*d + 2*Sqrt[b^2 - 4*a*c]*f) - 2*a*c*(10*c*d + 3*Sqrt[b^2 -
4*a*c]*f))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I
*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])] - 15*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
```

```
)]*(-2*c*e + b*g)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4]])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(480*c^(5/2)*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(475) = 950$.
time = 0.07, size = 1042, normalized size = 2.06

method	result
elliptic	$\frac{g x^4 \sqrt{c x^4+b x^2+a}}{6}+\frac{f x^3 \sqrt{c x^4+b x^2+a}}{5}+\frac{\left(\frac{bg}{6}+ce\right)x^2 \sqrt{c x^4+b x^2+a}}{4c}+\frac{\left(\frac{bf}{5}+cd\right)x \sqrt{c x^4+b x}}{3c}$
default	$g\left(\frac{(c x^4+b x^2+a)^{\frac{3}{2}}}{6c}-\frac{b x^2 \sqrt{c x^4+b x^2+a}}{8c}-\frac{b^2 \sqrt{c x^4+b x^2+a}}{16c^2}-\frac{b \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)a}{8c^{\frac{3}{2}}}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] g*(1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2/c
^2*(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^
2+a)^(1/2))*a+1/32*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2)))+f*(1/5*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c*x*(c*x^4+b*x^2+a)^(1/2)-1/60
*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(2/5*a-2/15/c*b^2)*a*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e*(1/8*(
2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))*b^2)+d*(1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)
```

```
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-1/6*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + x*e + d), x)
```

Fricas [A]
time = 0.26, size = 574, normalized size = 1.14

$$\frac{32\sqrt{\frac{1}{2}}\left((5bc^2d-2(b^2c-3ac^2)f)x\sqrt{\frac{b^2-4ac}{c^2}}-(5b^2cd-2(b^3-3abc)f)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}\,E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)\middle|\,\frac{bc\sqrt{\frac{b^2-4ac}{c^2}}+b^2-2ac}{2ac}\right)-32\sqrt{\frac{1}{2}}\left((5(bc^2-2c^3)d-(2b^2c-(6a+b)c^2)f)x\sqrt{\frac{b^2-4ac}{c^2}}-(5(b^2c+2bc^2)d-(2b^3-(6ab-b^2)c)f)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}\,F\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)\middle|\,\frac{bc\sqrt{\frac{b^2-4ac}{c^2}}+b^2-2ac}{2ac}\right)+15(2(b^2c-4ac^2)e-(b^3-4abc)g)\sqrt{c}\,x\log\left(8c^2x^4+8bcx^2+b^2-4\sqrt{cx^4+bx^2+a}(2cx^2+b)\sqrt{c}+4ac\right)+4(40c^3gx^5+48c^3fx^4+80bc^2d+10(6c^3e+bc^2g)x^3+16(5c^3d+bc^2f)x^2-32(b^2c-3ac^2)f+5(6bc^2e-(3b^2c-8ac^2)g)x)\sqrt{cx^4+bx^2+a}}{960c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] 1/960*(32*sqrt(1/2)*((5*b*c^2*d - 2*(b^2*c - 3*a*c^2)*f)*x*sqrt((b^2 - 4*a*
c)/c^2) - (5*b^2*c*d - 2*(b^3 - 3*a*b*c)*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/
c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 3
2*sqrt(1/2)*((5*(b*c^2 - 2*c^3)*d - (2*b^2*c - (6*a + b)*c^2)*f)*x*sqrt((b^
2 - 4*a*c)/c^2) - (5*(b^2*c + 2*b*c^2)*d - (2*b^3 - (6*a*b - b^2)*c)*f)*x)*
sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)
*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^
2) + b^2 - 2*a*c)/(a*c)) + 15*(2*(b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)*g)*s
qrt(c)*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(c) + 4*a*c) + 4*(40*c^3*g*x^5 + 48*c^3*f*x^4 + 80*b*c^2*d + 10
*(6*c^3*e + b*c^2*g)*x^3 + 16*(5*c^3*d + b*c^2*f)*x^2 - 32*(b^2*c - 3*a*c^2
)*f + 5*(6*b*c^2*e - (3*b^2*c - 8*a*c^2)*g)*x)*sqrt(c*x^4 + b*x^2 + a))/(c^
3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx^2+cx^4}\,\left(d+ex+fx^2+gx^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)*(d + e*x + f*x**2 + g*x**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(g*x^3 + f*x^2 + x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+b\,x^2+a}\,\left(g\,x^3+f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)*(d + e*x + f*x^2 + g*x^3),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(1/2)*(d + e*x + f*x^2 + g*x^3), x)
```

3.105 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=359

$$\frac{g\sqrt{a+bx^2+cx^4}}{2c}+\frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{c}\,x^2\right.}{}$$

[Out] 1/4*(-b*g+2*c*e)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)+1/2*g*(c*x^4+b*x^2+a)^(1/2)/c+f*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*f*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(f+d*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1687, 1211, 1117, 1209, 1261, 654, 635, 212}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}+\frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{g\sqrt{a+bx^2+cx^4}}{2c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]

[Out] (g*Sqrt[a + b*x^2 + c*x^4])/(2*c) + (f*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*c*e - b*g)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)) - (a^(1/4)*f*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+fx^2}{\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{x(e+gx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{e+gx}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}\,f\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} + \Big(d +$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2 -}{(\sqrt{a}+\sqrt{}}}}{c^{3/4}}$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,f\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2 -}{(\sqrt{a}+\sqrt{}}}}{c^{3/4}}$$

$$= \frac{g\sqrt{a+bx^2+cx^4}}{2c} + \frac{fx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{(2ce-bg)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2}}\right.}{4c^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.84, size = 525, normalized size = 1.46

$$\frac{i\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)f\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\sqrt{2}\sqrt{c}\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)f\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(2\sqrt{c}\,g(a+bx^2+cx^4)+(-2ce+bg)\sqrt{a+bx^2+cx^4}\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{4c^{3/2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*f*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*Sqrt[2]*S
qrt[c]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*f)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*(2*Sqrt[c]*g*(a + b*x^2 + c*x^4) + (-2*c*e + b*g)*Sqrt[a
```

```
 + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4
*c^(3/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.06, size = 454, normalized size = 1.26

method	result
elliptic	$\frac{g\sqrt{c\,x^4+b\,x^2+a}}{2c}+\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+}}$
default	$g\left(\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}\right)-\frac{fa\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}}{}$
risch	$\frac{g\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{fa\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x}{}\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2)))-1/2*f*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))/c^(1/2)+1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [A]
time = 0.21, size = 376, normalized size = 1.05

$$\frac{4\sqrt{\frac{1}{2}}\left(acfx\sqrt{\frac{b^2-4ac}{c^2}}-abfx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)|\frac{bc\sqrt{\frac{b^2-4ac}{c^2}}+b^2-2ac}{2ac})+4\sqrt{\frac{1}{2}}\left((c^2d-acf)x\sqrt{\frac{b^2-4ac}{c^2}}+(bcd+abf)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}F(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)|\frac{bc\sqrt{\frac{b^2-4ac}{c^2}}+b^2-2ac}{2ac})-(2ace-abg)\sqrt{c}\,x\log\left(8c^2x^4+8bcx^2+b^2-4\sqrt{cx^4+bx^2+a}(2cx^2+b)\sqrt{c}+4ac\right)+4\sqrt{cx^4+bx^2+a}(acgx+2acf)}{8ac^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(1/2)*(a*c*f*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*f*x)*sqrt(c)*sqrt((
c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a
*c)/(a*c)) + 4*sqrt(1/2)*((c^2*d - a*c*f)*x*sqrt((b^2 - 4*a*c)/c^2) + (b*c*
d + a*b*f)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(ar
csin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b
^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - (2*a*c*e - a*b*g)*sqrt(c)*x*log(8*
c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
 + 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*(a*c*g*x + 2*a*c*f))/(a*c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.106 $\int \frac{d+ex+fx^2+gx^3}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=447

$$\frac{x\left(b^2d-2acd-abf+c(bd-2af)x^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{be-2ag+(2ce-bg)x^2}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\,(bd-2af)x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{c}}{}$$

```
[Out] x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(
1/2)+(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-(-2*a
*f+b*d)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2)
)+c^(1/4)*(-2*a*f+b*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(-f*a^
(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(3/4)/c^(1/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1687, 1192, 1211, 1117, 1209, 1261, 650}

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}(bd-2af)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d-\sqrt{a}\,f\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}(bd-2af)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x\left(cx^2(bd-2af)-abf-2acd+b^2d\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/((b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - (Sqrt[c]*(b*d - 2*a*f)*x*Sqrt[a + b*x^2 + c*x^4])/(a*
(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(b*d - 2*a*f)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2
- 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d - Sqrt[a]*f)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2
*Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
```

```
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{d+fx^2}{(a+bx^2+cx^4)^{3/2}}\,dx + \int \frac{x(e+gx^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{a\,(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right.$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{a\,(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{be-2ag+(2ce-bg)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(\sqrt{c}\,(ba}{}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{a\,(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{be-2ag+(2ce-bg)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}\,(bd}{a\,(b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.98, size = 513, normalized size = 1.15

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-2a^2g-bdx(b+cx^2)+2acx(d+x(e+fx))+ab(e+x(f-gx))\right)+i\left(-b+\sqrt{b^2-4ac}\right)(bd-2af)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2d+4acd+b\sqrt{b^2-4ac}\,d-2a\sqrt{b^2-4ac}\,f\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{4a\,(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-2*a^2*g - b*d*x*(b + c*x^2) + 2*a
*c*x*(d + x*(e + f*x)) + a*b*(e + x*(f - g*x))) + I*(-b + Sqrt[b^2 - 4*a*c]
)*(b*d - 2*a*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-(b^2*d) + 4*a*c*d + b*Sqrt[b^2 -
4*a*c]*d - 2*a*Sqrt[b^2 - 4*a*c]*f)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(440) = 880.
time = 0.05, size = 1005, normalized size = 2.25

method	result
elliptic	$-\frac{2c\left(-\frac{(2fa-bd)x^3}{2a\left(4ac-b^2\right)}+\frac{(bg-2ce)x^2}{2c\left(4ac-b^2\right)}-\frac{\left(abf+2acd-b^2d\right)x}{2ac\left(4ac-b^2\right)}+\frac{2ag-eb}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{d}{a}-\frac{abf+2acd-b^2d}{a\left(4ac-b^2\right)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+\right.}}{a}}}{}$
default	$-\frac{g\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}+f\left(-\frac{2c\left(-\frac{x^3}{4ac-b^2}-\frac{bx}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{b\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -g/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+f*(-2*c*(-1/(4*a*c-b^2)*x^
3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)-1/4*b/(4*a*c-b^2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipti
cF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e/(c*x^4+b*x^2+a)^(1/2)*(2*c
*x^2+b)/(4*a*c-b^2)+d*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a
*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2)
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ellip
ticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 723, normalized size = 1.62

$$-\frac{\sqrt{\frac{1}{2}}\left(ab^2cd-2a^2bcf+(b^2c^2d-2abc^2f)x^4+(b^3cd-2ab^2cf)x^2-(a^2bcd-2a^3cf+(abc^2d-2a^2c^2f)x^4+(ab^2cd-2a^2bcf)x^2)\sqrt{\frac{b^2-4ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4ac}{a^2}}+b^2-2ac}{2ac}\right)-\sqrt{\frac{1}{2}}\left(((2ab+b^2)c^2d-(ab^2c+2abc^2)f)x^4+(2a^2b+ab^2)cd+((2ab^2+b^3)cd-(ab^3+2ab^2c)f)x^2-(a^2b^2+2a^2bc)f+(((2a^2-ab)c^2d-(a^2bc-2a^2c^2)f)x^4+(2a^3-a^2b)cd+((2a^2b-ab^2)cd-(a^2b^2-2a^2bc)f)x^2-(a^3b-2a^3c)f)\sqrt{\frac{b^2-4ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}F\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4ac}{a^2}}+b^2-2ac}{2ac}\right)+2(a^2bce-2a^3cg-(abc^2d-2a^2c^2f)x^3+(2a^2c^2e-a^2bcg)x^2+(a^2bcf-(ab^2c-2a^2c^2)d)x)\sqrt{cx^4+bx^2+a}}{2(a^3b^2c-4a^4c^2+(a^2b^2c^2-4a^3c^3)x^4+(a^2b^3c-4a^3bc^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*(a*b^2*c*d - 2*a^2*b*c*f + (b^2*c^2*d - 2*a*b*c^2*f)*x^4 +
(b^3*c*d - 2*a*b^2*c*f)*x^2 - (a^2*b*c*d - 2*a^3*c*f + (a*b*c^2*d - 2*a^2*c
^2*f)*x^4 + (a*b^2*c*d - 2*a^2*b*c*f)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt
((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^
2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((2*a*b + b^2)*c^2*d - (a*b^2*c + 2*a*b*c^2)
*f)*x^4 + (2*a^2*b + a*b^2)*c*d + ((2*a*b^2 + b^3)*c*d - (a*b^3 + 2*a*b^2*c
)*f)*x^2 - (a^2*b^2 + 2*a^2*b*c)*f + (((2*a^2 - a*b)*c^2*d - (a^2*b*c - 2*a
^2*c^2)*f)*x^4 + (2*a^3 - a^2*b)*c*d + ((2*a^2*b - a*b^2)*c*d - (a^2*b^2 -
2*a^2*b*c)*f)*x^2 - (a^3*b - 2*a^3*c)*f)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*s
qrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((
a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2
- 2*a*c)/(a*c)) + 2*(a^2*b*c*e - 2*a^3*c*g - (a*b*c^2*d - 2*a^2*c^2*f)*x^3
+ (2*a^2*c^2*e - a^2*b*c*g)*x^2 + (a^2*b*c*f - (a*b^2*c - 2*a^2*c^2)*d)*x)*
sqrt(c*x^4 + b*x^2 + a))/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)
*x^4 + (a^2*b^3*c - 4*a^3*b*c^2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^2 + gx^3}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.107 $\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx$

Optimal. Leaf size=680

$$\frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\,(b^2-4ac)\,(a+bx^2+cx^4)^{3/2}}-\frac{be-2ag+(2ce-bg)x^2}{3\,(b^2-4ac)\,(a+bx^2+cx^4)^{3/2}}+\frac{4(2ce-bg)\,(b+2cx^2)}{3\,(b^2-4ac)^2\,\sqrt{a+bx^2+cx^4}}+\frac{x(2b$$

```
[Out] 1/3*x*(b^2*d-2*a*c*d-a*b*f+c*(-2*a*f+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)^(3/2)+1/3*(-b*e+2*a*g-(-b*g+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(3/
2)+4/3*(-b*g+2*c*e)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/3*x*
(2*b^4*d-17*a*b^2*c*d+20*a^2*c^2*d+a*b^3*f+4*a^2*b*c*f+c*(12*a^2*c*f+a*b^2*
f-16*a*b*c*d+2*b^3*d)*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/3*(12
*a^2*c*f+a*b^2*f-16*a*b*c*d+2*b^3*d)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-
4*a*c+b^2)^2/(a^(1/2)+x^2*c^(1/2))+1/3*c^(1/4)*(12*a^2*c*f+a*b^2*f-16*a*b*c
*d+2*b^3*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(7/4)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*(2*b^2*d-10*a*c*d+a*b*f+6*a^(3/2)*f*c^(1/2)-3*b*d*a^(1/2)*c^
(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*a*c+b^2)
/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 680, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1687, 1192, 1211, 1117, 1209, 1261, 652, 627}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(6a^{3/2}\sqrt{c}\,f-3\sqrt{a}\,b\sqrt{c}\,d+abf-10acd+2b^2d\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\,x\sqrt{a+bx^2+cx^4}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)}{3a^2\left(b^2-4ac\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{x(cx^2(12a^2cf+ab^2f-16abcd+2b^3d)+4a^2bcf+20a^2c^2d+ab^3f-17ab^2cd+2b^4d)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(12a^2cf+ab^2f-16abcd+2b^3d\right)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}+\frac{x(cx^2(bd-2af)-abf-2acd+b^2d)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}+\frac{4(b+2cx^2)\,(2ce-bg)}{3\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}}-\frac{-2ag+x^2(2ce-bg)+be}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*f + c*(b*d - 2*a*f)*x^2))/(3*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^(3/2)) - (b*e - 2*a*g + (2*c*e - b*g)*x^2)/(3*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^(3/2)) + (4*(2*c*e - b*g)*(b + 2*c*x^2))/(3*(b^2 - 4*a
*c)^2*Sqrt[a + b*x^2 + c*x^4]) + (x*(2*b^4*d - 17*a*b^2*c*d + 20*a^2*c^2*d
+ a*b^3*f + 4*a^2*b*c*f + c*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x
^2))/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(2*b^3*d -
16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(b^2 -
 4*a*c)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (c^(1/4)*(2*b^3*d - 16*a*b*c*d + a*b^2
```

```
*f + 12*a^2*c*f)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(3*a^(7/4)*(b^2 - 4*a*c)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4
)*(2*b^2*d - 3*Sqrt[a]*b*Sqrt[c]*d - 10*a*c*d + a*b*f + 6*a^(3/2)*Sqrt[c]*f
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
6*a^(7/4)*(b - 2*Sqrt[a]*Sqrt[c])*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{(a+bx^2+cx^4)^{5/2}}\,dx = \int \frac{d+fx^2}{(a+bx^2+cx^4)^{5/2}}\,dx + \int \frac{x(e+gx^2)}{(a+bx^2+cx^4)^{5/2}}\,dx$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{1}{2}\text{Subst}\left(\int \frac{e+gx}{(a+bx+cx^2)^{5/2}}\,dx, x, x^2\right.$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{x(2b}{}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{4}{3\left(b^2\right.}$$

$$= \frac{x(b^2d-2acd-abf+c(bd-2af)x^2)}{3a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} - \frac{be-2ag+(2ce-bg)x^2}{3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^{3/2}} + \frac{4}{3\left(b^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.61, size = 598, normalized size = 0.88

$$\frac{-4a(b^2-4ac)\left(-2a^2g-bdx(b+cx^2)+2acx(d+x(e+fx))+ab(e+x(f-gx))\right)+4(a+bx^2+cx^4)\left(2b^3dx(b+cx^2)+abx(-17bcd+b^2f-16c^2dx^2+bcfx^2)+4a^2(-b^2g+c^2x(5d+x(4e+3fx))+bc(2e+x(f-2gx)))\right)+\frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(a+bx^2+cx^4)\left(-\left(\left(-b+\sqrt{b^2-4ac}\right)(2b^3d-16abcd+ab^2f+12a^2cf)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)+\left(-2b^4d+b^3\left(2\sqrt{b^2-4ac}d-af\right)+4abc\left(-4\sqrt{b^2-4ac}d+af\right)+ab^2\left(18cd+\sqrt{b^2-4ac}f\right)+4a^2c\left(-10cd+3\sqrt{b^2-4ac}f\right)\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{12a^2(b^2-4ac)^2(a+bx^2+cx^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x]
```

```
[Out] (-4*a*(b^2 - 4*a*c)*(-2*a^2*g - b*d*x*(b + c*x^2) + 2*a*c*x*(d + x*(e + f*x
)) + a*b*(e + x*(f - g*x))) + 4*(a + b*x^2 + c*x^4)*(2*b^3*d*x*(b + c*x^2)
+ a*b*x*(-17*b*c*d + b^2*f - 16*c^2*d*x^2 + b*c*f*x^2) + 4*a^2*(-(b^2*g) +
c^2*x*(5*d + x*(4*e + 3*f*x)) + b*c*(2*e + x*(f - 2*g*x)))) + (I*Sqrt[2]*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-((-b + Sqrt[b^2 - 4*a*
c])*(2*b^3*d - 16*a*b*c*d + a*b^2*f + 12*a^2*c*f)*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])]) + (-2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d - a*f) + 4*a*b*c*(-4
*Sqrt[b^2 - 4*a*c]*d + a*f) + a*b^2*(18*c*d + Sqrt[b^2 - 4*a*c]*f) + 4*a^2*
c*(-10*c*d + 3*Sqrt[b^2 - 4*a*c]*f))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])
)/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]])/(12*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x
^4)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(654) = 1308$.
time = 0.09, size = 1395, normalized size = 2.05

method	result
elliptic	$\frac{\left(\frac{(2fa-bd)x^3}{3ca\left(4ac-b^2\right)}-\frac{(bg-2ce)x^2}{3\left(4ac-b^2\right)c^2}+\frac{\left(abf+2acd-b^2d\right)x}{3a\left(4ac-b^2\right)c^2}-\frac{2ag-eb}{3\left(4ac-b^2\right)c^2}\right)\sqrt{c\,x^4+b\,x^2+a}}{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)^2}-\frac{2c\left(-\frac{\left(12a^2cf+a\,b^2f-16abcd+2b^3d\right)x^3}{6a^2\left(4ac-b^2\right)^2}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*g*(8*b*c^2*x^6+12*b^2*c*x^4+12*a*b*c*x^2+3*b^3*x^2+8*a^2*c+2*a*b^2)/(c
*x^4+b*x^2+a)^(3/2)/(16*a^2*c^2-8*a*b^2*c+b^4)+f*((2/3/c/(4*a*c-b^2)*x^3+1/
3*b/(4*a*c-b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+b/c*x^2+a/c)^2-2*c*(-1/6*
(12*a*c+b^2)/(4*a*c-b^2)^2/a*x^3-1/6*b*(4*a*c+b^2)/a/(4*a*c-b^2)^2/c*x)/((x
^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(-1/3*b/a/(4*a*c-b^2)-1/3*b*(4*a*c+b^2)/a/(4*a
```

```
*c-b^2)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/6*c*(12*a*c+b^2)/(4*a*c-b^2)^2*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)))+1/3*e*(16*c^3*x^6+24*b*c^2*x^4+24*a*c^2*x^2+6*b^2*c*x^2+12*a*b*c-b^3)/(
c*x^4+b*x^2+a)^(3/2)/(16*a^2*c^2-8*a*b^2*c+b^4)+d*((-1/3*b/a/(4*a*c-b^2)/c*
x^3+1/3*(2*a*c-b^2)/a/(4*a*c-b^2)/c^2*x)*(c*x^4+b*x^2+a)^(1/2)/(x^4+b/c*x^2
+a/c)^2-2*c*(1/3*b*(8*a*c-b^2)/(4*a*c-b^2)^2/a^2*x^3-1/6*(20*a^2*c^2-17*a*b
^2*c+2*b^4)/a^2/(4*a*c-b^2)^2/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(2/3*(5*
a*c-b^2)/(4*a*c-b^2)/a^2-1/3*(20*a^2*c^2-17*a*b^2*c+2*b^4)/a^2/(4*a*c-b^2)^
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*
EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b*c*(8*a*c-b^2)/(4*a*c-b^2)^2/a*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. 2(652) = 1304.
time = 0.12, size = 1948, normalized size = 2.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] -1/6*(sqrt(1/2)*((2*(b^4*c^2 - 8*a*b^2*c^3)*d + (a*b^3*c^2 + 12*a^2*b*c^3)*
f)*x^8 + 2*(2*(b^5*c - 8*a*b^3*c^2)*d + (a*b^4*c + 12*a^2*b^2*c^2)*f)*x^6 +
```

```
(2*(b^6 - 6*a*b^4*c - 16*a^2*b^2*c^2)*d + (a*b^5 + 14*a^2*b^3*c + 24*a^3*b
*c^2)*f)*x^4 + 2*(2*(a*b^5 - 8*a^2*b^3*c)*d + (a^2*b^4 + 12*a^3*b^2*c)*f)*x
^2 + 2*(a^2*b^4 - 8*a^3*b^2*c)*d + (a^3*b^3 + 12*a^4*b*c)*f - ((2*(a*b^3*c^
2 - 8*a^2*b*c^3)*d + (a^2*b^2*c^2 + 12*a^3*c^3)*f)*x^8 + 2*(2*(a*b^4*c - 8*
a^2*b^2*c^2)*d + (a^2*b^3*c + 12*a^3*b*c^2)*f)*x^6 + (2*(a*b^5 - 6*a^2*b^3*
c - 16*a^3*b*c^2)*d + (a^2*b^4 + 14*a^3*b^2*c + 24*a^4*c^2)*f)*x^4 + 2*(2*(
a^2*b^4 - 8*a^3*b^2*c)*d + (a^3*b^3 + 12*a^4*b*c)*f)*x^2 + 2*(a^3*b^3 - 8*a
^4*b*c)*d + (a^4*b^2 + 12*a^5*c)*f)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((
a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqr
t((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a
*c)/(a*c)) + sqrt(1/2)*(((4*(5*a^2*b + 4*a*b^2)*c^3 - (a*b^3 + 2*b^4)*c^2)*
d - (12*a^2*b*c^3 + (8*a^2*b^2 + a*b^3)*c^2)*f)*x^8 + 2*((4*(5*a^2*b^2 + 4*
a*b^3)*c^2 - (a*b^4 + 2*b^5)*c)*d - (12*a^2*b^2*c^2 + (8*a^2*b^3 + a*b^4)*c
)*f)*x^6 - ((a*b^5 + 2*b^6 - 8*(5*a^3*b + 4*a^2*b^2)*c^2 - 6*(3*a^2*b^3 + 2
*a*b^4)*c)*d + (8*a^2*b^4 + a*b^5 + 24*a^3*b*c^2 + 2*(8*a^3*b^2 + 7*a^2*b^3
)*c)*f)*x^4 - 2*((a^2*b^4 + 2*a*b^5 - 4*(5*a^3*b^2 + 4*a^2*b^3)*c)*d + (8*a
^3*b^3 + a^2*b^4 + 12*a^3*b^2*c)*f)*x^2 - (a^3*b^3 + 2*a^2*b^4 - 4*(5*a^4*b
 + 4*a^3*b^2)*c)*d - (8*a^4*b^2 + a^3*b^3 + 12*a^4*b*c)*f + (((4*(5*a^3 - 4
*a^2*b)*c^3 - (a^2*b^2 - 2*a*b^3)*c^2)*d + (12*a^3*c^3 - (8*a^3*b - a^2*b^2
)*c^2)*f)*x^8 + 2*((4*(5*a^3*b - 4*a^2*b^2)*c^2 - (a^2*b^3 - 2*a*b^4)*c)*d
+ (12*a^3*b*c^2 - (8*a^3*b^2 - a^2*b^3)*c)*f)*x^6 - ((a^2*b^4 - 2*a*b^5 - 8
*(5*a^4 - 4*a^3*b)*c^2 - 6*(3*a^3*b^2 - 2*a^2*b^3)*c)*d + (8*a^3*b^3 - a^2*
b^4 - 24*a^4*c^2 + 2*(8*a^4*b - 7*a^3*b^2)*c)*f)*x^4 - 2*((a^3*b^3 - 2*a^2*
b^4 - 4*(5*a^4*b - 4*a^3*b^2)*c)*d + (8*a^4*b^2 - a^3*b^3 - 12*a^4*b*c)*f)*
x^2 - (a^4*b^2 - 2*a^3*b^3 - 4*(5*a^5 - 4*a^4*b)*c)*d - (8*a^5*b - a^4*b^2
- 12*a^5*c)*f)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
- b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*((2*(a
*b^3*c^2 - 8*a^2*b*c^3)*d + (a^2*b^2*c^2 + 12*a^3*c^3)*f)*x^7 + 8*(2*a^3*c^
3*e - a^3*b*c^2*g)*x^6 + ((4*a*b^4*c - 33*a^2*b^2*c^2 + 20*a^3*c^3)*d + 2*(
a^2*b^3*c + 8*a^3*b*c^2)*f)*x^5 + 12*(2*a^3*b*c^2*e - a^3*b^2*c*g)*x^4 + (2
*(a*b^5 - 7*a^2*b^3*c)*d + (a^2*b^4 + 3*a^3*b^2*c + 20*a^4*c^2)*f)*x^3 + 3*
(2*(a^3*b^2*c + 4*a^4*c^2)*e - (a^3*b^3 + 4*a^4*b*c)*g)*x^2 - (a^3*b^3 - 12
*a^4*b*c)*e - 2*(a^4*b^2 + 4*a^5*c)*g + (8*a^4*b*c*f + (3*a^2*b^4 - 23*a^3*
b^2*c + 28*a^4*c^2)*d)*x)*sqrt(c*x^4 + b*x^2 + a))/(a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b^5*c
 - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)
*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**4+b*x**2+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^4+b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((g*x^3 + f*x^2 + x*e + d)/(c*x^4 + b*x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{{\left(c\,x^4+b\,x^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x^2 + c*x^4)^(5/2), x)
```

3.108 $\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=19

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] g*x/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1602}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{ag-cgx^4}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.04, size = 19, normalized size = 1.00

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.04, size = 938, normalized size = 49.37

method	result
gosper	$\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$
trager	$\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$
default	$g\left(-c\left(-\frac{2c\left(\frac{b\,x^{3}}{2c\left(4ac-b^{2}\right)}+\frac{ax}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}}{2\left(4ac-b^{2}\right)\sqrt{\frac{-b+\sqrt{-4a}}{a}}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] g*(-c*(-2*c*(1/2*b/c/(4*a*c-b^2)*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*
c)^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))))+a*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/
((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))-1/2*b*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))))
```

Maxima [A]
time = 0.32, size = 17, normalized size = 0.89

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Fricas [A]
time = 0.42, size = 17, normalized size = 0.89

$$\frac{gx}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] g*x/sqrt(c*x^4 + b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-g\left(\int\left(-\frac{a}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx+\int\frac{cx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -g*(Integral(-a/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x
**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) + Integral(c*x**4/(a*sqrt(a +
b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**
2 + c*x**4)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.
time = 6.01, size = 60, normalized size = 3.16

$$\frac{(b^4g-8\,ab^2cg+16\,a^2c^2g)x}{\sqrt{cx^4+bx^2+a}\,(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)*x/(sqrt(c*x^4 + b*x^2 + a)*(b^4 - 8*a*
b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 0.99, size = 17, normalized size = 0.89

$$\frac{g\,x}{\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] (g*x)/(a + b*x^2 + c*x^4)^(1/2)
```

3.109 $\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] g*x/(c*x^4+b*x^2+a)^(1/2)-e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1687, 1602, 12, 1121, 627}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
```

```
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ag+ex-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx &= \int \frac{ex}{(a+bx^2+cx^4)^{3/2}}\,dx + \int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} + e\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} - \frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [A]
time = 10.10, size = 51, normalized size = 0.89

$$
\frac{-be+b^2gx-4acgx-2cex^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(b*e) + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 +
c*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.05, size = 974, normalized size = 17.09

method	result
gosper	$\frac{4acgx-b^2gx+2ce\,x^2+eb}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
trager	$\frac{4acgx-b^2gx+2ce\,x^2+eb}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$

elliptic	$\frac{e(2cx^2+b)}{\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}+\frac{gx}{\sqrt{cx^4+bx^2+a}}$
default	$-cg\left(-\frac{2c\left(\frac{bx^3}{2c(4ac-b^2)}+\frac{ax}{c(4ac-b^2)}\right)}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}}{2(4ac-b^2)\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*g*(-2*c*(1/2*b/c/(4*a*c-b^2)*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c
)^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))))+a*g*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)
/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-1/2*b*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e/(c*x^4+b*x^2+a)^(1/2)*(2*c*x
^2+b)/(4*a*c-b^2)
```

Maxima [A]
time = 0.33, size = 53, normalized size = 0.93

$$-\frac{2\,cx^2e-(b^2g-4\,acg)x+be}{\sqrt{cx^4+bx^2+a}\,(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*c*x^2*e - (b^2*g - 4*a*c*g)*x + b*e)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4*
a*c))
```

Fricas [A]
time = 0.40, size = 82, normalized size = 1.44

$$-\frac{\sqrt{cx^4+bx^2+a}\,(2\,cex^2-(b^2-4\,ac)gx+be)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*e*x^2 - (b^2 - 4*a*c)*g*x + b*e)/((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{ex}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x*
*4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-e*x/(a*sqrt(a + b*x
**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 +
 c*x**4)), x) - Integral(c*g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqr
t(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(54) = 108.
time = 4.65, size = 142, normalized size = 2.49

$$-\frac{\left(\frac{2\,(b^2ce-4\,ac^2e)x}{b^4-8\,ab^2c+16\,a^2c^2}-\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{b^3e-4\,abce}{b^4-8\,ab^2c+16\,a^2c^2}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((2*(b^2*c*e - 4*a*c^2*e)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - (b^4*g - 8*a*
b^2*c*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (b^3*e - 4*a*b*
c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B]
time = 0.93, size = 51, normalized size = 0.89

$$\frac{-g\,b^2\,x+e\,b+2\,c\,e\,x^2+4\,a\,c\,g\,x}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] (b*e + 2*c*e*x^2 - b^2*g*x + 4*a*c*g*x)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)^
(1/2))
```

3.110 $\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] g*x/(c*x^4+b*x^2+a)^(1/2)+f*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1687, 1602, 12, 1128, 650}

$$\frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{gx}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] + (f*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b
*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
```

```
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{ag+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx &= \int \frac{fx^3}{(a+bx^2+cx^4)^{3/2}}\,dx + \int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} + f\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} + \frac{1}{2}f\mathrm{Subst}\left(\int \frac{x}{(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{gx}{\sqrt{a+bx^2+cx^4}} + \frac{f(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [A]
time = 10.10, size = 48, normalized size = 0.84

$$\frac{bx(bg+fx)+2a(f-2cgx)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
[Out] (b*x*(b*g + f*x) + 2*a*(f - 2*c*g*x))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.05, size = 976, normalized size = 17.12

method	result
gosper	$\frac{4acgx-b^2gx-bf\,x^2-2fa}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$

trager	$\frac{4acgx-b^{2}gx-bf\,x^{2}-2fa}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}$
elliptic	$-\frac{f(b\,x^{2}+2a)}{\sqrt{c\,x^{4}+b\,x^{2}+a}\,(4ac-b^{2})}+\frac{gx}{\sqrt{c\,x^{4}+b\,x^{2}+a}}$
default	$-cg\left(-\frac{2c\left(\frac{b\,x^{3}}{2c(4ac-b^{2})}+\frac{ax}{c(4ac-b^{2})}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}}{2(4ac-b^{2})\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*g*(-2*c*(1/2*b/c/(4*a*c-b^2)*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c
)^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))))-f/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+a*g*(-2*c*(1/2/a*b/(4
*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2
)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b*c/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))
```

Maxima [A]
time = 0.33, size = 49, normalized size = 0.86

$$\frac{bfx^{2}+2\,af+(b^{2}g-4\,acg)x}{\sqrt{cx^{4}+bx^{2}+a}\,(b^{2}-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (b*f*x^2 + 2*a*f + (b^2*g - 4*a*c*g)*x)/(sqrt(c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c))
```

Fricas [A]
time = 0.40, size = 80, normalized size = 1.40

$$\frac{\sqrt{cx^4+bx^2+a}\,(bfx^2+(b^2-4\,ac)gx+2\,af)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*f*x^2 + (b^2 - 4*a*c)*g*x + 2*a*f)/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{fx^3}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+f*x**3+a*g)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x*
*4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-f*x**3/(a*sqrt(a +
b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**
2 + c*x**4)), x) - Integral(c*g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*
sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(53) = 106.
time = 3.94, size = 136, normalized size = 2.39

$$\frac{\left(\frac{(b^3f-4\,abcf)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{2\,(ab^2f-4\,a^2cf)}{b^4-8\,ab^2c+16\,a^2c^2}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^3*f - 4*a*b*c*f)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) + (b^4*g - 8*a*b^2*c
*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 2*(a*b^2*f - 4*a^2*c
*f)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B]
time = 0.96, size = 51, normalized size = 0.89

$$-\frac{g\,b^2\,x+f\,b\,x^2-4\,a\,c\,g\,x+2\,a\,f}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] -(2*a*f + b*f*x^2 + b^2*g*x - 4*a*c*g*x)/((4*a*c - b^2)*(a + b*x^2 + c*x^4)
^(1/2))
```

3.111 $\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=69

$$\frac{gx}{\sqrt{a+bx^2+cx^4}} - \frac{be-2af+(2ce-bf)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] g*x/(c*x^4+b*x^2+a)^(1/2)+(-b*e+2*a*f-(-b*f+2*c*e)*x^2)/(-4*a*c+b^2)/(c*x^4
+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1687, 1602, 1261, 650}

$$\frac{gx}{\sqrt{a+bx^2+cx^4}} - \frac{-2af+x^2(2ce-bf)+be}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (g*x)/Sqrt[a + b*x^2 + c*x^4] - (b*e - 2*a*f + (2*c*e - b*f)*x^2)/((b^2 - 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\begin{aligned}\int \frac{ag+ex+fx^3-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx &= \int \frac{x(e+fx^2)}{(a+bx^2+cx^4)^{3/2}}\,dx+\int \frac{ag-cgx^4}{(a+bx^2+cx^4)^{3/2}}\,dx\\ &= \frac{gx}{\sqrt{a+bx^2+cx^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{e+fx}{(a+bx+cx^2)^{3/2}}\,dx,x,x^2\right)\\ &= \frac{gx}{\sqrt{a+bx^2+cx^4}}-\frac{be-2af+(2ce-bf)x^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A]
time = 10.12, size = 61, normalized size = 0.88

$$\frac{-be+2af+b^2gx-4acgx-2cex^2+bfx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(b*e) + 2*a*f + b^2*g*x - 4*a*c*g*x - 2*c*e*x^2 + b*f*x^2)/((b^2 - 4*a*c)
*Sqrt[a + b*x^2 + c*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 2.
time = 0.05, size = 1012, normalized size = 14.67

method	result
gosper	$\frac{4acgx-b^2gx-bf\,x^2+2ce\,x^2-2fa+eb}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
trager	$\frac{4acgx-b^2gx-bf\,x^2+2ce\,x^2-2fa+eb}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}$
elliptic	$-\frac{bf\,x^2-2ce\,x^2+2fa-eb}{\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}+\frac{gx}{\sqrt{c\,x^4+b\,x^2+a}}$

default	$-cg\left(-\frac{2c\left(\frac{b\,x^3}{2c\left(4ac-b^2\right)}+\frac{ax}{c\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}}{2(4ac-b^2)\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c*g*(-2*c*(1/2*b/c/(4*a*c-b^2)*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+a/c)*c
)^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))))-f/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+a*g*(-2*c*(1/2/a*b/(4
*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+a/c)*c)^(1/2
)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b*c/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))+e/(c*x^4+b*x^2+a)^(1/2)*(2*c*x^2+b)/(4*a*c-b^2)
```

Maxima [A]
time = 0.34, size = 94, normalized size = 1.36

$$\frac{\sqrt{cx^4+bx^2+a}\,\left((bf-2\,ce)x^2+2\,af+(b^2g-4\,acg)x-be\right)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*((b*f - 2*c*e)*x^2 + 2*a*f + (b^2*g - 4*a*c*g)*x -
b*e)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Fricas [A]
time = 0.40, size = 92, normalized size = 1.33

$$\frac{\sqrt{cx^4+bx^2+a}\,\left((b^2-4\,ac)gx-(2\,ce-bf)x^2-be+2\,af\right)}{(b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fric
as")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*((b^2 - 4*a*c)*g*x - (2*c*e - b*f)*x^2 - b*e + 2*a*
f)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{ag}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{ex}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\left(-\frac{fx^3}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cgx^4}{a\sqrt{a+bx^2+cx^4}+bx^2\sqrt{a+bx^2+cx^4}+cx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x**4+f*x**3+a*g+e*x)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -Integral(-a*g/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x*
*4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(-e*x/(a*sqrt(a + b*x
**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 +
 c*x**4)), x) - Integral(-f*x**3/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt
(a + b*x**2 + c*x**4) + c*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*
g*x**4/(a*sqrt(a + b*x**2 + c*x**4) + b*x**2*sqrt(a + b*x**2 + c*x**4) + c*
x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(66) = 132.
time = 4.48, size = 166, normalized size = 2.41

$$\frac{\left(\frac{(b^3f-4\,abcf-2\,b^2ce+8\,ac^2e)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{b^4g-8\,ab^2cg+16\,a^2c^2g}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{2\,ab^2f-8\,a^2cf-b^3e+4\,abce}{b^4-8\,ab^2c+16\,a^2c^2}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*g*x^4+f*x^3+a*g+e*x)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac
")
```

```
[Out] (((b^3*f - 4*a*b*c*f - 2*b^2*c*e + 8*a*c^2*e)*x/(b^4 - 8*a*b^2*c + 16*a^2*c
^2) + (b^4*g - 8*a*b^2*c*g + 16*a^2*c^2*g)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*
```

```
x + (2*a*b^2*f - 8*a^2*c*f - b^3*e + 4*a*b*c*e)/(b^4 - 8*a*b^2*c + 16*a^2*c
^2))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [B]
time = 0.98, size = 62, normalized size = 0.90

$$-\frac{g\,b^2\,x+f\,b\,x^2-e\,b-2\,c\,e\,x^2-4\,a\,c\,g\,x+2\,a\,f}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^4+b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g + e*x + f*x^3 - c*g*x^4)/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] -(2*a*f - b*e + b*f*x^2 - 2*c*e*x^2 + b^2*g*x - 4*a*c*g*x)/((4*a*c - b^2)*(
a + b*x^2 + c*x^4)^(1/2))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```